

### Practice Problems for Exam 3

1. Expand each of the following sums as shown in the example. (You don't need to provide the final result)

$$\sum_{k=1}^4 (5 + 2^k) = (5 + 2^1) + (5 + 2^2) + (5 + 2^3) + (5 + 2^4) = 7 + 9 + 13 + 21$$

a.  $\sum_{k=1}^4 3k^2$       b.  $\sum_{i=0}^3 3^i$       c.  $\sum_{j=1}^4 \frac{j!}{3!}$

d.  $\sum_{k=2}^5 (k^2 + k - 1)$       e.  $\sum_{i=0}^3 ak^i$       f.  $\sum_{j=3}^6 a_j$

2. For each of the following sums, give a formula in terms of  $n$  for the sum, as shown in the example.

$$\sum_{k=1}^n 3k = \frac{3n(n+1)}{2}$$

a.  $\sum_{k=1}^n 2k$       b.  $\sum_{k=0}^n (5k + 2)$       c.  $\sum_{k=1}^n 4 \cdot 3^k$

d.  $\sum_{k=1}^n (ak + b)$       e.  $\sum_{j=0}^n ar^j, r \neq 1$       f.  $\sum_{j=0}^n ar^j, r = 1$

3. Set-Builder Notation.

Rewrite each set showing all its elements, for example,  $\{x \in \mathbb{Z} \mid 1 < x < 5\} = \{2, 3, 4\}$ .

- a.  $\{x \in \mathbb{N} \mid x < 6\}$   
 b.  $\{x \in \mathbb{Z} \mid |x| < 3\}$   
 c.  $\{y \in \mathbb{Z} \mid |y| < 8 \text{ and } y \bmod 3 = 1\}$   
 d.  $\{x \in \mathbb{Z} \mid 0 \leq x/3 \leq 2\}$   
 e.  $\{y \in \mathbb{Z} \mid y \cdot (-2) < 10\}$   
 f.  $\{z \in \mathbb{N} \mid z/(-3) > -4\}$

4. Set Operations and Vann Diagram

Let  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{x \in \mathbb{N} \mid 3 < x < 7\}$ ,  $C = \{y \in \mathbb{Z} \mid y > -4\}$

And the universe  $U = \{k \in \mathbb{Z} \mid |k| \leq 8\}$ .

- a.  $A \cap B = ?$ ,  $B \cap C = ?$ ,  $A \cap C = ?$   
 b.  $A \cup B = ?$ ,  $B \cup C = ?$ ,  $A \cup C = ?$   
 c.  $\bar{A} = ?$ ,  $\bar{B} = ?$ ,  $\bar{C} = ?$   
 d.  $A - B = ?$ ,  $B - C = ?$ ,  $C - A = ?$   
 e.  $(A \cap B) \cup C = ?$ ,  $A \cap (B \cup C) = ?$   
 f. Draw Vann diagram of  $A \cup (B \cap C)$  and  $\overline{A \cap (B \cup C)}$

5. Cartesian Product, Power Sets, subsets and cardinality

Let  $A = \{2, 4, 6, 8\}$ ,  $B = \{a, b, c\}$ ,  $C = \{\#, *, \&\}$

- a.  $A \times B$       b.  $B \times C$       c.  $C \times A$   
 d.  $P(A)$       e.  $P(B)$       f.  $P(C)$

g.  $A \times B \times C$

h.  $|A \times B|$

i.  $|P(C)|$

j.  $|P(A) \times P(C)|$

k.  $|P(P(C))|$

l. List all the subsets of  $B \times B$  that have two elements.

m. How many subsets does  $A$  have? And how many subsets does  $P(A)$  have?

## 6. Permutations and Combinations

Show the formulas you used as well as the final numbers.

Give the values of each of these quantities:

a.  $P(6, 2)$

b.  $P(6, 4)$

c.  $C(6, 2)$

d.  $C(6, 4)$

e.  $P(7, 3)$

f.  $C(7, 3)$

g.  $P(7, 4)$

h.  $C(7, 4)$

i.  $C(5, 5)$

j.  $C(5, 0)$

k.  $P(6, 0)$

l.  $P(6, 6)$

## 7. Counting

Show your work as well as the final numbers.

How many positive integers between 2000 to 4999

a. are divisible by 7

b. have distinct digits

c. are divisible by 5 or 9

d. are divisible by 6 or 9

e. are NOT divisible by **either 5 or 7**

f. are divisible by 5 but not by 7

g. are divisible by 5 and 7

## 8. Probability

Show your work as well as the final numbers.

If you roll a pair of dice, one red and one white, what is the probability of

a. number on the red is 5

b. number on the white is 4

c. 5 on red and 4 on white

d. 5 on red or 4 on white

e. the product of the numbers rolled is 12

f. the sum of the numbers rolled is 11

g. the number on the white one is greater than the number on the red one?

## 9. Review all the problems of Online Homework 7, 8, 9