CSU200 Discrete Structures	Profs. Aslam & Fell
Fall 2006	November 28, 2006

Written Homework 04

Assigned: Tue 28 Nov 2006
Due: Mon 04 Dec 2006

Instructions:

- The assignment is due at the *beginning* of class on the due date specified, i.e., 1:35pm for Prof. Aslam's section and 4:35pm for Prof. Fell's section. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.

Problem 1 [40 pts;, (20,10,10)]: Comparisons of Functions

In the problems that follow, you will compare three algorithms for search that we discussed in class: Ordered-Linear-Search, Chunk-Search, and Binary-Search. Let $T_1(n)$, $T_2(n)$, and $T_3(n)$, respectively, be the number of element examinations[1] required by these algorithms when run on a list whose length is n. Ignoring floors, ceilings, and lower order terms, we have

$$\begin{aligned} T_1(n) &= n \\ T_2(n) &= 2\sqrt{n} \\ T_3(n) &= \log_2(n). \end{aligned}$$

i. On a *single* sheet of graph paper, plot the number of element examinations required for each of the three algorithms when run on lists of length $n = 1$, 2, 4, 8, 16, and 32. For each algorithm, connect the plot points with a smooth, hand-drawn curve. See the plots given in the "Exponentials and Logs" handout for examples of what you should do. You may print a piece of graph paper from the PDF located at the following URL:

`http://www.pdfpad.com/graphpaper/`

(If you view this assignment on-line, you may simply click on the above hyperlink.)

ii. Suppose that you were given a budget of 30 element examinations. For each of the three algorithms, determine the largest array length such that the number of element examinations made is guaranteed to be at most 30.

iii. How many times larger is the array that Binary-Search can handle, as compared to the arrays that Chunk-Search and Ordered-Linear-Search can handle? How many times larger is the array that Chunk-Search can handle, as compared to the array that Ordered-Linear-Search can handle?

[1] worst-case...

Problem 2 [20 pts, (10,10)]**: Recurrences**

In each of the following problems, solve the recurrence using the method described in class and in the text. *You must show your work.*

Assume a base case of $T(1) = 1$. As part of your solution, you will need to establish a pattern for what the recurrence looks like after the k-th iteration. For this assignment, you need *not* formally prove that your patterns are correct via induction, though you will lose points if your patterns are not correct. Your solutions may involve n raised to a power and/or logarithms of n. For example, a solution of the form $9^{\log_3 n}$ is unacceptable; this should be simplified as $n^{\log_3 9} = n^2$.

i. $T(n) = 8\,T(n/2) + n$.

ii. $T(n) = 8\,T(n/2) + n^3$.

Problem 3 [40 pts, (5 each)]**: Graphs**

A (simple) graph consists of a set of vertices V and a set of (undirected) edges E. Let $V_1 = \{\texttt{a}, \texttt{b}, \texttt{c}\}$, i.e., a set of three vertices labeled `a`, `b`, and `c`. Now consider all (simple) graphs which can be formed from these three vertices; one obtains different graphs by having different sets of edges.

i. Draw all possible graphs that can be constructed from the vertices $V_1 = \{\texttt{a}, \texttt{b}, \texttt{c}\}$.

ii. How many such graphs have no edges? How many such graphs have exactly one edge? How many such graphs have exactly two edges? How many such graphs have all three edges? How many total graphs are there?

Now consider a vertex set of size 4, $V_2 = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$.

iii. How many possible edges exist over V_2?

iv. How many unique graphs can be constructed from V_2? *Hint:* In any such graph, each edge is either present or absent.

v. How many unique graphs can be constructed from V_2 with exactly three edges? *Hint:* One must *choose* where to place the three edges...

Now generalize these results for a vertex set of size n, i.e., $V_3 = \{\texttt{a}, \texttt{b}, \texttt{c}, \ldots\}$ where $|V_3| = n$.

vi. How many possible edges exist over V_3? Justify your answer.

vii. How many unique graphs can be constructed from V_3? Justify your answer.

viii. How many unique graphs can be constructed from V_3 using exactly k edges? Justify your answer.