# NLP \& Linguistics 

Natural Language Processing
CS 4120/6120—Spring 2017
Northeastern University

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## Engineering vs. Science?

- One story
- NLP took formal language theory and generative linguistics (same source?),
- Built small Al systems for a while,
- Then added statistics/machine learning (from speech recognition).
- What now?
- Shouldn't AI tell us about natural intelligence?
- Are all NLP models lousy linguistics?


## Zipf's Law

The Roots of Quantitative Linguistics

## Zipf's Law

- Distribution of word frequencies is very skewed - a few words occur very often, many words hardly ever occur
- e.g., two most common words ("the", "of") make up about $10 \%$ of all word occurrences in text documents
- Zipf's "law" (more generally, a "power law"):
- observation that rank ( $r$ ) of a word times its frequency $(f)$ is approximately a constant (k)
- assuming words are ranked in order of decreasing frequency
- i.e., $r . f \approx k$ or r. $P_{r} \approx c$, where $P_{r}$ is probability of word occurrence and $c \approx 0.1$ for English


## Zipf's Law



## News Collection (AP89) Statistics

Total documents<br>84,678<br>Total word occurrences 39,749,179<br>Vocabulary size 198,763<br>Words occurring > 1000 times 4,169<br>Words occurring once 70,064

| Word | Freq. | $r$ | $\operatorname{Pr}(\%)$ |  | $r . P r$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| assistant | 5,095 | 1,021 | .013 | 0.13 |  |
| sewers | 100 | 17,110 | $2.56 \times 10-4$ | 0.04 |  |
| toothbrush | 10 | 51,555 | $2.56 \times 10-5$ | 0.01 |  |
| hazmat | 1 | 166,945 | $2.56 \times 10-6$ | 0.04 |  |

## Top 50 Words from AP89

| Word | Freq. | $r$ | $P_{r}(\%)$ | $r . P_{r}$ | Word | Freq | $r$ | $P_{r}(\%)$ | $r . P_{r}$ |
| :--- | ---: | ---: | ---: | ---: | :--- | ---: | ---: | ---: | ---: |
| the | $2,420,778$ | 1 | 6.49 | 0.065 | has | 136,007 | 26 | 0.37 | 0.095 |
| of | $1,045,733$ | 2 | 2.80 | 0.056 | are | 130,322 | 27 | 0.35 | 0.094 |
| to | 968,882 | 3 | 2.60 | 0.078 | not | 127,493 | 28 | 0.34 | 0.096 |
| a | 892,429 | 4 | 2.39 | 0.096 | who | 116,364 | 29 | 0.31 | 0.090 |
| and | 865,644 | 5 | 2.32 | 0.120 | they | 111,024 | 30 | 0.30 | 0.089 |
| in | 847,825 | 6 | 2.27 | 0.140 | its | 111,021 | 31 | 0.30 | 0.092 |
| said | 504,593 | 7 | 1.35 | 0.095 | had | 103,943 | 32 | 0.28 | 0.089 |
| for | 363,865 | 8 | 0.98 | 0.078 | will | 102,949 | 33 | 0.28 | 0.091 |
| that | 347,072 | 9 | 0.93 | 0.084 | would | 99,503 | 34 | 0.27 | 0.091 |
| was | 293,027 | 10 | 0.79 | 0.079 | about | 92,983 | 35 | 0.25 | 0.087 |
| on | 291,947 | 11 | 0.78 | 0.086 | i | 92,005 | 36 | 0.25 | 0.089 |
| he | 250,919 | 12 | 0.67 | 0.081 | been | 88,786 | 37 | 0.24 | 0.088 |
| is | 245,843 | 13 | 0.65 | 0.086 | this | 87,286 | 38 | 0.23 | 0.089 |
| with | 223,846 | 14 | 0.60 | 0.084 | their | 84,638 | 39 | 0.23 | 0.089 |
| at | 210,064 | 15 | 0.56 | 0.085 | new | 83,449 | 40 | 0.22 | 0.090 |
| by | 209,586 | 16 | 0.56 | 0.090 | or | 81,796 | 41 | 0.22 | 0.090 |
| it | 195,621 | 17 | 0.52 | 0.089 | which | 80,385 | 42 | 0.22 | 0.091 |
| from | 189,451 | 18 | 0.51 | 0.091 | we | 80,245 | 43 | 0.22 | 0.093 |
| as | 181,714 | 19 | 0.49 | 0.093 | more | 76,388 | 44 | 0.21 | 0.090 |
| be | 157,300 | 20 | 0.42 | 0.084 | after | 75,165 | 45 | 0.20 | 0.091 |
| were | 153,913 | 21 | 0.41 | 0.087 | us | 72,045 | 46 | 0.19 | 0.089 |
| an | 152,576 | 22 | 0.41 | 0.090 | percent | 71,956 | 47 | 0.19 | 0.091 |
| have | 149,749 | 23 | 0.40 | 0.092 | up | 71,082 | 48 | 0.19 | 0.092 |
| his | 142,285 | 24 | 0.38 | 0.092 | one | 70,266 | 49 | 0.19 | 0.092 |
| but | 140,880 | 25 | 0.38 | 0.094 | people | 68,988 | 50 | 0.19 | 0.093 |

## Zipf’s Law for AP89



- Log-log plot: Note problems at high and low frequencies


## Zipf’s Law

- What is the proportion of words with a given frequency?
- Word that occurs $n$ times has rank $r_{n}=k / n$
- Number of words with frequency $n$ is
- $r_{n}-r_{n+1}=k / n-k /(n+1)=k / n(n+1)$
- Proportion found by dividing by total number of words = highest rank $=k$
- So, proportion with frequency $n$ is $1 / n(n+1)$


## Zipf's Law

- Example word frequency ranking

| Rank | Word | Frequency |
| ---: | ---: | ---: |
| 1000 | concern | 5,100 |
| 1001 | spoke | 5,100 |
| 1002 | summit | 5,100 |
| 1003 | bring | 5,099 |
| 1004 | star | 5,099 |
| 1005 | immediate | 5,099 |
| 1006 | chemical | 5,099 |
| 1007 | african | 5,098 |

- To compute number of words with frequency 5,099 - rank of "chemical" minus the rank of "summit" $-1006-1002=4$


## E×an? Pe

| Number of <br> Occurrences <br> $(n)$ | Predicted <br> Proportion <br> $(1 / n(n+1))$ | Actual <br> Proportion | Actual <br> Number of <br> Words |
| :---: | :---: | :---: | :---: |
| 1 | .500 | .402 | 204,357 |
| 2 | .167 | .132 | 67,082 |
| 3 | .083 | .069 | 35,083 |
| 4 | .050 | .046 | 23,271 |
| 5 | .033 | .032 | 16,332 |
| 6 | .024 | .024 | 12,421 |
| 7 | .018 | .019 | 9,766 |
| 8 | .014 | .016 | 8,200 |
| 9 | .011 | .014 | 6,907 |
| 10 | .009 | .012 | 5,893 |

- Proportions of words occurring $n$ times in 336,310 TREC documents
- Vocabulary size is 508,209


## Vocabulary Growth

- As corpus grows, so does vocabulary size - Fewer new words when corpus is already large
- Observed relationship (Heaps' Law):

$$
v=k . n^{B}
$$

where $v$ is vocabulary size (number of unique words),
$n$ is the number of words in corpus,
$k, B$ are parameters that vary for each corpus (typical values given are $10 \leq k \leq 100$ and $B \approx 0.5$ )

AP89 Example


## Heaps' Law Predictions

- Predictions for TREC collections are accurate for large numbers of words
- e.g., first 10,879,522 words of the AP89 collection scanned
- prediction is 100,151 unique words
- actual number is 100,024
- Predictions for small numbers of words (i.e. < 1000) are much worse


## GOV2 (Web) Example



## Ever Upwards

- Heaps' Law works with very large corpora - new words occurring even after seeing 30 million!
- parameter values different than typical TREC values
- New words come from a variety of sources
- spelling errors, invented words (e.g. product, company names), code, other languages, email addresses, etc.
- Language models (and other NLP and IR systems) need to handle open, growing vocabulary


## Power-Law Distributions

- For discrete data (Clauset et al., 2009):

$$
p(x)=\operatorname{Pr}(X=x)=C x^{-\alpha}
$$

- which diverges at 0 , thus requiring a lower bound $x_{\min }>0$
- which normalizes to $p(x)=\frac{x^{-\alpha}}{\zeta\left(\alpha, x_{\text {min }}\right)}$
- with Hurwitz zeta $\zeta\left(\alpha, x_{\min }\right)=\sum_{n=0}^{\infty}\left(n+x_{\min }\right)^{-\alpha}$



## Power Laws Everywhere!


(Clauset et al., 2009)

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## Power Laws Everywhere?

| data set | $p$ | Poisson |  | log-normal |  | exponential |  | stretched exp. |  | power law + cut-off |  | support for power law |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | LR | $p$ | LR | $p$ | LR | $p$ | LR | $p$ | LR | $p$ |  |
| Internet | 0.29 | 5.31 | 0.00 | -0.807 | 0.42 | 6.49 | 0.00 | 0.493 | 0.62 | -1.97 | 0.05 | with cut-off |
| calls | 0.63 | 17.9 | 0.00 | -2.03 | 0.04 | 35.0 | 0.00 | 14.3 | 0.00 | -30.2 | 0.00 | with cut-off |
| citations | 0.20 | 6.54 | 0.00 | -0.141 | 0.89 | 5.91 | 0.00 | 1.72 | 0.09 | $-0.007$ | 0.91 | moderate |
| email | 0.16 | 4.65 | 0.00 | -1.10 | 0.27 | 0.639 | 0.52 | -1.13 | 0.26 | -1.89 | 0.05 | with cut-off |
| metabolic | 0.00 | 3.53 | 0.00 | -1.05 | 0.29 | 5.59 | 0.00 | 3.66 | 0.00 | 0.000 | 1.00 | none |
| papers | 0.90 | 5.71 | 0.00 | -0.091 | 0.93 | 3.08 | 0.00 | 0.709 | 0.48 | -0.016 | 0.86 | moderate |
| proteins | 0.31 | 3.05 | 0.00 | -0.456 | 0.65 | 2.21 | 0.03 | 0.055 | 0.96 | -0.414 | 0.36 | moderate |
| species | 0.10 | 5.04 | 0.00 | -1.63 | 0.10 | 2.39 | 0.02 | -1.59 | 0.11 | -3.80 | 0.01 | with cut-off |
| terrorism | 0.68 | 1.81 | 0.07 | -0.278 | 0.78 | 2.457 | 0.01 | 0.772 | 0.44 | $-0.077$ | 0.70 | moderate |
| words | 0.49 | 4.43 | 0.00 | 0.395 | 0.69 | 9.09 | 0.00 | 4.13 | 0.00 | -0.899 | 0.18 | good |

# Learning in the Limit Gold's Theorem 

## Observe some values of a function



## Guess the whole function



## Another guess: Just as good?



## More data needed to decide



## Poverty of the Stimulus

## Poverty of the Stimulus

- Never enough input data to completely determine the polynomial ...
- Always have infinitely many possibilities
... unless you know the order of the polynomial ahead of time.
- 2 points determine a line
- 3 points determine a quadratic
- etc.
- In language learning, is it enough to know that the target language is generated by a CFG?
" without knowing the size of the CFG?


## Language learning:

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Children listen to language [unsupervised]

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Children listen to language [unsupervised] Children are corrected?? [supervised]

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Children listen to language [unsupervised]
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Remember: Language = set of strings

## Poverty of the Stimulus (1957)

Children listen to language
Children are corrected??

- Children observe language in context

Children observe frequencies of language

## Poverty of the Stimulus (1957)

Chomsky: Just like polynomials: never enough data unless you know something in advance. So kids must be born knowing what to expect in language.

- Children listen to language
- Children are corrected??
- Children observe language in context
- Children observe frequencies of language


## Gold's Theorem (1967)

a simple negative result along these lines:
kids (or computers) can't learn much
without supervision, inborn knowledge, or statistics

- Children listen to language
- Children are corrected??
- Children observe language in context
- Children observe frequencies of language


## The Idealized Situation

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Mom talks

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- Mom talks
- Baby listens


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- Mom talks
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- 1. Mom outputs a sentence
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- Guarantee: Mom's language is in the set of hypotheses that Baby is choosing among


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- Mom talks
- Baby listens
- 1. Mom outputs a sentence
- 2. Baby hypothesizes what the language is (given all sentences so far)
- 3. Goto step 1
- Guarantee: Mom's language is in the set of hypotheses that Baby is choosing among
- Guarantee: Any sentence of Mom's language is eventually uttered by Mom (even if infinitely many)


## The Idealized Situation

- Mom talks
- Baby listens
- 1. Mom outputs a sentence
- 2. Baby hypothesizes what the language is (given all sentences so far)
- 3. Goto step 1
- Guarantee: Mom's language is in the set of hypotheses that Baby is choosing among
- Guarantee: Any sentence of Mom's language is eventually uttered by Mom (even if infinitely many)
- Assumption: Vocabulary (or alphabet) is finite.


## Can Baby learn under these conditions?

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- Learning in the limit:
- There is some point at which Baby's hypothesis is correct and never changes again. Baby has converged!
- Baby doesn't have to know that it's reached this point - it can keep an open mind about new evidence - but if its hypothesis is right, no such new evidence will ever come along.


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- Baby knows the class $C$ of possibilities, but not $L$.
- Is there a perfect finite-state Baby?
- Is there a perfect context-free Baby?


## Languages vs. Grammars

Does Baby have to get the right grammar?
(E.g., does VP have to be called VP?)

Assumption: Finite vocabulary.

## Conservative Strategy

- Baby's hypothesis should always be smallest language consistent with the data
- Works for finite languages? Let's try it ...
- Language 1: \{aa,ab,ac\}
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Mom
Baby

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Baby L3

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| Mom | aa | $a b$ | $a c$ | $a b$ | $a a$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Baby | $L 3$ | $L 1$ | $L 1$ | $L 1$ | $L 1$ |

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- Works for finite languages? Let's try it ...
- Language 1: \{aa,ab,ac\}
- Language 2: \{aa,ab,ac,ad,ae\}
- Language 3: \{aa,ac\}
- Language 4: \{ab\}

$\begin{array}{llllll}\text { Mom } & \text { aa } & a b & a c & a b & a a \\ \text { Baby } & L 3 & L 1 & L 1 & L 1 & L 1\end{array}$


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## Evil Mom

- To find out whether Baby is perfect, we have to see whether it gets 100\% even in the most adversarial conditions
- Assume Mom is trying to fool Baby
- although she must speak only sentences from L
- and she must eventually speak each such sentence
- Does Baby's strategy work?


## An Unlearnable Class

Class of languages:
= Let $L_{n}=$ set of all strings of length $<n$

- What is $\mathrm{L}_{0}$ ?
- What is $L_{1}$ ?
- What is $\mathrm{L}_{\infty}$ ?
- If the true language is $\mathrm{L}_{\infty}$, can Mom really follow rules?
-Must eventually speak every sentence of $\mathrm{L}_{\infty}$. Possible?
-Yes: $\varepsilon$; a, b; aa, ab, ba, bb; aaa, aab, aba, abb, baa, ...
- Our class is $C=\left\{L_{0}, L_{1}, \ldots L_{\infty}\right\}$


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- A perfect C-baby will distinguish among all of these depending on the input.
- But there is no perfect C-baby ...


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- So if Mom's longest sentence so far has 75 words, baby's hypothesis is $L_{76}$.


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- Suppose Baby adopts conservative strategy, always picking smallest possible language in C.
- So if Mom's longest sentence so far has 75 words, baby's hypothesis is $L_{76}$.
- This won't always work: What language can't a conservative Baby learn?


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- Agreed?


## Mom's Revenge

```
If longest sentence so far is 75 words, and Mom keeps talking
from L}\mp@subsup{L}{76}{}\mathrm{ , then eventually a perfect C-baby must actually return to the conservative guess \(L_{76}\).
```

- Suppose true language is $\mathrm{L}_{\infty}$.
- Evil Mom can prevent our supposedly perfect C-Baby from converging to it.
- If Baby ever guesses $\mathrm{L}_{\infty}$, say when the longest sentence is 75 words:
- Then Evil Mom keeps talking from $L_{76}$ until Baby capitulates and revises her guess to $L_{76}$ - as any perfect C-Baby must.
- So Baby has not stayed at $L_{\infty}$ as required.
- Then Mom can go ahead with longer sentences. If Baby ever guesses $L_{\infty}$ again, she plays the same trick again.


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- So Baby has not stayed at $\mathrm{L}_{\infty}$ as required.
- Conclusion: There's no perfect Baby that is guaranteed to converge to $L_{0}, L_{1}, \ldots$ or $L_{\infty}$ as appropriate. If it always succeeds on finite languages, Evil Mom can trick it on infinite lanquage.


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"After all, it includes all languages in C, and more, so learner has harder choice
- How about class of context-free languages?
- Not unless you limit it further (e.g., \# of rules)


## Punchline

- But class of probabilistic context-free languages is learnable in the limit!! (Horning, 1969)
- If Mom has to output sentences randomly with the appropriate probabilities,
- she's unable to be too evil
- there are then perfect Babies that are guaranteed to converge to an appropriate probabilistic CFG
- I.e., from hearing a finite number of sentences, Baby can correctly converge on a grammar that predicts an infinite number of sentences.
- Baby is generalizing! Just like real babies!


## Perfect fit to perfect, incomplete data



## Imperfect fit to noisy data



## Imperfect fit to noisy data



Will an ungrammatical sentence ruin baby forever?

## Imperfect fit to noisy data


$\pm$ Input Data
$-2 x^{\wedge} 2-6 x+6$

$-8 x^{\wedge} 6+72.8 x^{\wedge} 5-250^{*} x^{\wedge} 4+$
$401^{*} x^{\wedge} 3-297^{*} x^{\wedge} 2+79.2^{*} x+$
6

- More Input Data

Will an ungrammatical sentence ruin baby forever? (yes, under the conservative strategy ...)

## Imperfect fit to noisy data



Will an ungrammatical sentence ruin baby forever?
(yes, under the conservative strategy ...)
Or can baby figure out which data to (partly) ignore?

## Imperfect fit to noisy data



Will an ungrammatical sentence ruin baby forever? (yes, under the conservative strategy ...)
Or can baby figure out which data to (partly) ignore? Statistics can help again ... how?

## Frequencies and Probabilities in Natural Languages

Chris Manning and others

## Models for language

- Human languages are the prototypical example of a symbolic system
- From the beginning, logics and logical reasoning were invented for handling natural language understanding
- Logics and formal languages have a language-like form that draws from and meshes well with natural languages

- Where are the numbers?


## Dominant answer in linguistic theory: Nowhere

Chomsky again (1969: 57; also 1956, 1957, etc.):

- "It must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term."
Probabilistic models wrongly mix in world knowledge
- New York vs. Dayton, Ohio

They don't model grammaticality [also, Tesnière 1959]

- Colorless green ideas sleep furiously
- Furiously sleep ideas green colorless
- [But see Pereira 2005]


## Categorical linguistic

theories (GB, Minimalism, LFG, HPSG, CG, ...)

- Systems of variously rules, principles, and representations is used to describe an infinite set of grammatical sentences of the language
- Other sentences are deemed ungrammatical
- Word strings are given a (hidden) structure



## The need for frequencies / probability distributions

The motivation comes from two sides:

- Categorical linguistic theories claim too much:
- They place a hard categorical boundary of grammaticality, where really there is a fuzzy edge, determined by many conflicting constraints and issues of conventionality vs. human creativity
- Categorical linguistic theories explain too little:
- They say nothing at all about the soft constraints which explain how people choose to say things
- Something that language educators, computational NLP people - and historical linguists and sociolinguists dealing with real language - usually want to know about


## 1. The hard constraints of categorical grammars

- Sentences must satisfy all the rules of the grammar
- One group specifies the arguments that different verbs take - lexical subcategorization information
- Some verbs must take objects: *Kim devoured [ * means ungrammatical]
- Others do not: *Kim's lip quivered the straw
- Others take various forms of sentential complements
- In NLP systems, ungrammatical sentences don't parse
- But the problem with this model was noticed early on:
- "All grammars leak." (Sapir 1921: 38)


## Example: verbal clausal subcategorization frames

- Some verbs take various types of sentential complements, given as subcategorization frames:
- regard: __ NP[acc] as \{NP, AdjP\}
- consider: __ NP[acc] \{AdjP, NP, VP[inf]\}
- think: __ CP[that]; __ NP[acc] NP
- Problem: in context, language is used more flexibly than this model suggests
- Most such subcategorization 'facts' are wrong


## Subcat on the MBTA


?The Conductor of this train is responsible to ensure that your trip is both safe and enjoyable. ...responsible for ensuring...
?...responsible that it be ensured that ...

## Standard subcategorization rules (Pollard and Sag 1994)

- We consider Kim to be an acceptable candidate
- We consider Kim an acceptable candidate
- We consider Kim quite acceptable
- We consider Kim among the most acceptable candidates
- *We consider Kim as an acceptable candidate
- *We consider Kim as quite acceptable
- *We consider Kim as among the most acceptable candidates
- ?*We consider Kim as being among the most acceptable candidates


# Subcategorization facts from The New York Times 

Consider as:

- The boys consider her as family and she participates in everything we do.
- Greenspan said, "I don't consider it as something that gives me great concern.
- "We consider that as part of the job," Keep said.
- Although the Raiders missed the playoffs for the second time in the past three seasons, he said he considers them as having championship potential.
- Culturally, the Croats consider themselves as belonging to the "civilized" West, ...


# More subcategorization facts: regard 

Pollard and Sag (1994):

- *We regard Kim to be an acceptable candidate
- We regard Kim as an acceptable candidate

The New York Times:

- As 70 to 80 percent of the cost of blood tests, like prescriptions, is paid for by the state, neither physicians nor patients regard expense to be a consideration.
- Conservatives argue that the Bible regards homosexuality to be a sin.


## More subcategorization facts: turn out and end up

Pollard and Sag (1994):

- Kim turned out political
- *Kim turned out doing all the work

The New York Times:

- But it turned out having a greater impact than any of us dreamed.

Pollard and Sag (1994):

- Kim ended up political
- *Kim ended up sent more and more leaflets

The New York Times:

- On the big night, Horatio ended up flattened on the ground like a fried egg with the yolk broken.


## Probability mass functions: subcateqorization of reqard



## Leakage leads to change

- People continually stretch the 'rules' of grammar to meet new communicative needs, to better align grammar and meaning, etc.
- As a result language slowly changes
- while: used to be only a noun (That takes a while); now mainly used as a subordinate clause introducer (While you were out)
- e-mail: started as a mass noun like mail (most junk e-mail is annoying); it's moving to be a count noun (filling the role of e-letter): I just got an interesting email about that.


## Blurring of categories: "Marqinal prepositions"

- An example of blurring in syntactic category during linguistic change is so-called 'marginal prepositions' in English, which are moving from being participles to prepositions
- Some still clearly maintain a verbal existence, like following, concerning, considering; for some it is marginal, like according, excepting; for others their verbal character is completely lost, such as during [cf. endure], pending, notwithstanding.


## Verb (VBG) ? Preposition IN

As verbal participle, understood subject agrees with noun:

- They moved slowly, toward the main gate, following the wall
- Repeat the instructions following the asterisk

A temporal use with a controlling noun becomes common:

- This continued most of the week following that illstarred trip to church
Prep. uses (meaning is after, no controlling noun) appear
- He bled profusely following circumcision
- Following a telephone call, a little earlier, Winter had said ...


# Mapping the recent change of following: participle $\rightarrow$ prep. 

- Fowler (1926): "there is a continual change going on by which certain participles or adjectives acquire the character of prepositions or adverbs, no longer needing the prop of a noun to cling to ... [we see] a development caught in the act"
- Fowler (1926) -- no mention of following in particular
- Fowler [Gowers] (1948): "Following is not a preposition. It is the participle of the verb follow and must have a noun to agree with"
- Fowler [Gowers] (1954): generally condemns temporal usage, but says it can be justified in certain circumstances


## 2. Explaining more: What do people say?

- What people do say has two parts:
- Contingent facts about the world
- People in Minnesota have talked a lot about snow falling, not stocks falling, lately
- The way speakers choose to express ideas using the resources of their language
- People don't often put that-clauses pre-verbally:
- That we will have to revise this program is almost certain
- The latter is properly part of people's Knowledge of Language-i.e., part of linguistics.


## What do people say?

- Simply delimiting a set of grammatical sentences provides only a very weak description of a language, and of the ways people choose to express ideas in it
- Probability densities over sentences and sentence structures can give a much richer view of language structure and use
- In particular, we find that the same soft generalizations and tendencies of one language often appear as (apparently) categorical constraints in other languages
- A syntactic theory should be able to uniformly capture these constraints, rather than only recognizing them when they are categorical


## Example: Bresnan, Dingare

## \& Manning

- Project modeling English diathesis alternations (active/passive, locative inversion, etc.)
- In some languages passives are categorically restricted by person considerations:
- In Lummi (Salishan, Washington state), $1 / 2$ person must be the subject if other argument is 3rd person. There is variation if both arguments are 3rd person. (Jelinek and Demers 1983) [cf. also Navajo, etc.]
- *That example was provided by me
- *He likes me
- ?I am liked by him


## Bresnan, Dingare \&

## Manning

- In English, there is no such categorical constraint, but we can still see it at work as a soft constraint.
- Collected data from verbs with an agent and patient argument (canonical transitives) from treebanked portions of the Switchboard corpus of conversational American English, analyzing for person and act/pass

|  | Active | Passive |
| :--- | ---: | ---: |
| $1 / 2 \mathrm{Ag}, 1 / 2 \mathrm{Pt}$ | 158 | $0(0.0 \%)$ |
| $1 / 2 \mathrm{Ag}, 3 \mathrm{Pt}$ | 5120 | $1(0.0 \%)$ |
| $3 \mathrm{Ag}, 1 / 2 \mathrm{Pt}$ | 552 | $16(2.8 \%)$ |
| $3 \mathrm{Ag}, 3 \mathrm{Pt}$ | 3307 | $46(1.4 \%)$ |

## Bresnan, Dingare \&

## Manning

- While person is only a small part of the picture in determining the choice of active/passive in English (information structure, genre, etc. is more important), there is nonetheless a highly significant ( $\mathrm{X}^{2} \mathrm{p}<$ 0.0001 ) effect of person on active/passive choice
- The exact same hard constraint of Lummi appears as a soft constraint in English
- This behavior is predicted by the universal hierarchies within a stochastic OT model (which extends existing OT approaches to valence - Aissen 1999, Lødrup 1999)
- Conversely linguistic model predicts that no "antiEnglish" [which is just the opposite] exists


# Syntactic Matching 

Roger Levy

## Conclusions

- There are many phenomena in language that cry out for non-categorical and probabilistic modeling and explanation
- Probabilistic models can be applied on top of one's favorite sophisticated linguistic representations!
- Frequency evidence can enrich linguistic theory by revealing soft constraints at work in language use


## What Next?

- Courses you could take
- Machine Learning
- Information Retrieval
- Data Mining
- Special Topics


## What Next?

- People you could talk to
- LuWang
- Byron Wallace
- Jay Aslam
- Tim Bickmore
- People in network science, the social sciences, the humanities, and linguistics working on language data
nothing but the the history of the lat if ever we did rties would be so $r$ the defeat of an he defeated party leserved calamity, var in which we ever likely to continent more when it began, stance of a purely e to have even a for the purpose of ppression, and not retation of some it as independent n. If it be posnd America, it is of doing so. It 5 what we want. ength is what we letween two great revent, or greatly us, hopeless, and ivil war in characssies of national
, it would seem good opportunity - more a cordial $t$ least if we may the other side of
events, unless the accounts from many quarters as to General Schenck's instructions are utterly belied, the new American Ambassador will bring us quite reasonable, though not perhaps wholly admissible demands,-demands which we certainly ought to consider most gravely, and of which we should do well to yield frankly and freely all that we should ourselves feel called upon, in the same circumstances, to press. If we do so, General Schenck's mission may make England safer and stronger than she has ever been since the close of the Civil War in 1865, and will give her a reputation for moderation and candour as well.


## ENGLISH PUBLIC OPINION ON THE WAR.

Some of the philosophers should turn their attention from the subject of spectroscopic investigations and the invention of electrometers, galvanometers, hygremeters, and so forth, to the far more difficult problem of inventing a mode of measuring the intensity and diffusion of political wishes and convictions. No task at present is more difficult for a Statesman than this. There are, indeed, all sorts of shades of difference between the character of really prevalent and preponderant public opinions, of which no man, however acute, ever forms more than a purely conjectural impression, and of which, nevertheless, any respectably-accurate measure would be a matter of the highest political importance. For instance, there is at times a public opinion on one side of a question which is very widely diffused, but of very slight in-tensity,-which, in fact, amounts to nothing more than a wish in a particular direction without a will, and still more without any intention of submitting to a considerable sacrifice rather than not carry out the will into action. Again, there is such a thing as

