CS 7180, Fall 2012
Homework 4—Decision Theory and Game Theory

Assigned: November 30, 2012
Due: December 7, 2012,

1. Consider the Allais paradox on page 620 of your text book: an agent who prefers B over A (taking the sure thing), and C over D (taking the higher EMV) is not acting rationally according to utility theory. Do you think this indicates a problem for the agent, a problem for the theory, or no problem at all? Explain.

2. In 1713, Nicolas Bernoulli stated a puzzle, now called the St. Petersburg paradox, which works as follows. You have the opportunity to play a game in which a fair coin is tossed repeatedly until it comes up heads. If the first heads appears on the nth toss, you win \$$2^n$.

   (a) Show that the expected monetary value of this game in infinite.

   (b) How much would you, personally, pay to play the game?

   (c) Nicolas's cousin Daniel Bernoulli resolved the paradox in 1738 by finding that the utility of money is logarithmic (i.e., $U(S_n) = alog_2 n + b$ where S_n is the state of having \$$n$). What is the expected utility of the game under this assumption?

   (d) What is the maximum amount it would be rational to pay to play the game, assuming you have initial wealth \$$k$?

3. The following payoff matrix (from Blinder, 1983 and Bernstein, 1996) shows a game between politicians and the Federal Reserve (the central banking system of the United States).

	Fed: contract	Fed: do nothing	Fed: expand
Pol: contract	$F = 7$, $P = 1$	$F = 9$, $P = 4$	$F = 6, P = 6$
Pol: do nothing	$F = 8$, $P = 2$	$F = 5$, $P = 5$	$F = 4, P = 9$
Pol: expand	$F = 3$, $P = 3$	$F = 2$, $P = 7$	$F = 1, P = 8$

Politicians can expand or contract fiscal policy (i.e., run a deficit or surplus in government revenue), while the Fed can expand or contract monetary policy (i.e., lower or raise interest rates). Of course, either side can also choose to do nothing. Each side also has a preferences for who should do what—neither wants to be the bad guy. The payoffs shown are simply the rank orderings: 9 for the first choice, 1 for the last. Find the Nash equilibrium of the game in pure strategies. Is this a Pareto-optimal solution?