

## Special Functions

We define here some functions of numbers that you will frequently use in computing practice and theory.

### factorial function, $n!$

If  $n$  is a positive integer, the *factorial*  $n!$  is defined to be

$$n! = n(n-1)\cdots 2 \cdot 1.$$

So, for example

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

We also define

$$0! = 1.$$

This fits with the combinatorial formulas we will derive later in the semester.

### floor and ceiling

The functions *floor* and *ceiling* (or *ceil*) are functions that take real arguments and give integer values.

$$\text{floor}(x) = \lfloor x \rfloor = \text{the greatest integer less than or equal to } x.$$

$$\text{ceiling}(x) = \lceil x \rceil = \text{the least integer greater than or equal to } x.$$

**procedure:** (floor *real*)

**returns:** the integer closest to *real* toward  $-\infty$

(floor 19)  $\Rightarrow$  19  
(floor 2/3)  $\Rightarrow$  0  
(floor -2/3)  $\Rightarrow$  -1  
(floor 17.3)  $\Rightarrow$  17.0  
(floor -17/2)  $\Rightarrow$  -9

**procedure:** (ceiling *real*)

**returns:** the integer closest to *real* toward  $+\infty$

(ceiling 19)  $\Rightarrow$  19  
(ceiling 2/3)  $\Rightarrow$  1  
(ceiling -2/3)  $\Rightarrow$  0  
(ceiling 17.3)  $\Rightarrow$  18.0  
(ceiling -17/2)  $\Rightarrow$  -8

These and other Scheme examples are from [The Scheme Programming Language](#), *Second Edition* © 1996.

### truncate and round

The functions *truncate* (or *trunc*) and *round* are also functions that take real arguments and give integer values. You probably learned about *round* back in elementary school.

**procedure:** (round *real*)

**returns:** the integer closest to *real*

If *real* is exactly between two integers, the closest even integer is returned.

(round 19)  $\Rightarrow$  19  
(round 2/3)  $\Rightarrow$  1  
(round -2/3)  $\Rightarrow$  -1  
(round 17.3)  $\Rightarrow$  17.0  
(round -17/2)  $\Rightarrow$  -8  
(round 2.5)  $\Rightarrow$  2.0  
(round 3.5)  $\Rightarrow$  4.0

**procedure:** (truncate real)  
**returns:** the integer closest to real toward zero

(truncate 19)  $\Rightarrow$  19  
(truncate 2/3)  $\Rightarrow$  0  
(truncate -2/3)  $\Rightarrow$  0  
(truncate 17.3)  $\Rightarrow$  17.0  
(truncate -17/2)  $\Rightarrow$  -8

## absolute value

The *absolute value* of a real number is defined by

$$abs(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

**procedure:** (abs real)  
**returns:** the absolute value of *real*

(abs 1)  $\Rightarrow$  1  
(abs -3/4)  $\Rightarrow$  3/4  
(abs 1.83)  $\Rightarrow$  1.83

## References:

"Discrete Mathematics, second edition" by James L. Hein, pages 79, 84, Jones and Bartlett Mathematics, 2003.

[floor and ceiling functions](#)

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