

Written Homework 03

Assigned: Wed 24 Oct 2007

Due: Wed 31 Oct 2007

Instructions:

- The assignment is due at the *beginning* of class on the due date specified, i.e., 10:30am for Prof. Aslam's section and 1:35pm for Prof. Fell's section. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.

Problem 1 [30 pts;, (2,4,4,4,4,4,4,4)]: **Divisibility**

Consider the set of positive even integers $S = \{2, 4, \dots, 3000\}$.

- What is the cardinality of this set?
- How many of these integers are divisible by 3?
- How many of these integers are divisible by 5?
- How many of these integers are divisible by 3 AND by 5?
- How many of these integers are not divisible by 3 OR by 5?
- What is the least number of distinct integers that must be chosen from S so that at least one of them is divisible by 3?
- What is the least number of distinct integers that must be chosen from S so that at least one of them is divisible by 5?
- What is the least number of distinct integers that must be chosen from S so that at least one of them is divisible by 3 or 5?

Problem 2 [30 pts, (10 pts each)]: **Mule Trains**

A mule train heading down the Grand Canyon has a guide on the front mule and thirteen tourists, five men and eight women on the the other thirteen mules.

- In how many ways can the riders be arranged so that all the male tourists are together, one behind the other?
- In how many ways can the riders be arranged so that all the male tourists are together one behind the other, or all the women are together one behind the other (or both)?

- iii. In how many ways can the riders be arranged so that no two male tourists are adjacent? *Hint:* Consider arranging the women first and then placing the men's mules among the women's mules so as to ensure that no two men are adjacent.

Problem 3 [10 pts, (5 each)]: **Little League**

Twenty-five kids are on a Little League baseball team, 16 boys and 9 girls.

- i. How many ways can the coach choose a team of 9 players?
- ii. How many ways can the coach choose a team of 9 players that includes at least 3 girls and at least 3 boys?

Problem 4 [30 pts, (10 each)]: **Big League**

The Red Sox play the Yankees in a seven game series that ends when one team has won four games. We record the outcome of a game with a **W** for a Red Sox win and an **L** for a Red Sox loss, e.g. **WWWW**, **WLWLWLW**, or **WWLLLWW**.

- i. How many possible outcomes are there?
- ii. How many series would have to be played to be sure that the same outcome happens twice?
- iii. If the Red Sox win in four games with a total of 17 runs, how many ways could their runs be distributed among the four games? (Sample run distributions: (2, 7, 5, 3) (12, 1, 1, 3))