CSU200 Discrete Structures	Profs. Aslam & Fell
Fall 2006	November 2, 2006

Written Homework 03

Assigned: Thu 02 Nov 2006
Due: Thu 09 Nov 2006

Instructions:

- The assignment is due at the *beginning* of class on the due date specified, i.e., 1:35pm for Prof. Aslam's section and 4:35pm for Prof. Fell's section. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.

- In each problem, you *must* show all of your work, including the principles used (e.g., sum rule, product rule, pigeonhole principle, permutations, combinations, etc.) and a justification for their use. You will not receive any credit if you do not show and explain where your answers came from.

Problem 1 [28 pts;, (4 points each)]: Divisibility

Consider the set of integers $\{1, 2, \ldots, 3000\}$.

i. How many of these integers are divisible by 3?

ii. How many of these integers are divisible by 5?

iii. How many of these integers are divisible by 3 AND by 5?

iv. How many of these integers are not divisible by 3 OR by 5?

v. What is the least number of distinct integers that must be chosen so that at least one of them is divisible by 3?

vi. What is the least number of distinct integers that must be chosen so that at least one of them is divisible by 5?

vii. What is the least number of distinct integers that must be chosen so that at least one of them is divisible by 3 or 5?

Problem 2 [30 pts, (10 pts each)]: Rows

The Northeastern University College of Computer and Information Science has 5 women and 20 men who are full-time faculty members. Their photos are to be displayed in a single row at the entrance to their building.

i. In how many ways can the photos be displayed so that all of the women are together? In how many ways can the photos be displayed so that all of the women are together and all of the men are together?

ii. In how many ways can the photos be displayed so that all of the women are together or all of the men are together (or both)?

iii. In how many ways can the photos be displayed so that no two women are adjacent? *Hint:* Consider arranging the men's photos first and then placing the women's photos among the men's photos so as to ensure that no two women are adjacent.

Problem 3 [20 pts]: Exams

A discrete structures exam consists of 14 problems divided into two groups of 7 problems each. Students are required to answer any 8 problems out of the 14 but no more than 5 from either group. In how many ways can a student choose the 8 problems to solve?

Problem 4 [22 pts, (11 each)]: Cars

A marketing firm has been hired to conduct a survey of automobile purchases in the greater Boston area. The survey consists of recording, for each household surveyed, the number of cars presently owned by that household among the following six major manufacturers: GM, Ford, Daimler-Chrysler, Toyota, Honda, and Nissan. Thus, one household may have two GMs and a Ford, another household may have one Honda and one Nissan, etc.

i. Suppose that no household surveyed has more than four cars. In how many different ways can the survey sheet be filled out?

ii. There are approximately 1,200,000 households in the greater Boston area. Suppose that all households are surveyed, the resulting survey sheets are organized and tallied by type (e.g., two GMs and a Ford vs. one Honda and one Nissan, etc.), and the survey results are sorted by popularity (e.g., two GMs and a Ford is the most common, one Honda and one Nissan is the second most common, etc.). At a minimum, how large must be the count associated with the most popular survey result?