# Homework 02

**Assigned:** Wed 05 Oct 2005
**Due:** Wed 12 Oct 2005

**Instructions:**

- The assignment is due at the *beginning* of class on the due date specified, i.e., 10:30am for Prof. Fell's section and 1:35pm for Prof. Aslam's section. Late assignments submitted within 24 hours of the official due date and time will receive a 20% penalty. Additional 10% penalties will be assessed in each subsequent 24 hour period, up to a maximum of 50% as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.

**Problem 1** [20 pts, (4,6,4,6)]**:** Consider the arithmetic series

$$5 + 8 + 11 + 14 + \cdots + 125.$$

**i.** How many terms are in this series? Explain.

**ii.** Apply Gauss's trick to evaluate this sum. Show your work. You *must* give an answer in the form

$$\frac{x \cdot y}{2}$$

for some $x$ and $y$, and you may then use a calculator to evaluate this expression.

**iii.** Write this series as a summation in the form

$$\sum_{k=1}^{n}(a \cdot k + b)$$

for some $a$, $b$, and $n$.

**iv.** Rewrite this summation in the form

$$a \cdot \sum_{k=1}^{n} k + \sum_{k=1}^{n} b.$$

What is the value of

$$\sum_{k=1}^{n} b$$

for your values of $n$ and $b$? Apply the standard arithmetic summation formula to evaluate

$$\sum_{k=1}^{n} k$$

for your value of $n$. Finally, evaluate the original expression by using the values of these summations and your value of $a$. Show your work. You should obtain the same value as in part ii above, of course.

**Problem 2** [30 pts, (15,15)]**:** Solve the following recurrences via iteration as we did in class and in the recurrence handout. Assume a base case of $T(1) = 1$. As part of your solution, you will need to establish a pattern for what the recurrence looks like after the $k$-th iteration. For this assignment, you need *not* formally prove that your patterns are correct, though you will lose points if your pattern are not correct. Your solutions may involve $n$ raised to a power and/or logarithms of $n$. For example, a solution of the form $9^{\log_3 n}$ is unacceptable; this should be simplified as $n^{\log_3 9} = n^2$.

**i.** $T(n) = 4\,T(n/2) + n$.

**ii.** $T(n) = 4\,T(n/2) + n^2$.

**Problem 3** [30 pts, (15,15)]**:**

**i.** Consider the series

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \sum_{k=1}^{n} k(k+1).$$

Show that

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

by induction.

**ii.** Now consider the series

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)}.$$

Show that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}$$

by induction.

**Problem 4** [20 pts, (5,10,5)]**:** A *telescoping series* is a series of the form

$$\sum_{k=1}^{n} (a_k - a_{k+1})$$

for some sequence $a_1, a_2, a_3, \ldots$

**i.** Show that $\sum_{k=1}^{n} (a_k - a_{k+1}) = a_1 - a_{n+1}$.

**ii.** Show that $\sum_{k=1}^{n} \frac{1}{k(k+1)}$ is a telescoping series. What is the form of $a_k$ for any $k$?

*Hint:* Factor the ratio $\frac{1}{k(k+1)}$ into terms involving $\frac{1}{k}$ and $\frac{1}{k+1}$.

**iii.** Using parts i and ii above, show that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}.$$

Explain.