Solutions for Review 3

1. Expand each of the following sums as shown in the example. (You don't need to provide the final result)

a. $\sum_{k=1}^{4} 3k^2 = 3(1)^2 + 3(2)^2 + 3(3)^2 + 3(4)^2 = 3 + 12 + 27 + 48$

b. $\sum_{i=0}^{3} 3^i = 3^0 + 3^1 + 3^2 + 3^3 = 1 + 3 + 9 + 27$

c. $\sum_{j=1}^{4} j!/3! = (1!/3!) + (2!/3!) + (3!/3!) + (4!/3!) = (1/6) + (1/3) + 1 + 4$

d. $\sum_{k=2}^{5} (k^2 + k + 1) = (2^2 + 2 + 1) + (3^2 + 3 + 1) + (4^2 + 4 + 1) + (5^2 + 5 + 1) = 7 + 13 + 21 + 31$

e. $\sum_{i=0}^{3} ak^i = ak^0 + ak^1 + ak^2 + ak^3 = a + ak + ak^2 + ak^3$

f. $\sum_{j=3}^{6} a_j = a_3 + a_4 + a_5 + a_6$

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2. **For each of the following sums, give a formula in terms of *n* for the sum, as shown in the example.**

a. $\sum_{k=1}^{n} 2k = 2 \cdot \frac{n(n+1)}{2} = n(n+1)$

b. $\sum_{k=0}^{n} (5k+2) = \frac{(2 + 5n + 2)(n+1)}{2} = \frac{(5n+4)(n+1)}{2}$

$$c.\sum_{k=1}^{n} 4\cdot 3^k = 4\cdot\frac{(3^n-1)\cdot 3}{3-1} = 6(3^n-1) = 2\cdot 3^{n+1}-6$$

$$d.\sum_{k=1}^{n}(ak+b) = \frac{[(a+b)+(an+b)]n}{2} = \frac{(an+a+2b)n}{2}$$

$$e.\sum_{j=0}^{n} ar^j, (r=1) = (n+1)a$$

$$f.\sum_{j=0}^{n} ar^j, (r\neq 1) = \frac{a(r^{n+1}-1)}{r-1}$$

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3. Set-Builder Notation.

Rewrite each set showing all its elements, for example, {x e Z | 1 < x < 5} = {2, 3, 4}.

a. {x ∈ N | x < 6} = {0, 1, 2, 3, 4, 5}

b. {x ∈ Z | |x| < 3} = {-2, -1, 0, 1, 2}

c. {y ∈ Z | |y| < 8 and y mod 3 = 1} = {-5, -2, 1, 4, 7}

d. {x ∈ Z | 0 <= x/3 <=2} = {0, 1, 2, 3, 4, 5, 6}

e. {y ∈ Z^- | y*(-2) < 10} = {-1, -2, -3, -4}

f. {z ∈ N | z/ (-3) > -4} = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

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4. Set Operations and Vann Diagram

A = {1, 2, 3, 4, 5}
B= {x ∈ N | 3<x<7} = {4, 5, 6}
C= {y∈ Z^- | y > -4} = {-3, -2, -1}
U = {k ∈ Z | |k|≤8} = {-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8}

a. A∩B = {4, 5}, B∩C = {}, A∩C = {}

b. A U B = {1, 2, 3, 4, 5, 6}
B U C = {4, 5, 6, -3, -2, -1}

A U C = {-3, -2, -1, 1, 2, 3, 4, 5}

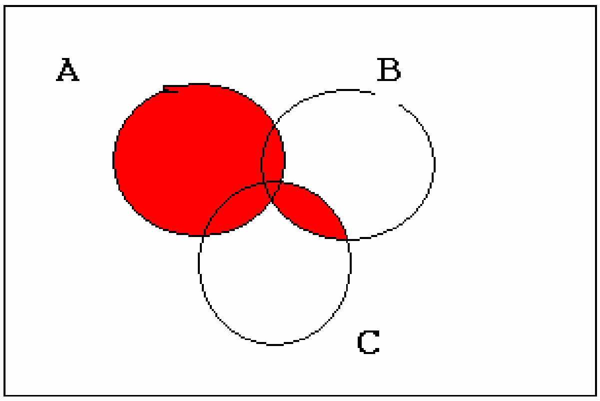
c. $\overline{A}$ = {-8, -7, -6, -5, -4, -3, -2, -1, 0, 6, 7, 8}
$\overline{B}$ = {-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 7, 8}
$\overline{C}$ = {-8, -7, -6, -5, -4, 0, 1, 2, 3, 4, 5, 6, 7, 8}

d. A-B = {1, 2, 3}
B-C= {4, 5, 6}
C-A = {-3, -2, -1}

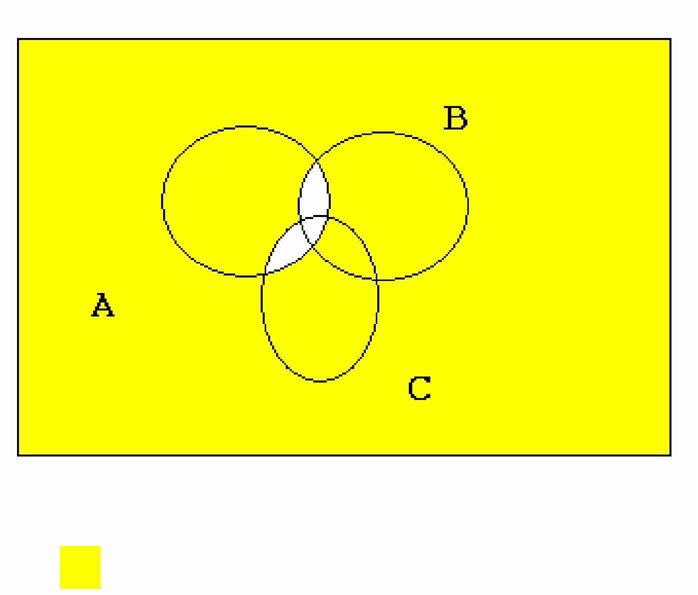
e. (A∩B) U C = {4, 5} U {-3, -2, -1} = {4, 5, -3, -2, -1}
A∩ (BUC) = {1, 2, 3, 4, 5} ∩ {4, 5, 6, -3, -2, -1} = {4, 5}

f. Draw Vann diagram of

A U (B∩C)



$\overline{A \cap (B \cup C)}$ = yellow region



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5. **Cartesian Product, Power Sets, subsets and cardinality**
Let A = {2, 4, 6, 8}, B = {a, b, c}, C = {#, *, &}

a. A x B = {(2,a),(2,b),(2,c),(4,a),(4,b), (4,c), (6,a), (6,b), (6,c), (8,a), (8,b), (8,c)}

b. B x C = { (a, #), (b,#), (c,#), (a,*), (b,*), (c,*), (a,&), (b,&), (c,&)}

c. C x A = {(#,2) (#,4), (#,6), (#,8), (*,2), (*,4), (*,6), (*,8),(&,2), (&,4), (&,6), (&,8)}

d. *P*(A) = {empty set, {2}, {4}, {6}, {8}, {2,4}, {2,6}, {2,8}, {4,6}, {4,8}, {6,8}, {2,4,6}, {2,4,8}, {4,6,8}, {2,4,6,8}}

e. *P*(B) = {empty set, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

f. *P*(C) = {empty set, {#}, {*}, {&}, {#, *}, {#, &}, {*, &}, {#, *, &}}

g. A x B x C = {(2, a, #), (2,a, *), (2, a, &), (2, b, #), (2,b, *), (2, b, &), (2, c, #), (2,c, *), (2, c, &), (4, a, #), (4, a ,*), (4, a, &), (4, b, #), (4,b, *), (4, b, &), (4, c, #), (4,c, *), (4, c, &), (6, a, #), (6,a, *), (6, a, &), (6, b, #), (6, b, *), (6, b, &), (6, c, #), (6, c, *), (6, c, &), (8, a, #), (8,a, *), (8, a, &), (8, b, #), (8, b, *), (8, b, &), (8, c, #), (8, c, *), (8, c, &)}

h. |A x B| = |A|*|B| = 4*3 = 12

i. $|P(C)| = 2^{|C|} = 2^3 = 8$

j. $|P(A) \times P(C)| = |P(A)|*|P(C)| = 2^{|A|}*2^{|C|} = 2^{|A|+|C|} = 2^{4+3} = 128$

k. $|P(P(C))| = 2^{|P(C)|} = 2^{2^{\wedge}|C|} = 2^{2^{\wedge}3} = 2^8 = 256$

l. List all the subsets of B x B that have two elements.
B x B = {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}
The SUBSETS of B x B that have two elements:
{(a, a), (a, b)}, {(a, a), (a, c)}, {(a, a), (b, a)}, {(a, a), (b, b)}, {(a, a), (b, c)}, {(a, a), (c, a)}, {(a, a), (c, b)}, {(a, a), (c, c)}, {(a, b), (a, c)}, {(a, b), (b, a)}, {(a, b), (b, b)}, {(a, b), (b, c)}, {(a, b), (c, a)}, {(a, b), (c, b)}, {(a, b), (c, c)}, {(a, c), (b, a)}, {(a, c), (b, b)}, {(a, c), (b, c)}, {(a, c), (c, a)}, {(a, c), (c, b)}, {(a, c), (c, c)}, {(b, a), (b, b)}, {(b, a), (b, c)}, {(b, a), (c, a)}, {(b, a), (c, b)}, {(b, a), (c, c)}, {(b, b), (b, c)}, {(b, b), (c, a)}, {(b, b), (c, b)}, {(b, b), (c, c)}, {(b, c), (c, a)}, {(b, c), (c, b)}, {(b, c), (c, c)}, {(c, a), (c, b)}, {(c, a), (c, c)}, {(c, b), (c, c)}.

m. How many subsets does A have? And how many subsets does *P* (A) have?
A has 4 elements. So it has $2^4 = 16$ subsets. P (A) has 2^{16} subsets.

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6. **Give the values of each of these quantities:**

a. P(6, 2) = 6 * 5 = 30

b. P(6, 4) = 6! / (6 – 4)! = 6 * 5 * 4 * 3 = 360

c. C(6, 2) = 6!/(2!4!) = 30/2 = 15

d. C(6, 4) = 6!/(4! 2!) = 15

e. P(7, 3) = 7!/4! = 35 * 6 = 210

f. C(7, 3) = 7!/(3!4!) = 7* 6*5 / 6 = 35

g. P(7, 4) = 7!/3! = 7 * 6 * 5 * 4 = 120 * 7 = 840

h. C(7, 4) = 7!/(4! (7-4)!) = 7 * 6 * 5 / (3 * 2 * 1) = 35

i. C(5, 5) = 5!/(5! (5 -5)!) = 1

j. C(5, 0) = 5!/(0! 5!) = 1

k. P(6, 0) = 6!/6! = 1

l. P(6, 6) = 6!/0! = 6! = 720

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7. Counting
Show your work as well as the final numbers.

How many positive integers between 2000 to 4999

a. are divisible by 7
Between 1 and 4999 => 714 numbers are divisible by 7
Between 1 and 1999 => 285 numbers are divisible by 7
Between 2000 and 4999 => 714 -285 = 429 numbers are divisible by 7

b. have distinct digits = 3 * 9 * 8 * 7 = 1512

c. are divisible by 5 or 9
Similar argument.
999-399 = 600 divisible by 5
555- 222= 333 divisible by 9
111 – 44 = 67 by both.
600+ 333 - 67 = 866 are divisible by 5 or 9

d. are divisible by 6 or 9
Similar argument.
833 – 333 = 500 divisible by 6
555- 222= 333 divisible by 9
(4999/18) – (1999/18) = 277 – 111 = 166 by both
500 + 333 – 166 = 667 are divisible by 6 or 9

e. are NOT divisible by either 5 or 7
142 – 57 = 85 are divisible by both 5 and 7 (divisible by 35)
(4999 – 1999) – 85 = 2915

f. are divisible by 5 but not by 7
Between 1 and 4999 => 999 numbers are divisible by 5
Between 1 and 1999 => 399 numbers are divisible by 5
Between 2000 and 4999 => 999 – 399 = 600 numbers are divisible by 5

142 – 57 = 85 are divisible by both 5 and 7 (divisible by 35). So, 600 – 85 = 515 are divisible by 5 but not by 7.

g. 142 – 57 = 85 are divisible by both 5 and 7 (divisible by 35)

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8. Probability

Total outcomes = 6 * 6 = 36

a. number on the red is 5 = 6/36 = 1/6

b. number on the white is 4 = 6/36 = 1/6

c. 5 on red and 4 on white = 1/36

d. 5 on red or 4 on white = 1/6 + 1/6 – 1/36 = 11/36

e. the product of the numbers rolled is 12
The numbers can be: 4 * 3 or 6 * 2
2/36 + 2/36 = 1/9

f. the sum of the numbers rolled is 11
The numbers are 6 and 5
The probability = 2/36 = 1/18

g. the number on the white one is greater than the number on the red one?
Possible outcomes are

White	Red
2	1
3	2
3	1
4	3
4	2
4	1
5	4
5	3
5	2
5	1
6	5
6	4
6	3
6	2
6	1

Total = 15 outcomes
The probability = 15/36