Log-Linear Models with Structured Outputs

Natural Language Processing
CS 4120/6120—Spring 2016
Northeastern University

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(some slides from Andrew McCallum)

Overview

- Sequence labeling task (cf. POS tagging)
- Independent classifiers
- HMMs
- (Conditional) Maximum Entropy Markov Models
- Conditional Random Fields
- Beyond Sequence Labeling

Sequence Labeling

- Inputs: $x = (x_1, \ldots, x_n)$
- Labels: $y = (y_1, \ldots, y_n)$
- Typical goal: Given x, predict y

- Example sequence labeling tasks
  - Part-of-speech tagging
  - Named-entity-recognition (NER)
    - Label people, places, organizations

NER Example:

Red Sox and Their Fans Let Loose

Elise Amendola/Associated Press

Fans of the slugger David Ortiz in Boston's Copley Square.

By PETE THAMEL
Published: October 31, 2007

BOSTON, Oct. 30 — Jonathan Papelbon turned Boston's World Series victory parade into a full-scale dance party Tuesday as the Red Sox put an exclamation point on the 2007 season.

E-MAIL
PRINT
REPRINTS
SAVE

First Solution: Maximum Entropy Classifier

- Conditional model $p(y|x)$.
  - Do not waste effort modeling $p(x)$, since x is given at test time anyway.
  - Allows more complicated input features, since we do not need to model dependencies between them.
- Feature functions $f(x,y)$:
  - $f_1(x,y)$ = { word is Boston & y=Location }
  - $f_2(x,y)$ = { first letter capitalized & y=Name }
  - $f_3(x,y)$ = { x is an HTML link & y=Location}

First Solution: MaxEnt Classifier

- How should we choose a classifier?
- Principle of maximum entropy
  - We want a classifier that:
    - Matches feature constraints from training data.
    - Predictions maximize entropy.
- There is a unique, exponential family distribution that meets these criteria.

First Solution: MaxEnt Classifier

- Problem with using a maximum entropy classifier for sequence labeling:
- It makes decisions at each position independently!

Second Solution: HMM

$$P(\mathbf{y},\mathbf{x}) = \prod_t P(y_t \mid y_{t-1}) P(x \mid y_t)$$

- Defines a generative process.
- Can be viewed as a weighted finite state machine.

Second Solution: HMM

- How can represent we multiple features in an HMM?
  - Treat them as conditionally independent given the class label?
    - The example features we talked about are not independent.
  - Try to model a more complex generative process of the input features?
    - We may lose tractability (i.e. lose a dynamic programming for exact inference).

Second Solution: HMM

- Let’s use a conditional model instead.

Third Solution: MEMM

- Use a series of maximum entropy classifiers that know the previous label.
- Define a Viterbi algorithm for inference.

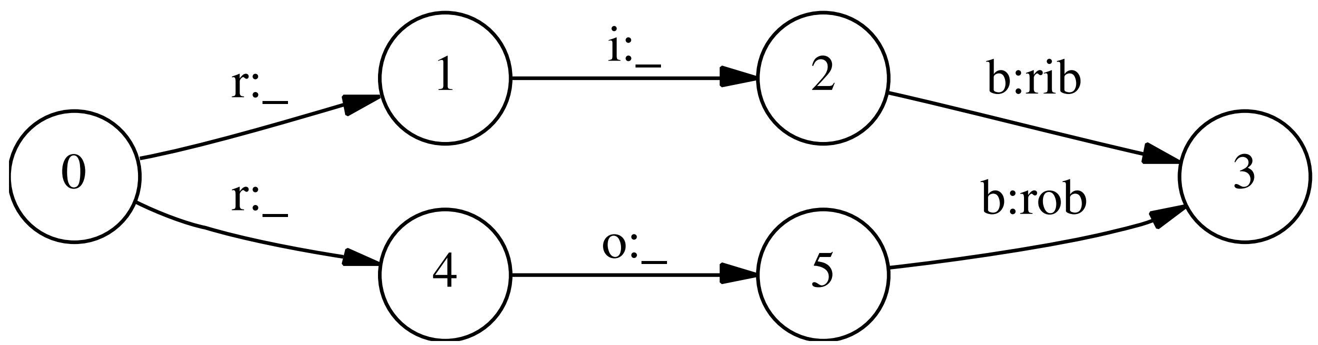
$$P(\mathbf{y} \mid \mathbf{x}) = \prod_t P_{y_{t-1}}(y_t \mid \mathbf{x})$$

Third Solution: MEMM

- Combines the advantages of maximum entropy and HMM!
- But there is a problem…

Problem with MEMMs: Label Bias

- In some state space configurations, MEMMs essentially completely ignore the inputs.



- This is not a problem for HMMs, because the input sequence is generated by the model.

Fourth Solution: Conditional Random Field

- Conditionally-trained, undirected graphical model.
- For a standard linear-chain structure:

$$P(\mathbf{y} \mid \mathbf{x}) = \prod_t \Psi_k(y_t, y_{t-1}, \mathbf{x})$$

$$\Psi_k(y_t, y_{t-1}, \mathbf{x}) = \exp\left(\sum_k \lambda_k f(y_t, y_{t-1}, \mathbf{x}) \right)$$

Fourth Solution: CRF

- Have the advantages of MEMMs, but avoid the label bias problem.
- CRFs are globally normalized, whereas MEMMs are locally normalized.
- Widely used and applied. CRFs give state-the-art results in many domains.

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Remember, Z is the normalization constant. How do we compute it?

CRF Applications

- Part-of-speech tagging
- Named entity recognition
- Document layout (e.g. table) classification
- Gene prediction
- Chinese word segmentation
- Morphological disambiguation
- Citation parsing
- Etc., etc.

NER as Sequence Tagging

The Phoenicians came from the Red Sea

NER as Sequence Tagging

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

NER as Sequence Tagging

Capitalized word

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

NER as Sequence Tagging

Capitalized word

Ends in “s”

O	B-E	O	O	O	B-L	I-L
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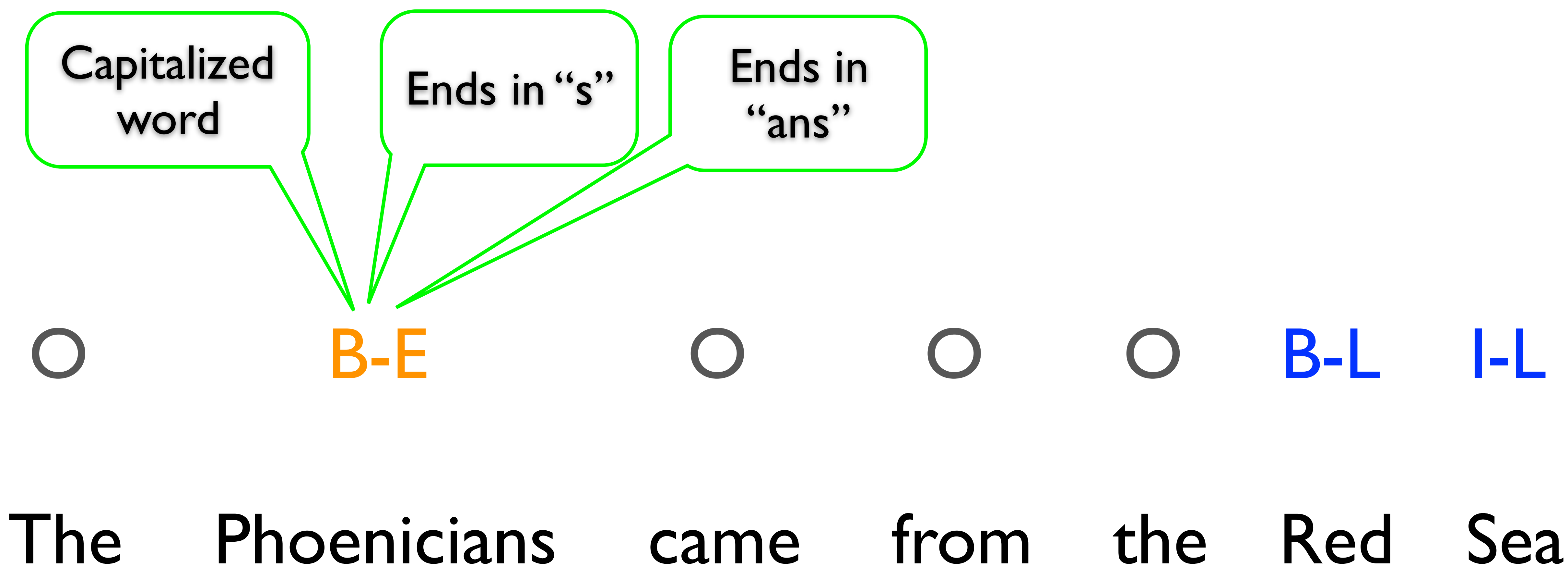
NER as Sequence Tagging



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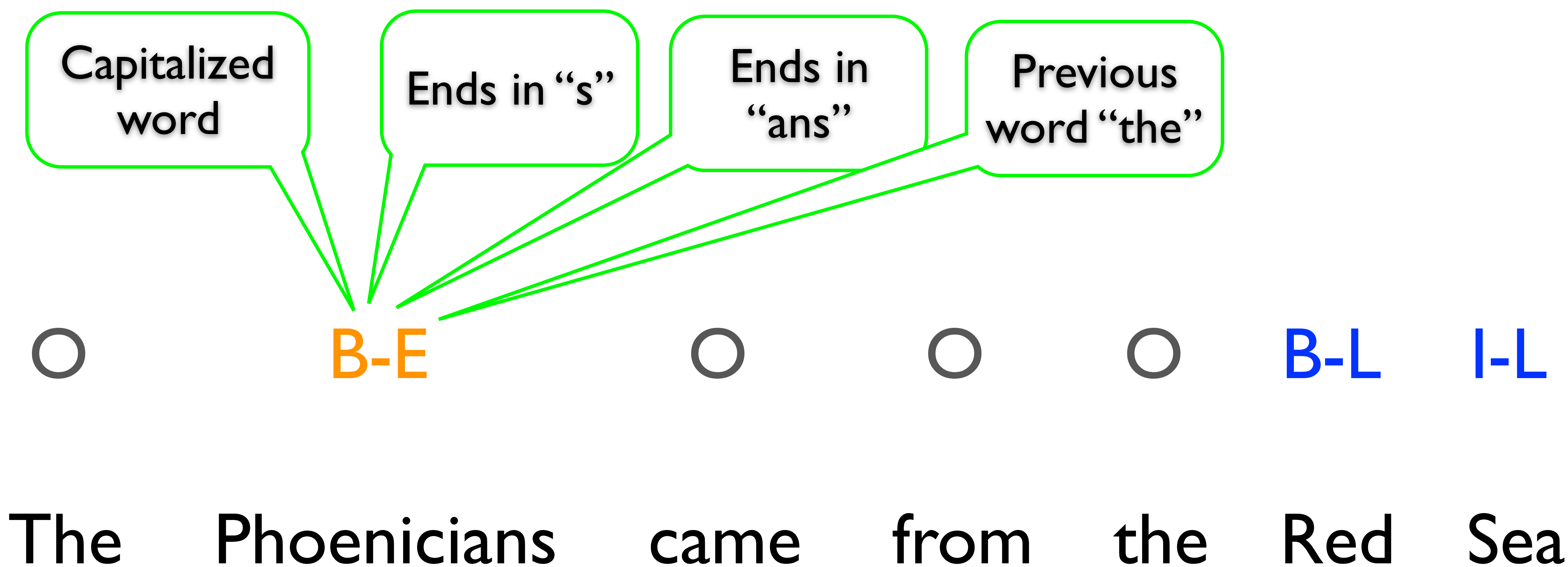

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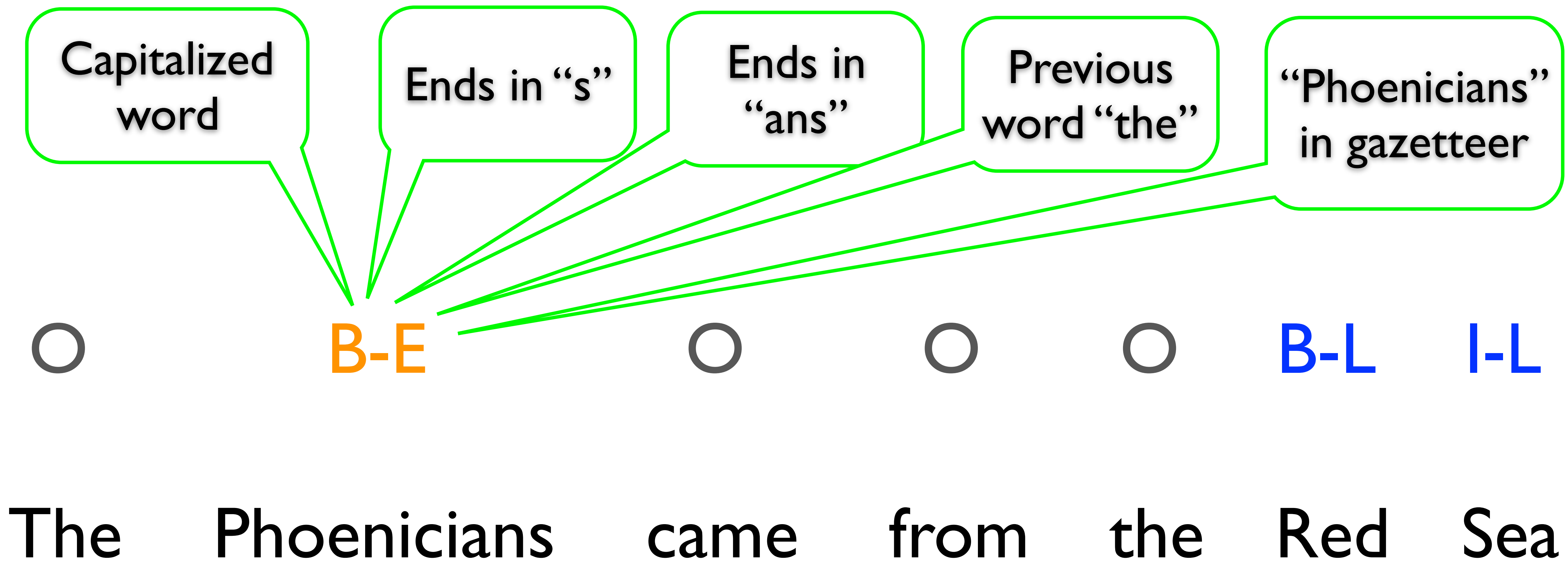

NER as Sequence Tagging

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

NER as Sequence Tagging

Not capitalized

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

NER as Sequence Tagging

Not capitalized

Tagged as "VB"

O	B-E	O	O	O	B-L	I-L
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NER as Sequence Tagging

B-E to right

Not capitalized

Tagged as "VB"

O	B-E	O	O	O	B-L	I-L
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NER as Sequence Tagging

O	B-E	O	O	O	B-L	I-L
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NER as Sequence Tagging

Word “sea”

O	B-E	O	O	O	B-L	I-L
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NER as Sequence Tagging

Word "sea" preceded by "the ADJ"

Word "sea"

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

NER as Sequence Tagging

Hard constraint: I-L must follow B-L or I-L

Word "sea" preceded by "the ADJ"

Word "sea"

O	B-E	O	O	O	B-L	I-L
The	Phoenicians	came	from	the	Red	Sea

Overview

- What computations do we need?
- Smoothing log-linear models
- MEMMs vs. CRFs again
  - Action-based parsing and dependency parsing

Recipe for Conditional Training of p(y | x)

1. Gather constraints/features from training data

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero
3. Classify training data with current parameters; calculate expectations $$E_\Theta[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_\Theta(y'|x_j) f_{iy}(x_j, y')$$
4. Gradient is $\tilde{E}[f_{iy}] - E_\Theta[f_{iy}]$
5. Take a step in the direction of the gradient
6. Repeat from 3 until convergence

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EM!

Where have we seen expected counts before?

Gradient-Based Training

- $\lambda := \lambda + \text{rate} * \text{Gradient}(F)$
- After all training examples? (batch)
- After every example? (on-line)
- Use second derivative for faster learning?
- A big field: numerical optimization

Parsing as Structured Prediction

Shift-reduce parsing

Stack	Input remaining	Action
()	Book that flight	shift
(Book)	that flight	reduce, Verb $\rightarrow$ book, (Choice #1 of 2)
(Verb)	that flight	shift
(Verb that)	flight	reduce, Det $\rightarrow$ that
(Verb Det)	flight	shift
(Verb Det flight)		reduce, Noun $\rightarrow$ flight
(Verb Det Noun)		reduce, NOM $\rightarrow$ Noun
(Verb Det NOM)		reduce, NP $\rightarrow$ Det NOM
(Verb NP)		reduce, VP $\rightarrow$ Verb NP
(Verb)		reduce, S $\rightarrow$ V
(S)		SUCCESS!

Ambiguity may lead to the need for backtracking.

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Train log-linear model of p(action | context)

Compare to an MEMM

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Train log-linear model of p(action | context)

Structured Log-Linear Models

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- Linear model for scoring structures

$$\mathrm{score}(out, in) = \theta \cdot \mathbf{features}(out, in)$$

Structured Log-Linear Models

- Linear model for scoring structures
- Get a probability distribution by normalizing

$$\text{score}(out, in) = \theta \cdot \mathbf{features}(out, in)$$

$$p(out \mid in) = \frac{1}{Z} e^{score(out,in)} \quad Z = \sum_{out' \in GEN(in)} e^{score(out',in)}$$

Structured Log-Linear Models

- Linear model for scoring structures
- Get a probability distribution by normalizing
  - Viz. logistic regression, Markov random fields, undirected graphical models

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Usually the bottleneck in NLP

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- Inference: sampling, variational methods, dynamic programming, local search, ...

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Structured Log-Linear Models

- Linear model for scoring structures
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  - Viz. logistic regression, Markov random fields, undirected graphical models
- Inference: sampling, variational methods, dynamic programming, local search, ...
- Training: maximum likelihood, minimum risk, etc.

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Structured Log-Linear Models

With latent variables

- Several layers of linguistic structure
- Unknown correspondences
- Naturally handled by probabilistic framework
- Several inference setups, for example:

$$p(out_1 \mid in) = \sum_{out_2, alignment} p(out_1, out_2, alignment \mid in)$$

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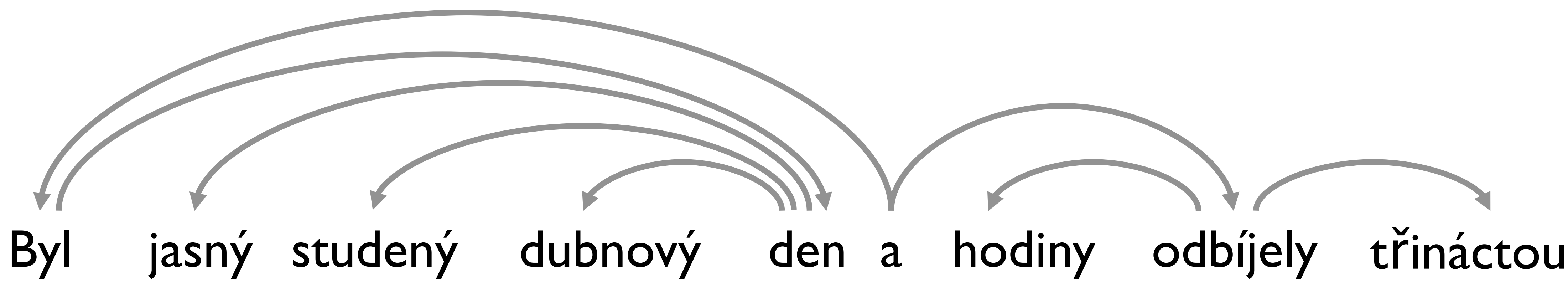
Another computational problem

Edge-Factored Parsers

- No <u>global</u> features of a parse *(McDonald et al. 2005)*
- Each feature is attached to some edge
- *MST or CKY-like DP for fast $O(n^2)$ or $O(n^3)$ parsing*



"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers

- Is this a good edge?

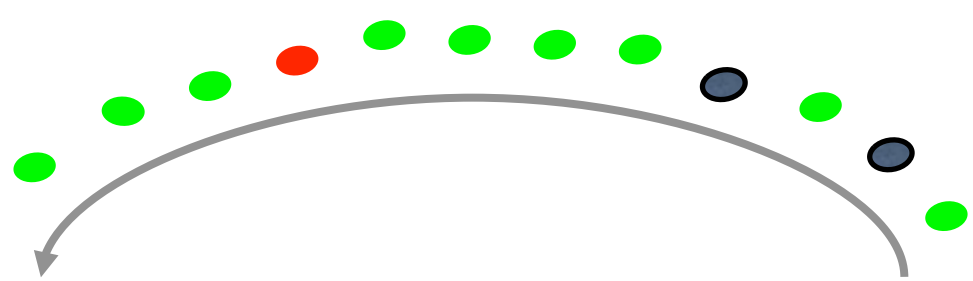
Byl jasný studený dubnový den a hodiny odbíjely třináctou

“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

- Is this a good edge?

yes, lots of positive features ...



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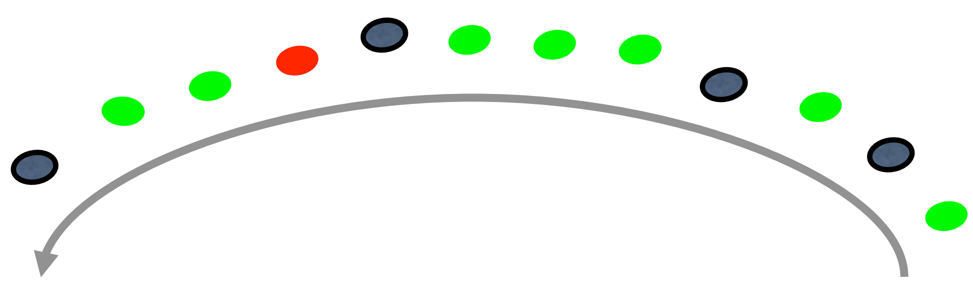

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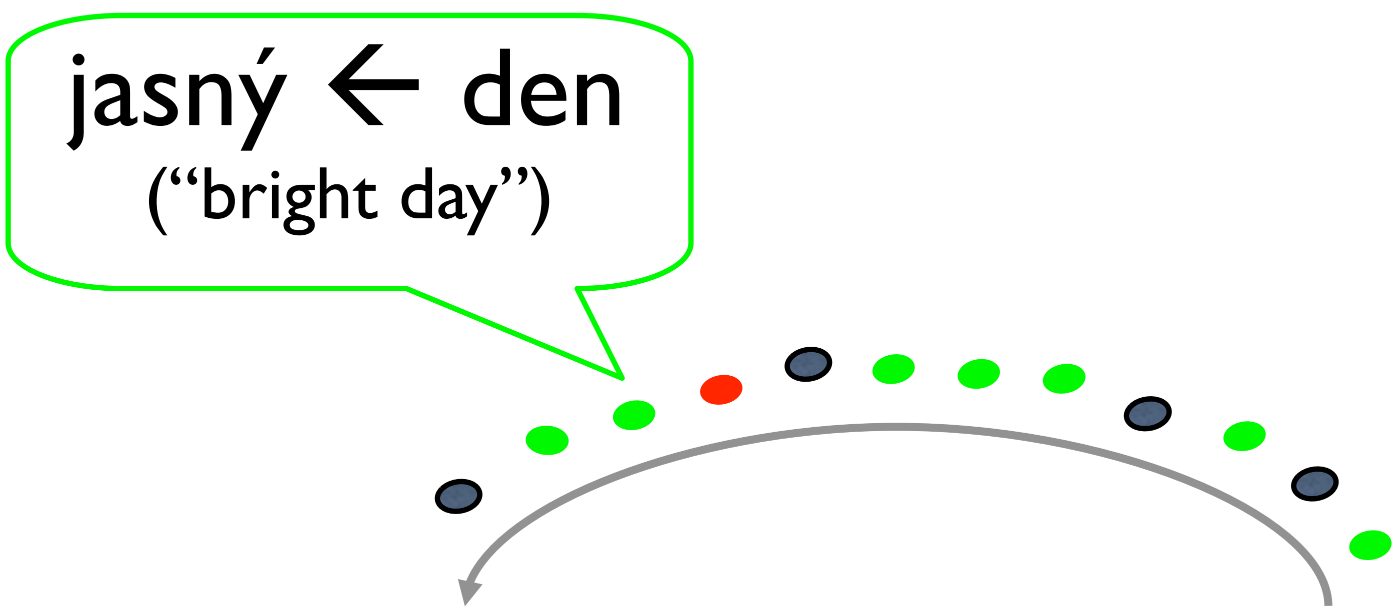

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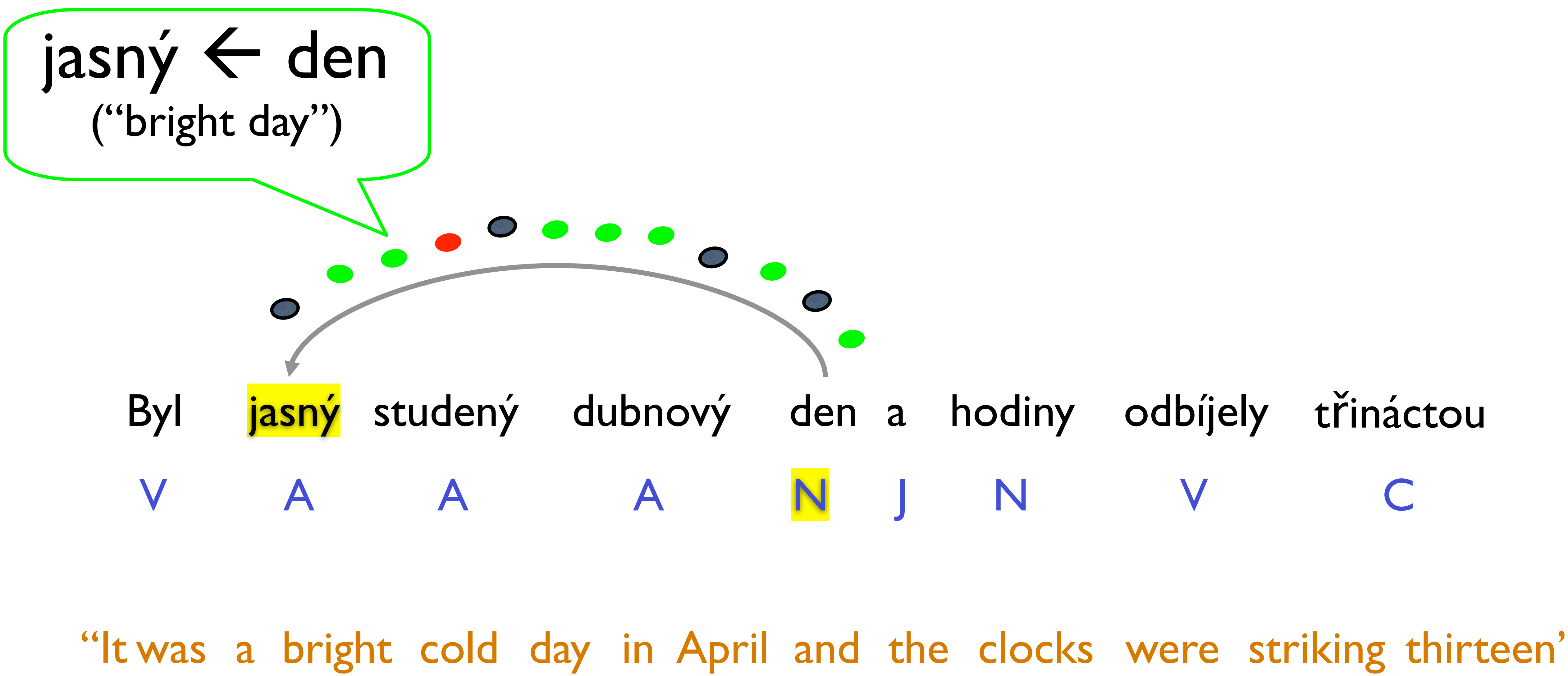

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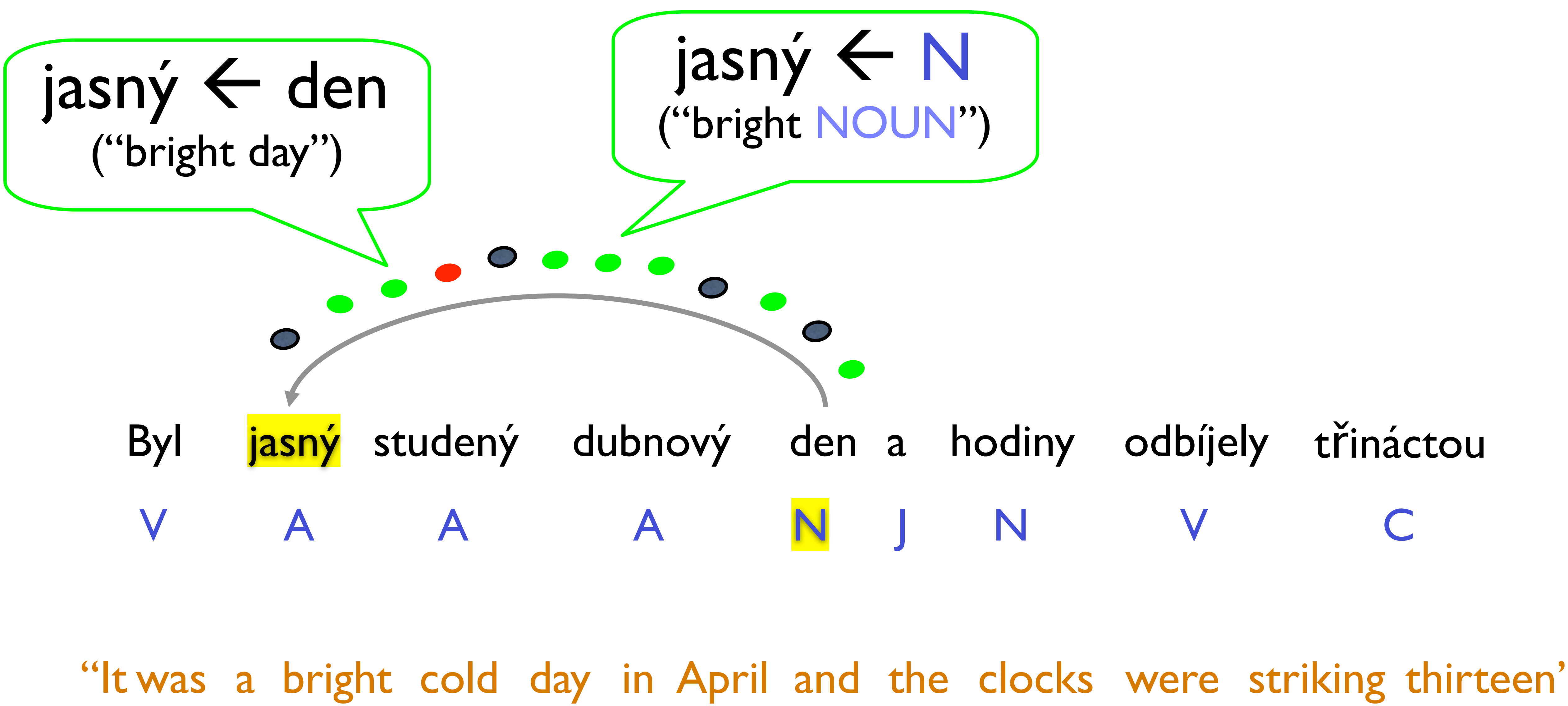

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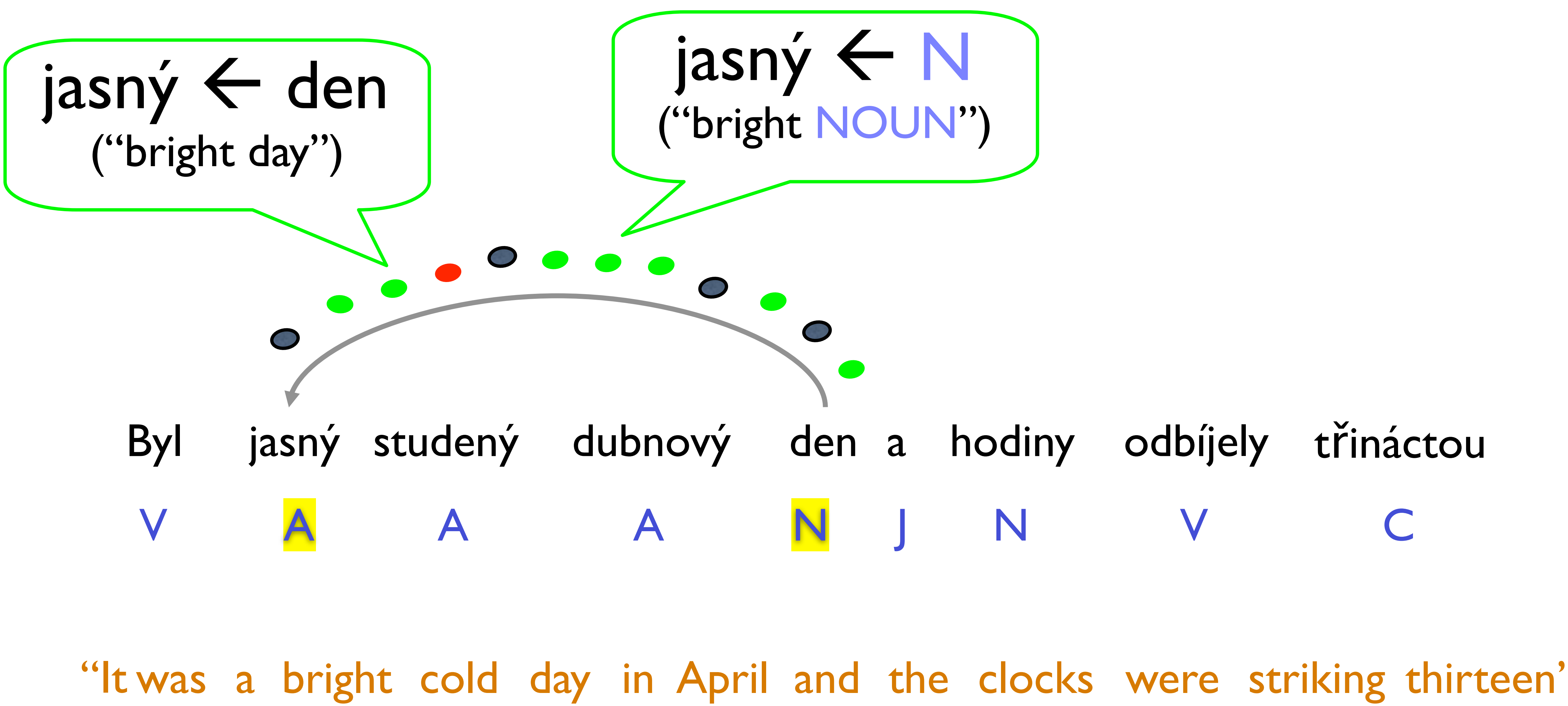

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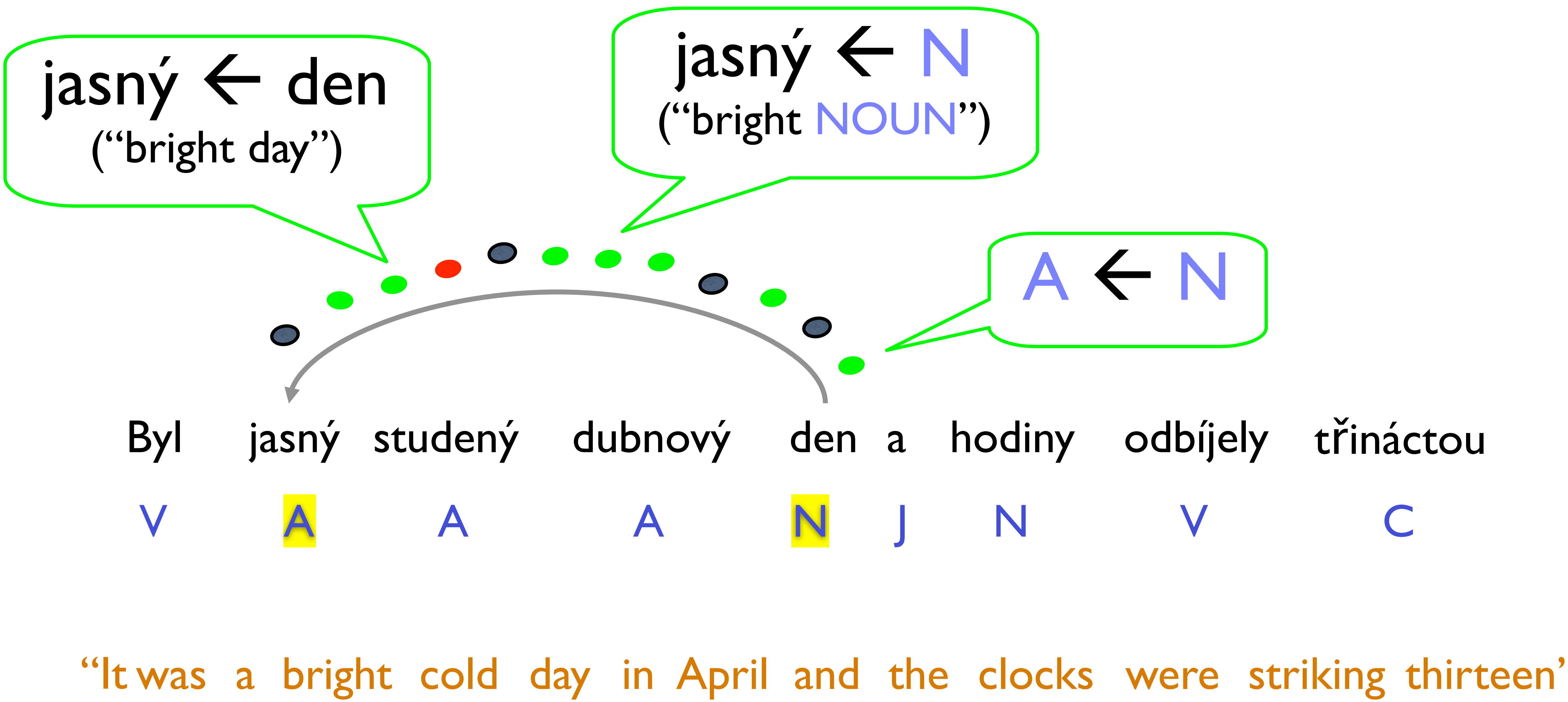

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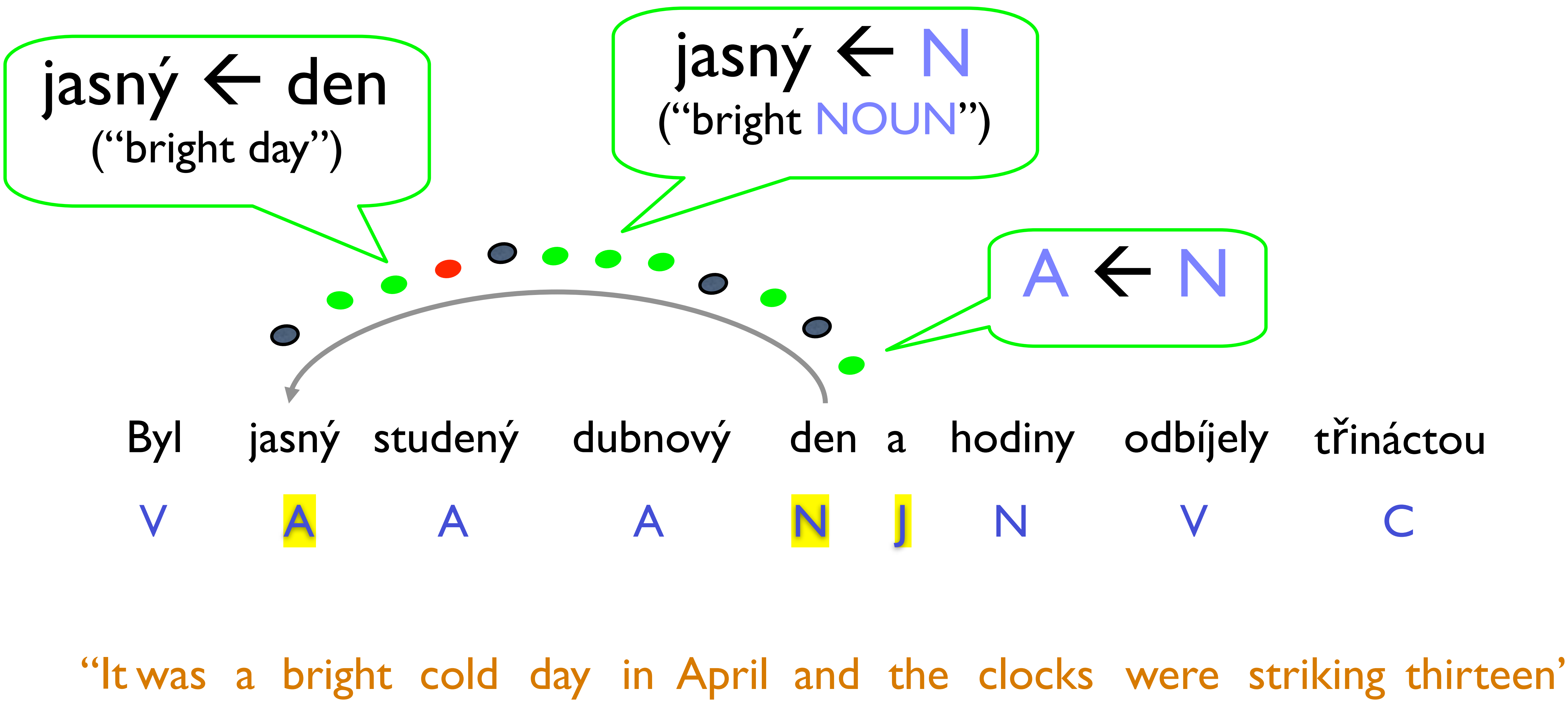

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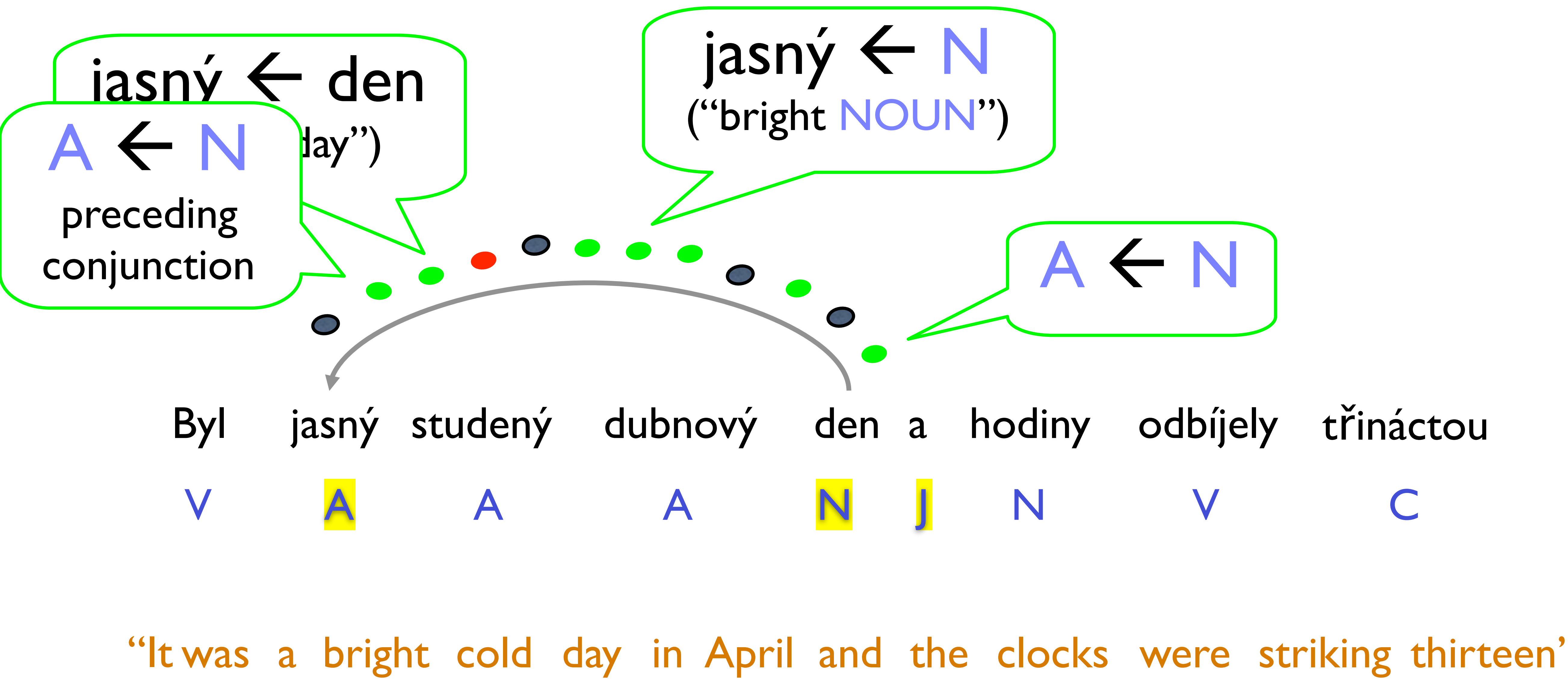


Edge-Factored Parsers

- How about this competing edge?

Byl jasný studený dubnový den a hodiny odbíjely třináctou

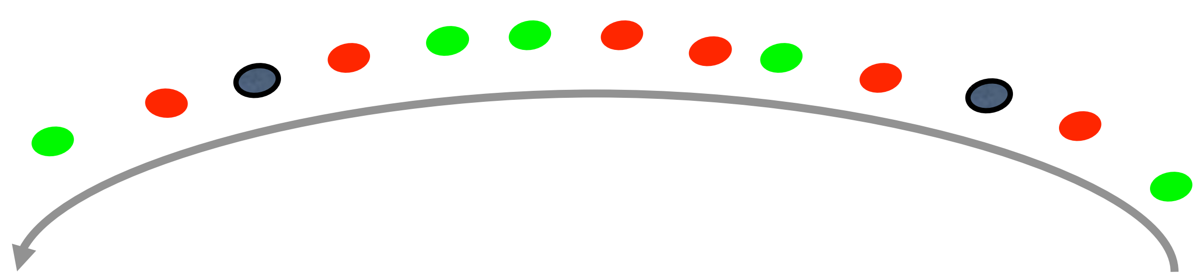
V A A A N J N V C

"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers

- How about this competing edge?

not as good, lots of red ...



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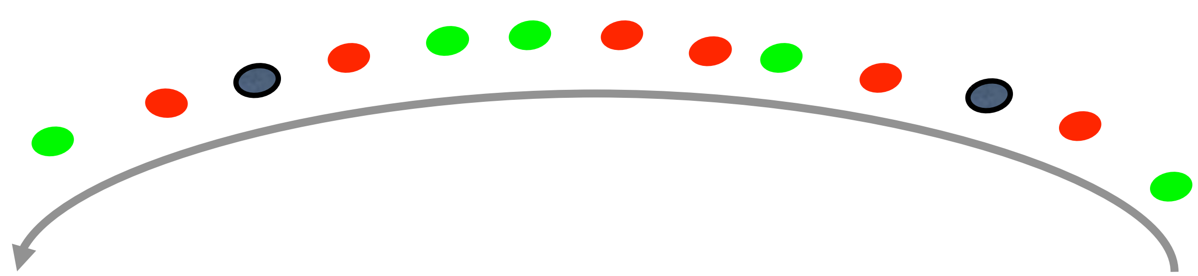

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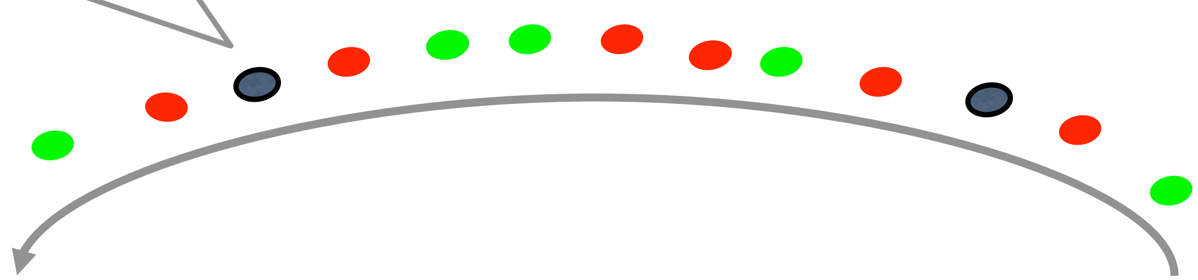
jasný ← hodiny
("bright clocks")



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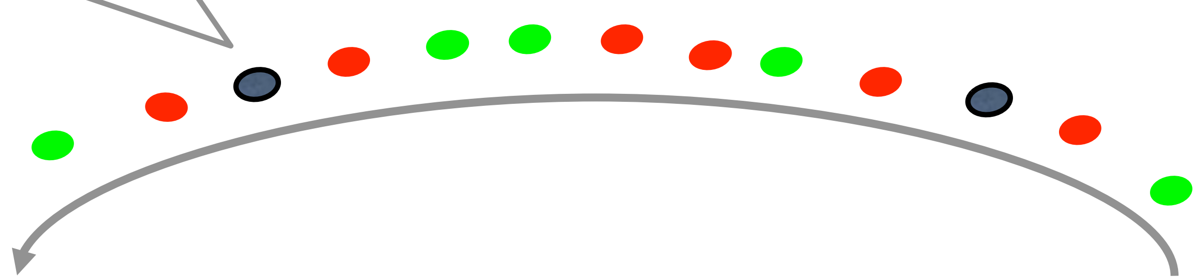
... undertrained ...



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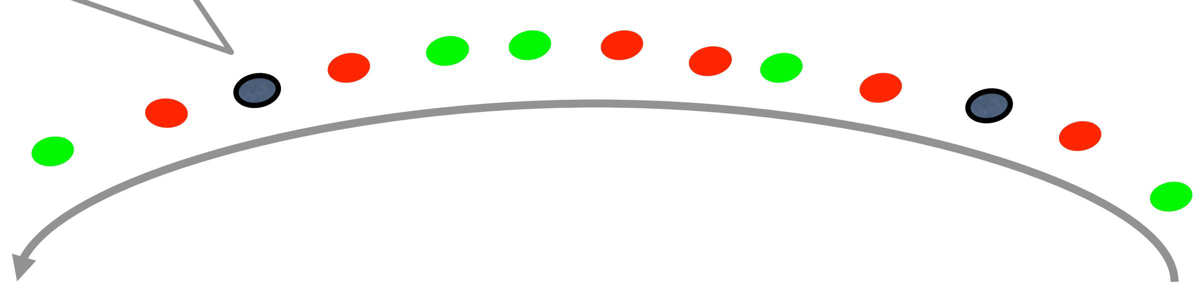

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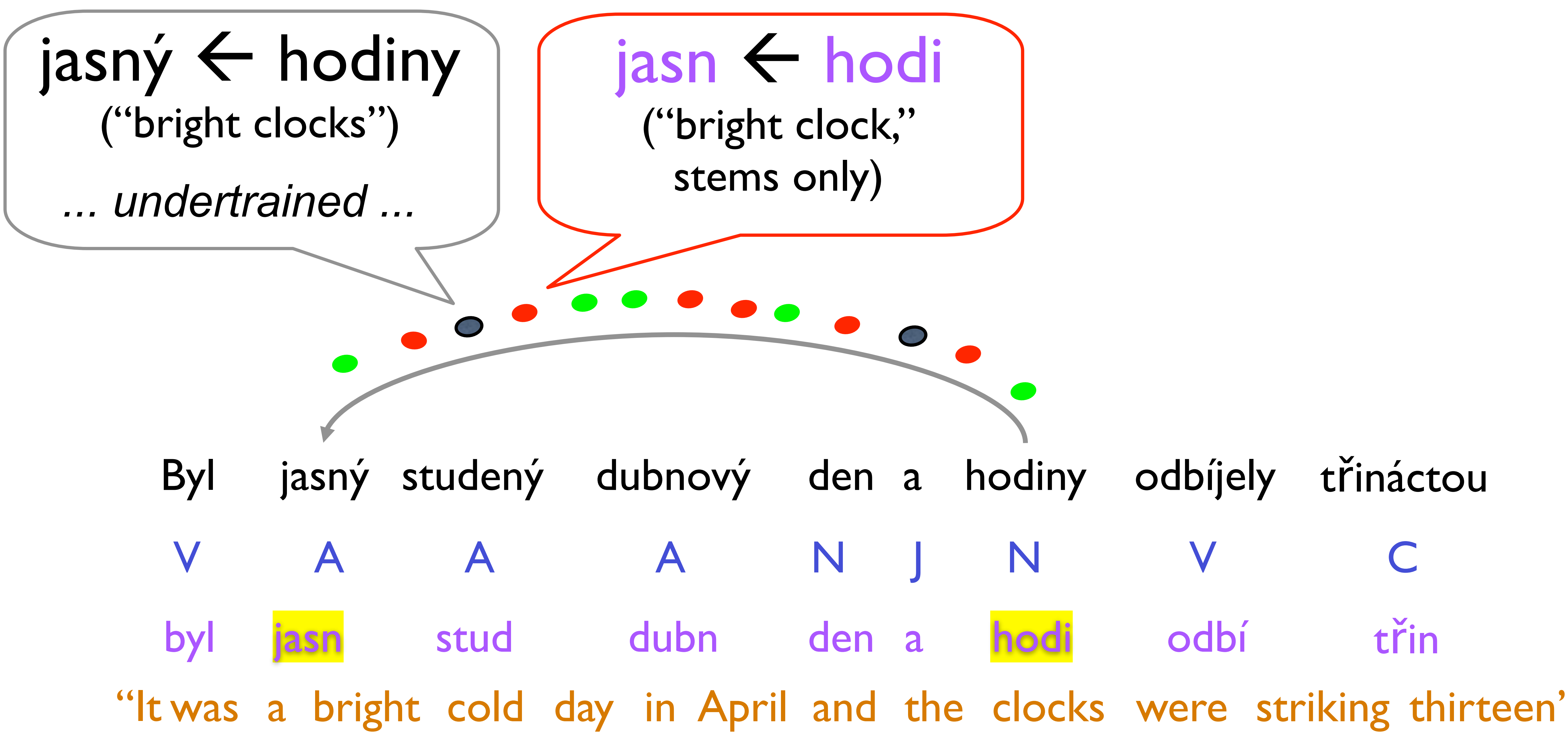

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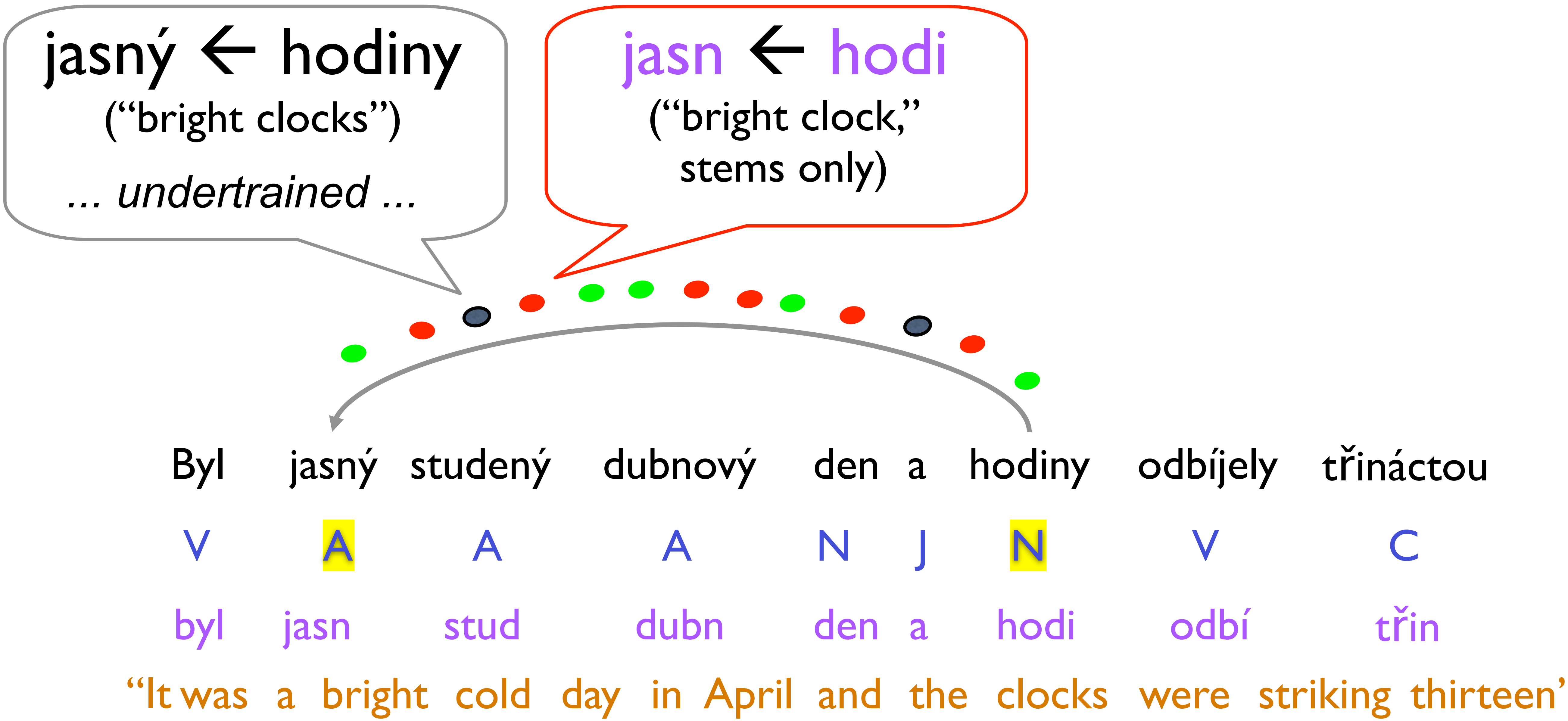


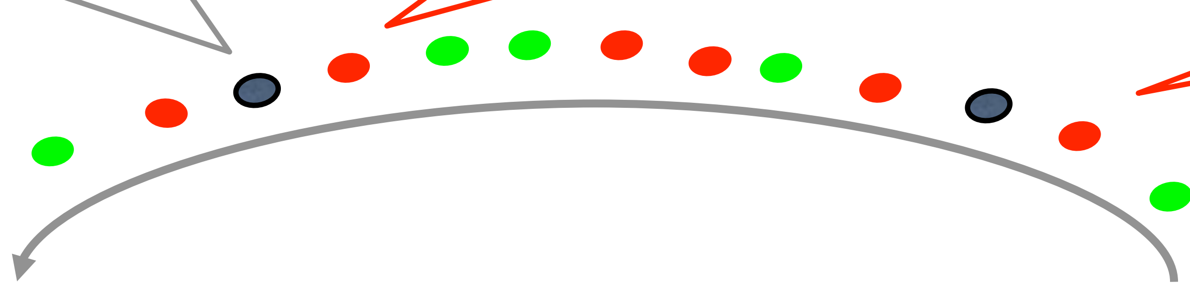
Edge-Factored Parsers

- How about this competing edge?

jasný ← hodiny
("bright clocks")
... undertrained ...

jasn ← hodi
("bright clock,"
stems only)

$A_{singular}$ ← N_{plural}



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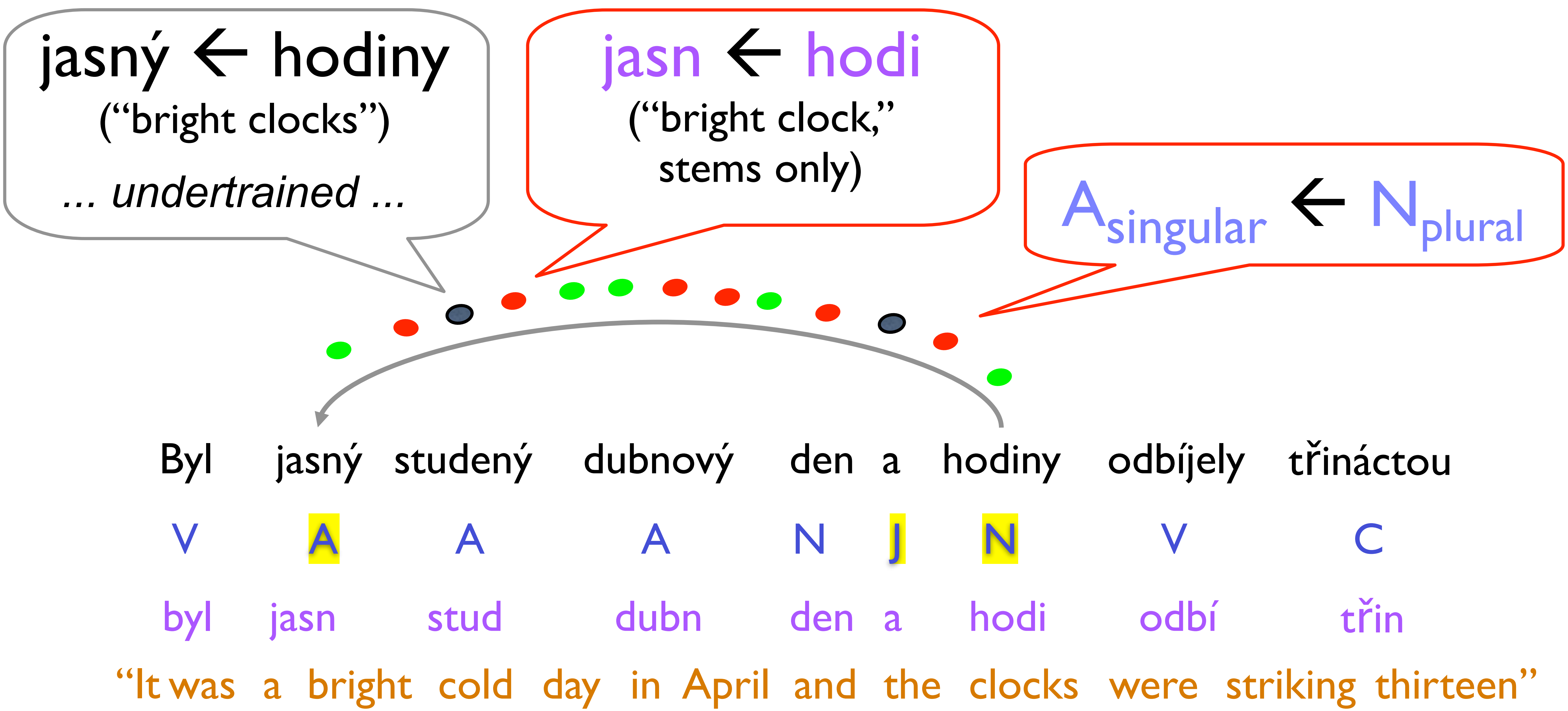

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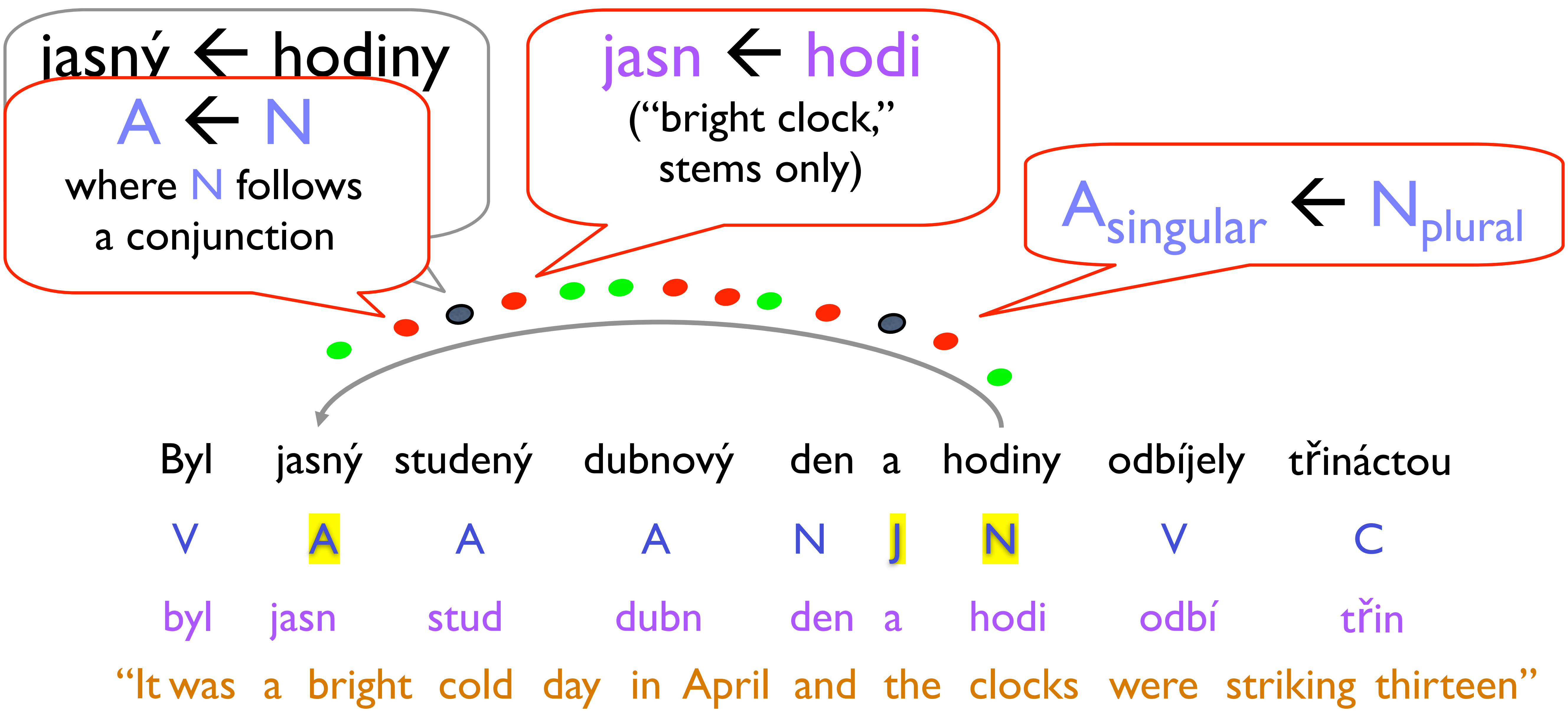
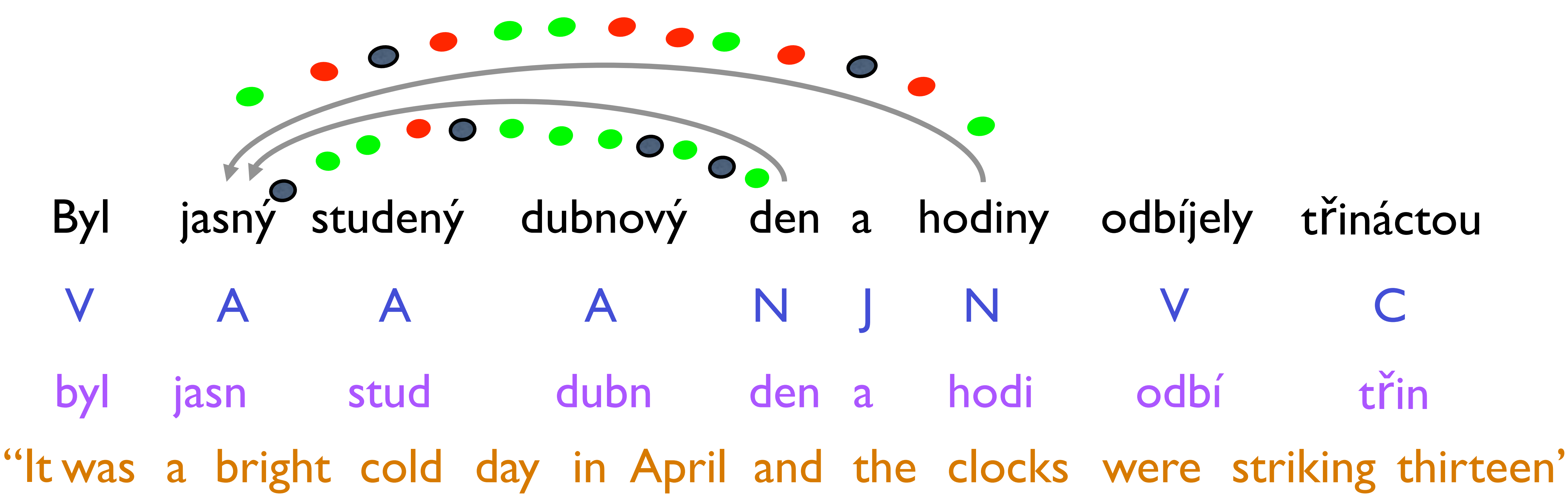


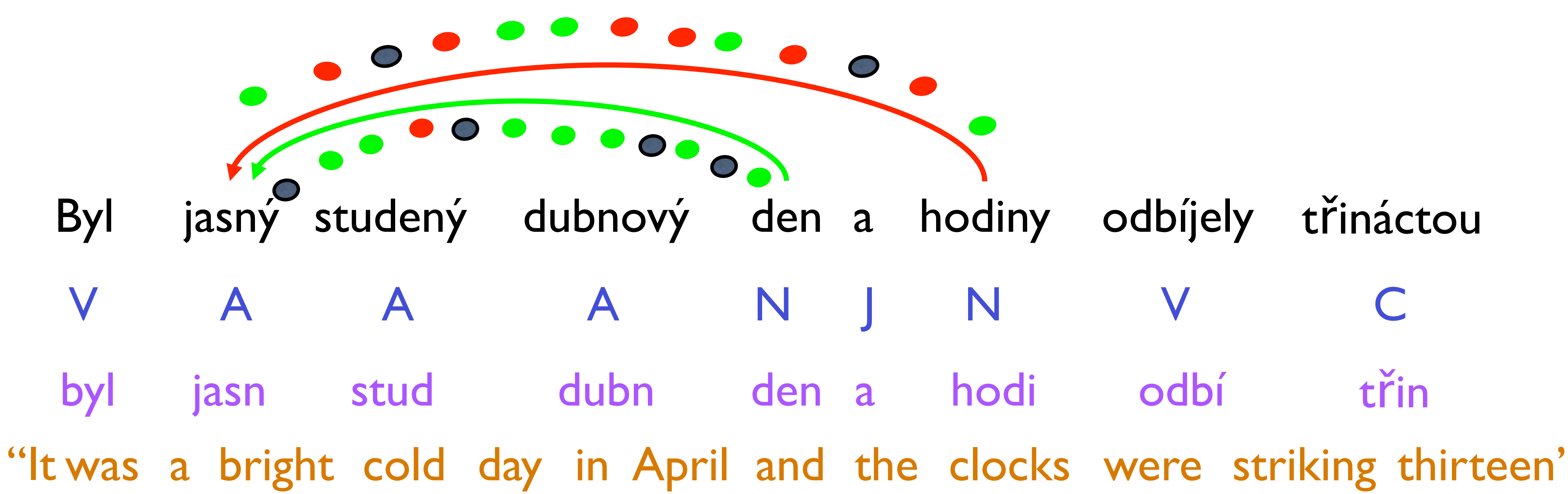
Edge-Factored Parsers

- Which edge is better?
  - "bright day" or "bright clocks"?



Edge-Factored Parsers

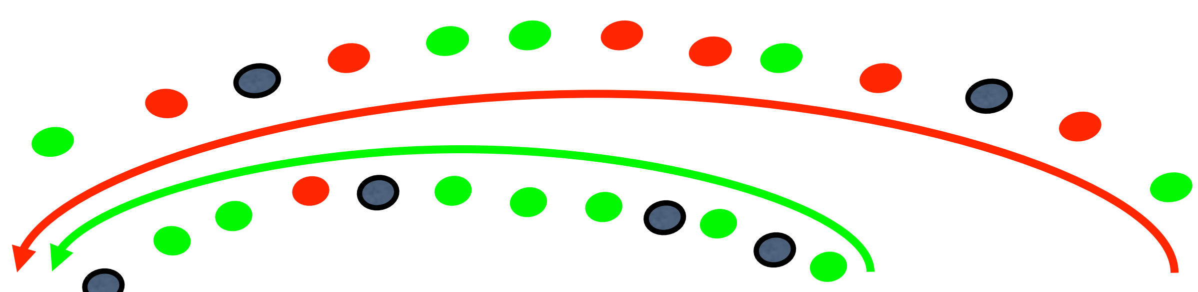
- Which edge is better?
- Score of an edge e = $\theta \cdot$ **features**(e)
- Standard algos ➔ valid parse with max <u>total</u> score



Edge-Factored Parsers

our current weight vector

- Which edge is better?
- Score of an edge e = $\theta \cdot$ **features**(e)
- Standard algos ➔ valid parse with max <u>total</u> score



Byl jasný studený dubnový den a hodiny odbíjely třináctou

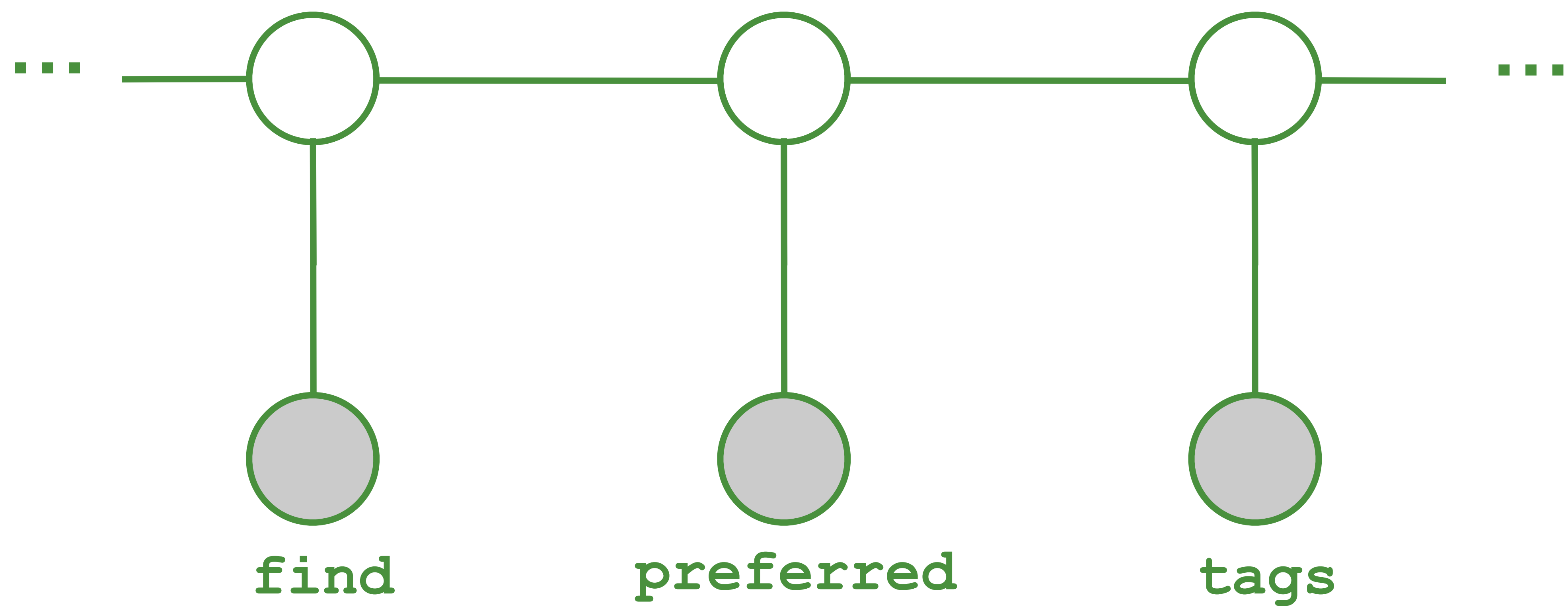
V A A A N J N V C

byl jasn stud dubn den a hodi odbí třin

"It was a bright cold day in April and the clocks were striking thirteen"

Local factors in a graphical model

- First, a familiar example
  - Conditional Random Field (CRF) for POS tagging



Local factors in a graphical model

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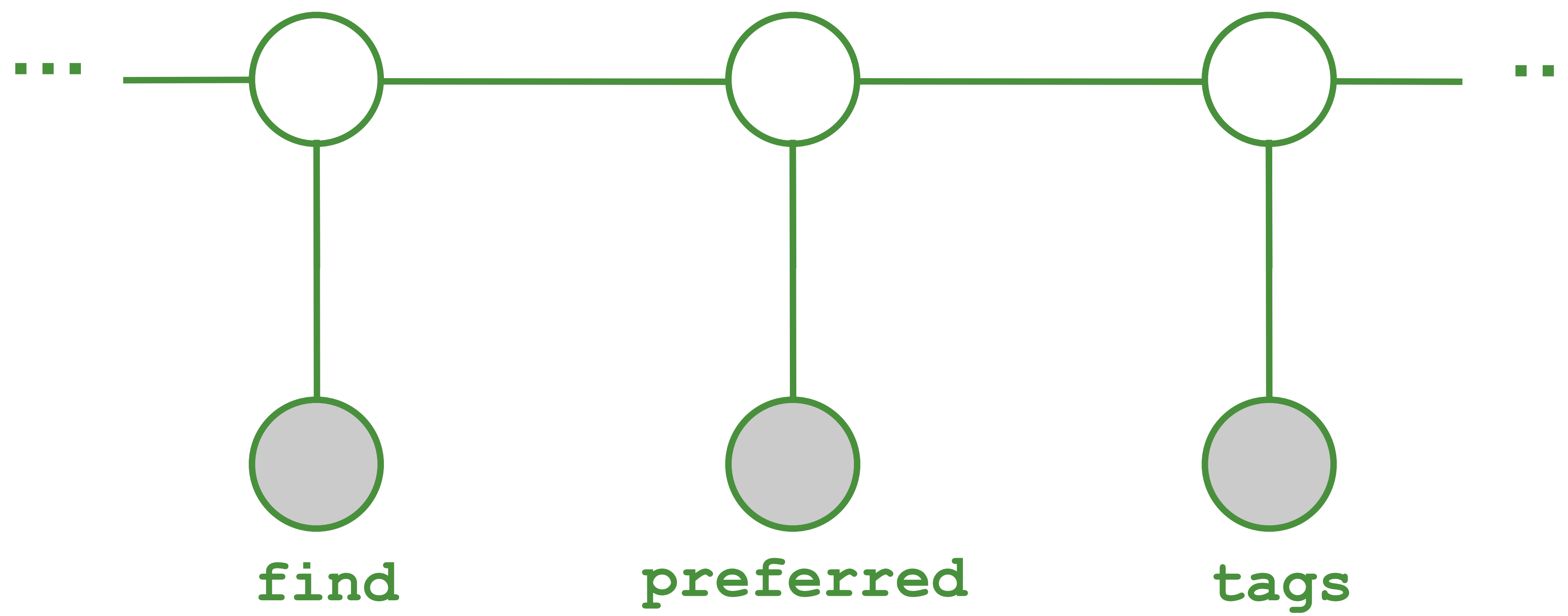

Observed input sentence (shaded)

Local factors in a graphical model

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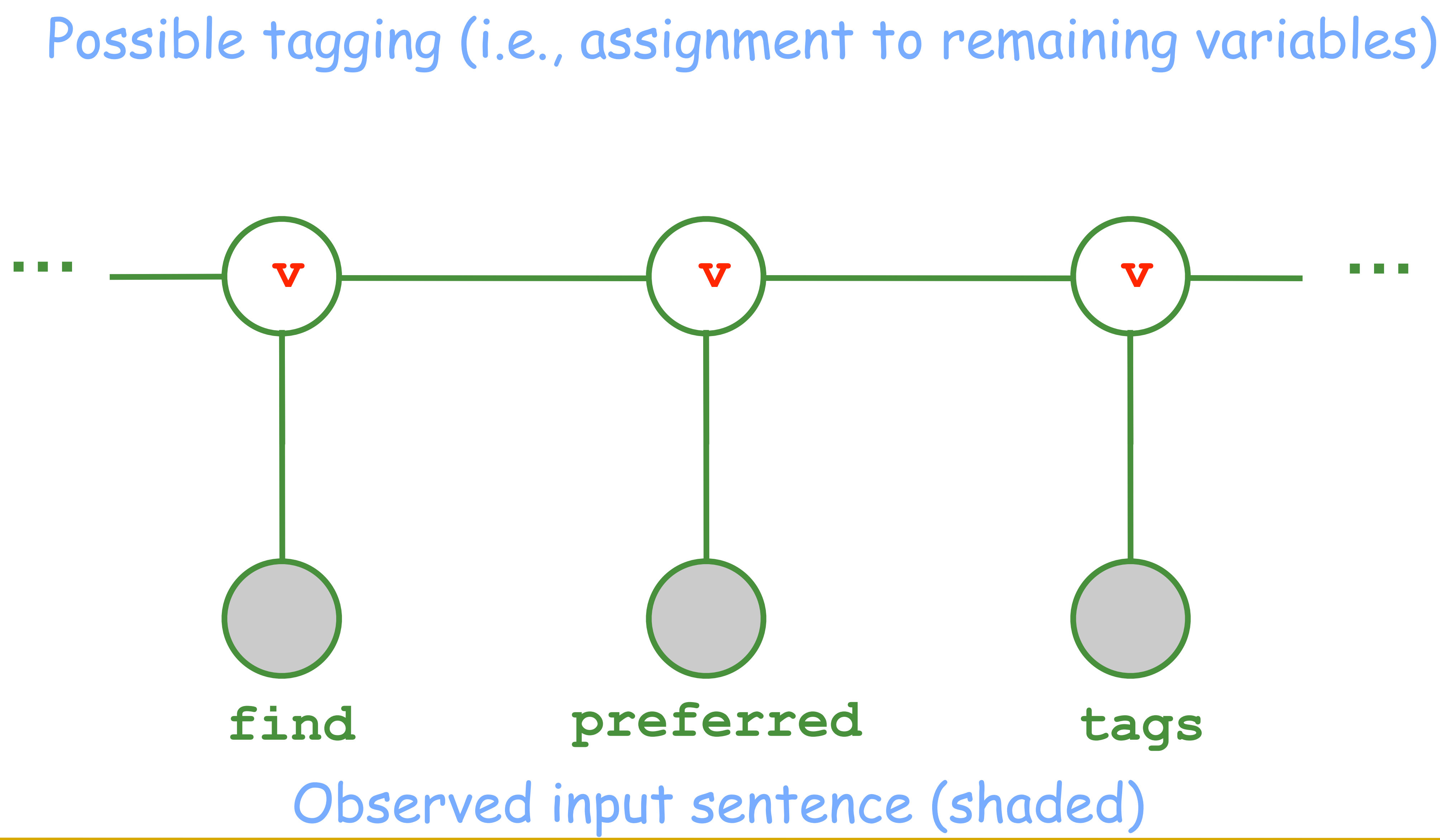
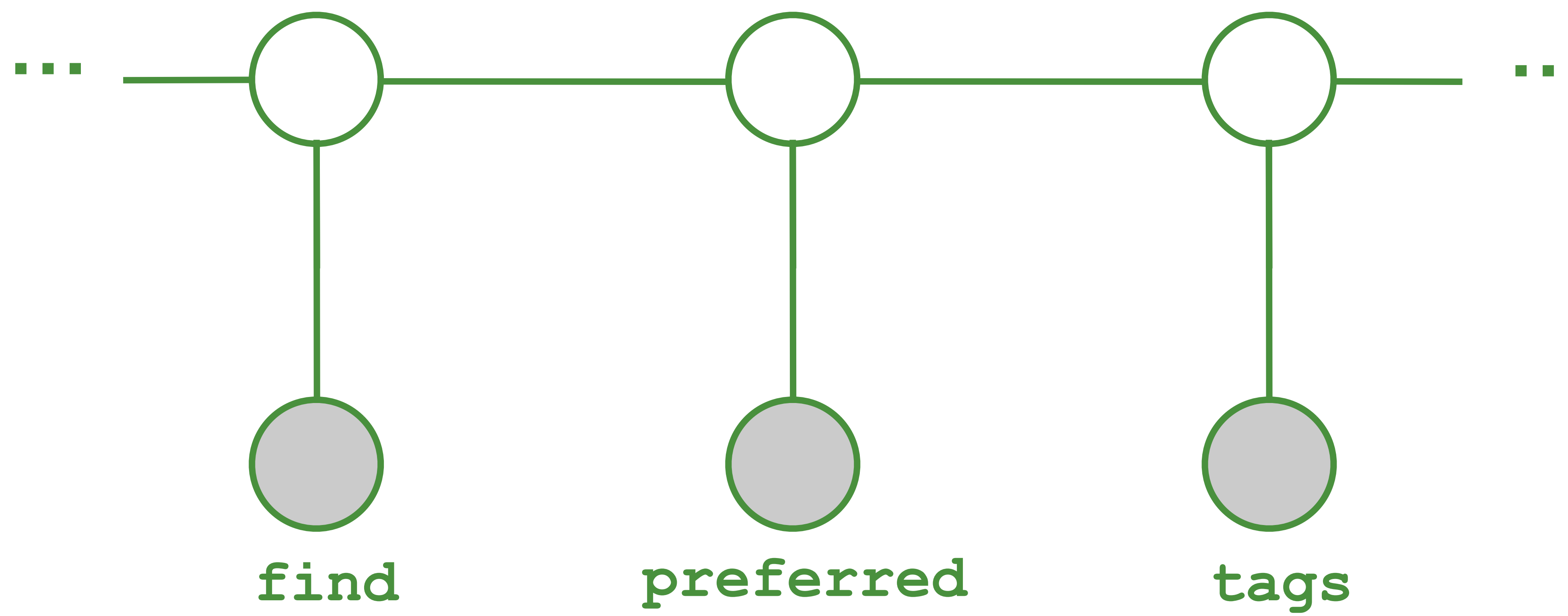


Local factors in a graphical model

- First, a familiar example
  - Conditional Random Field (CRF) for POS tagging

Possible tagging (i.e., assignment to remaining variables)
Another possible tagging



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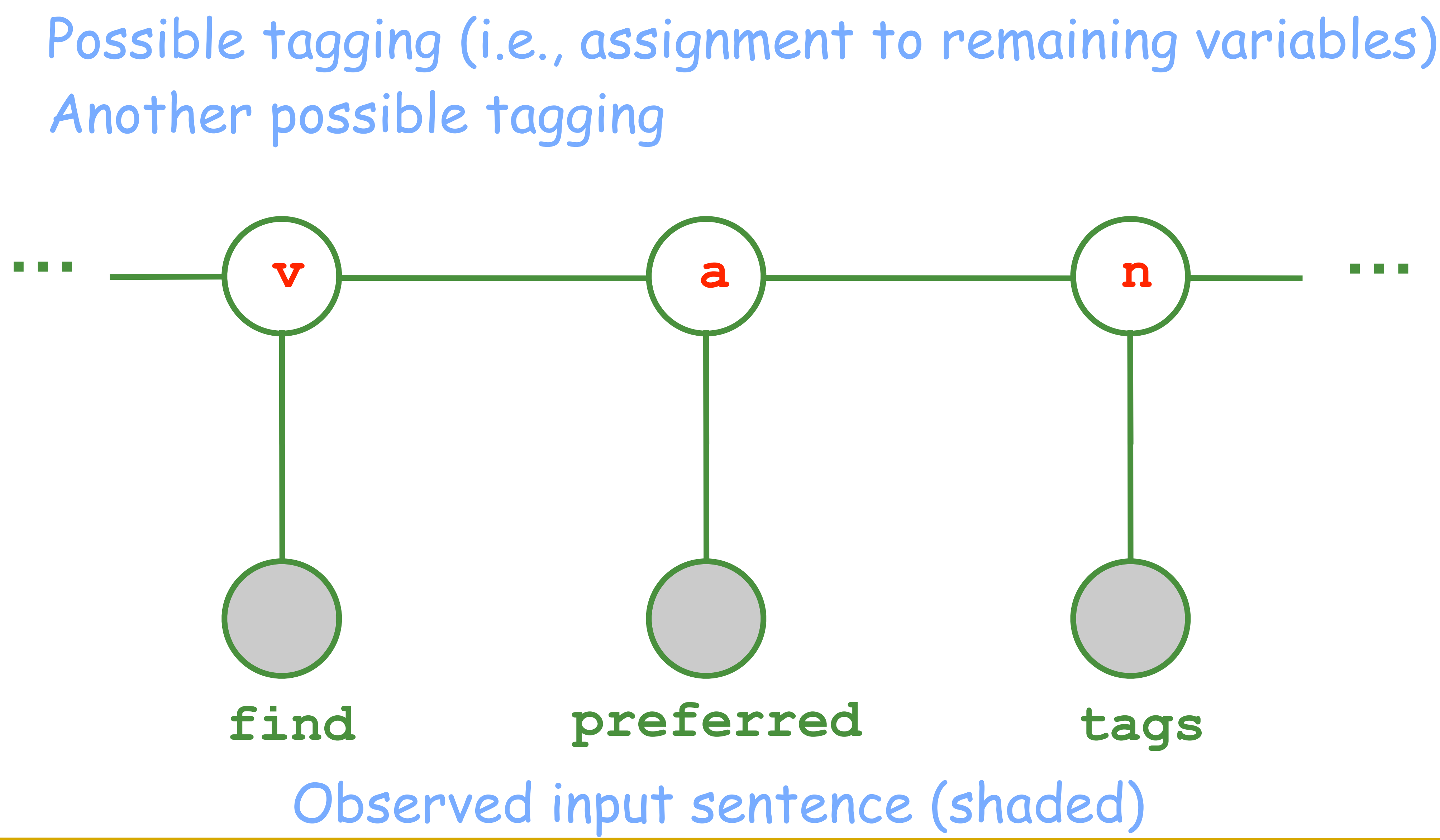

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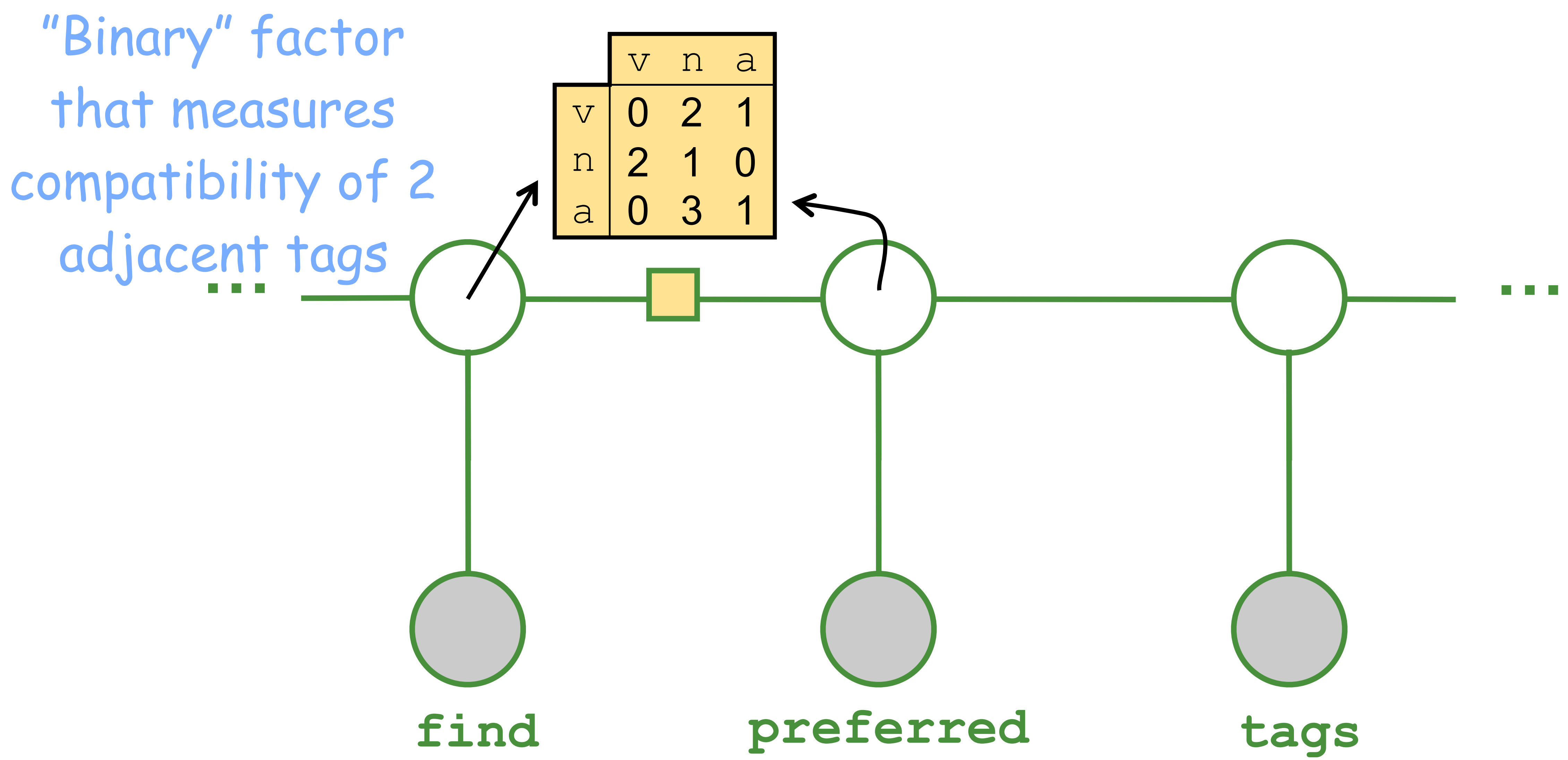

Local factors in a graphical model

- First, a familiar example
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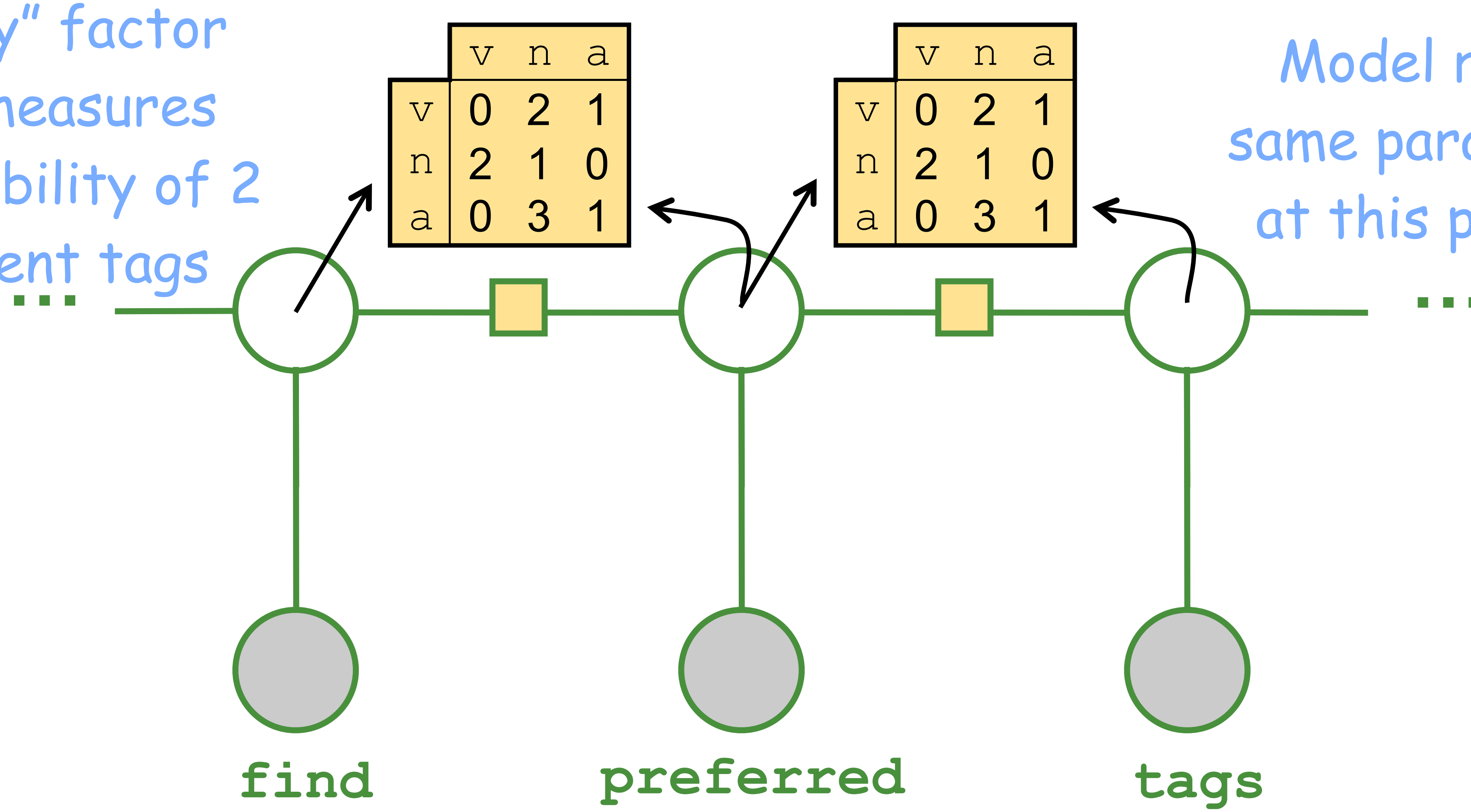
"Binary" factor that measures compatibility of 2 adjacent tags



Model reuses same parameters at this position

Local factors in a graphical model

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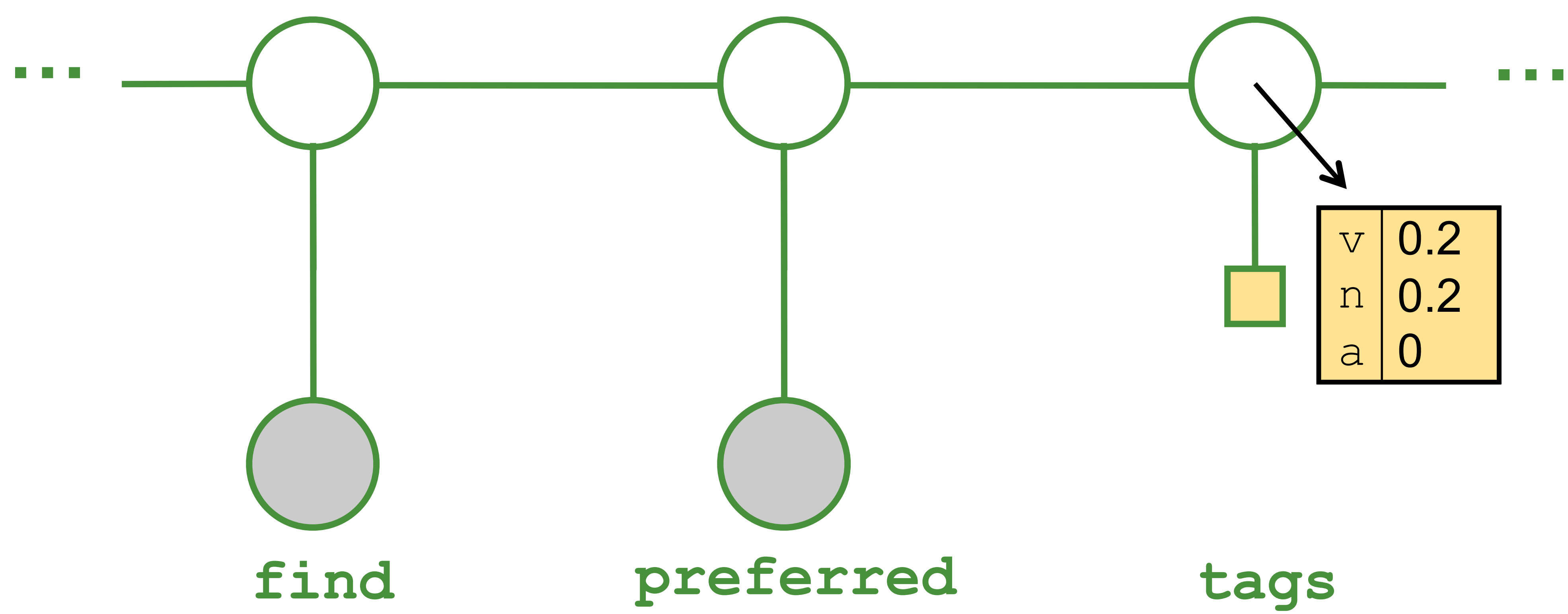

Local factors in a graphical model

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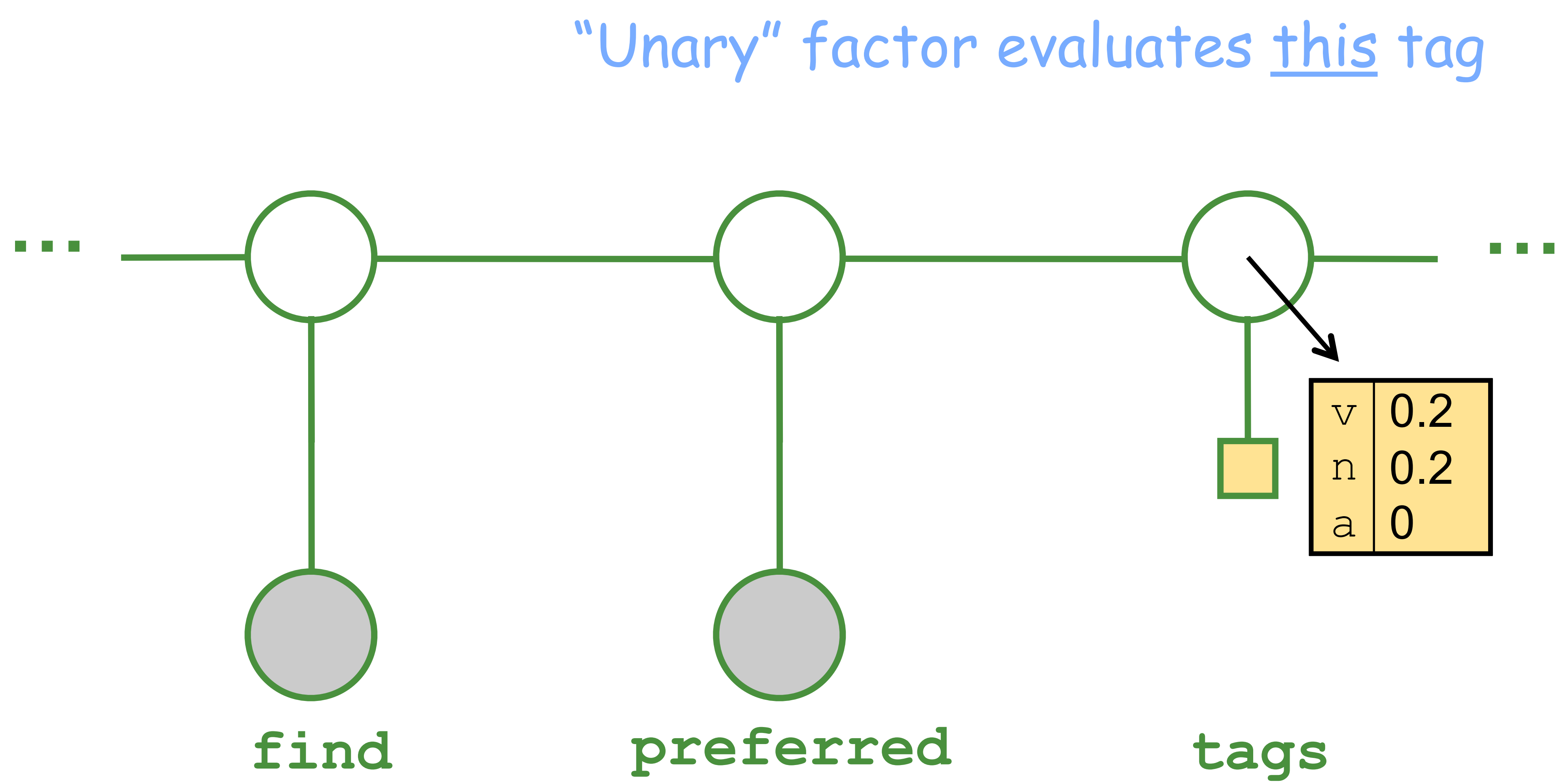

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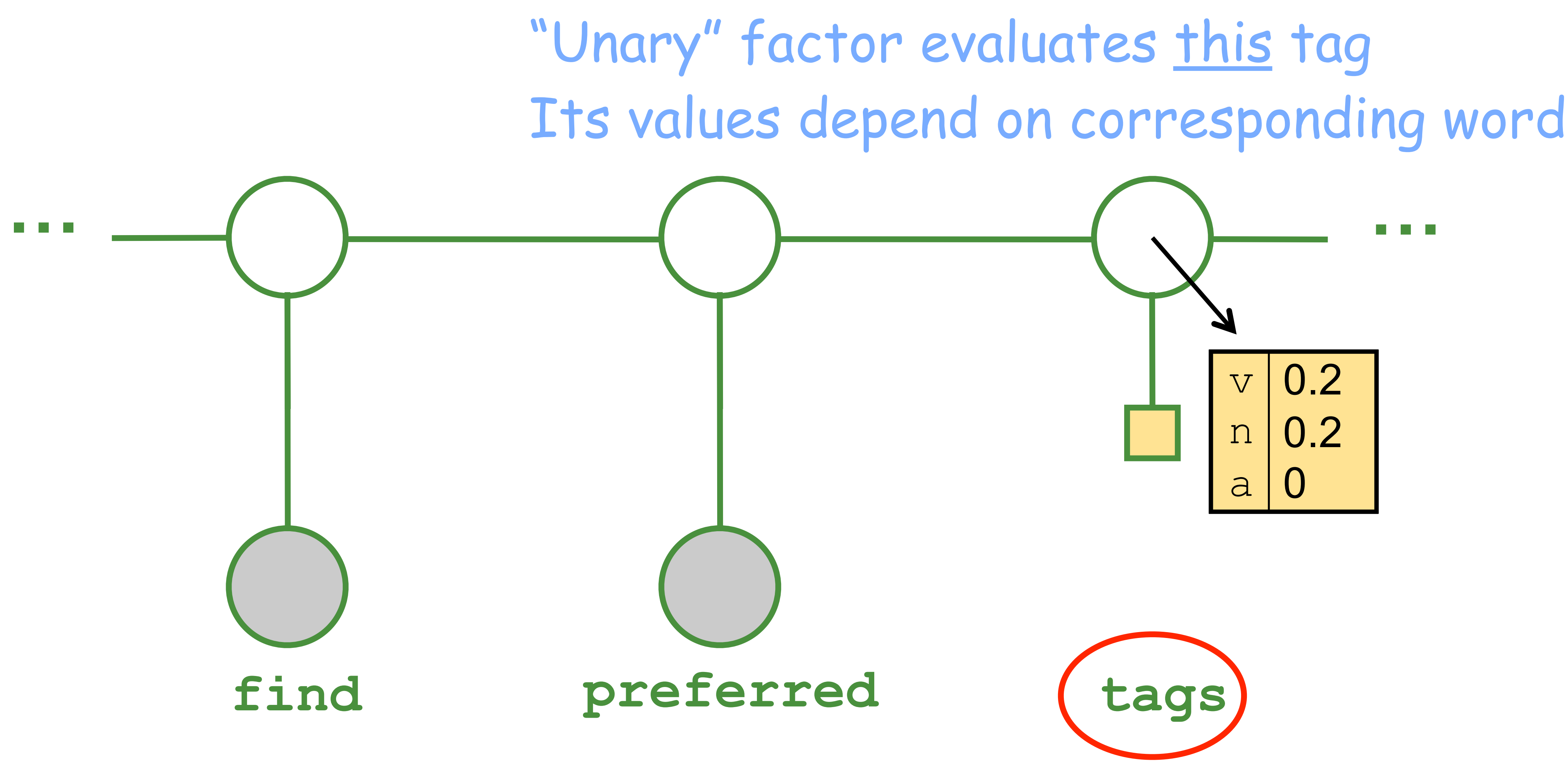

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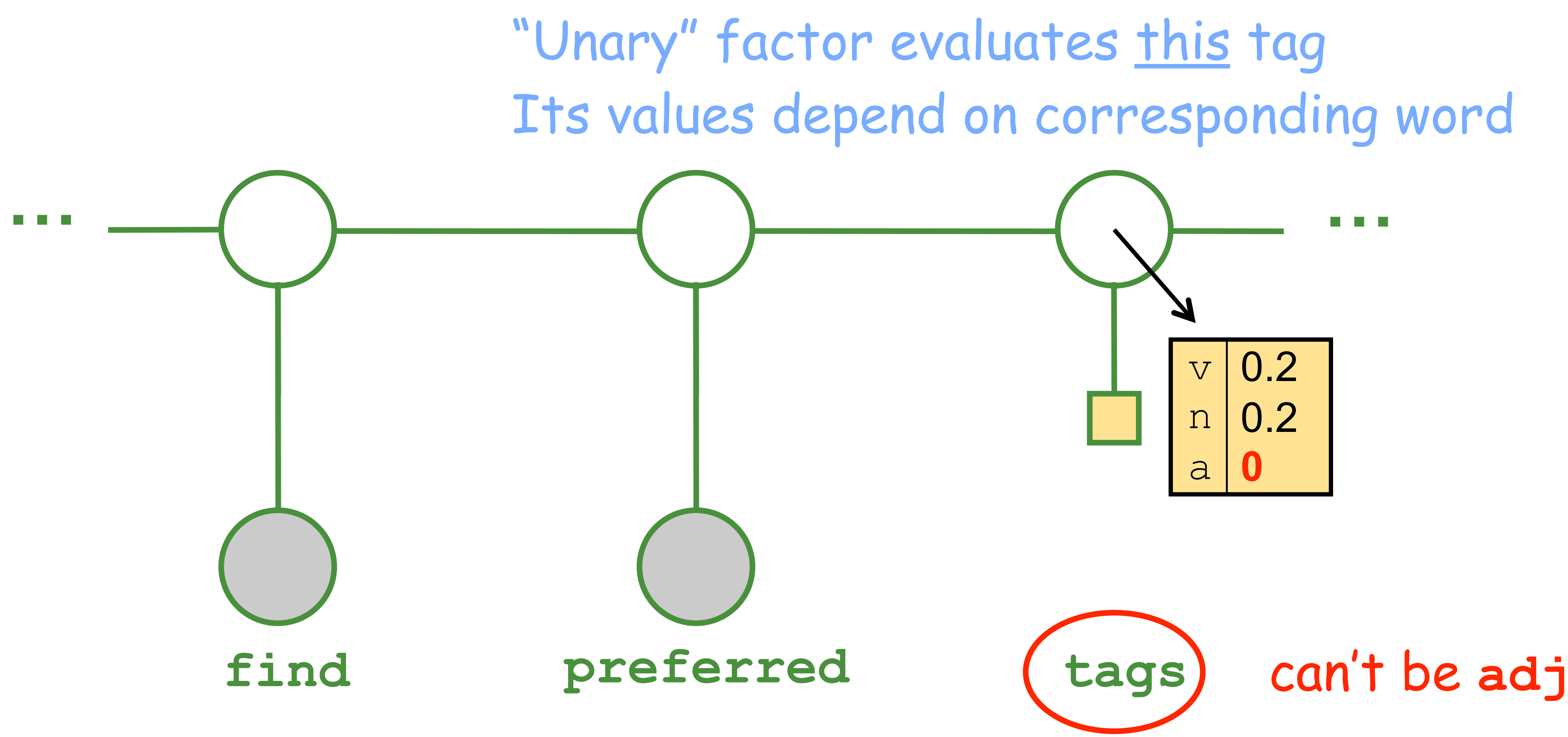

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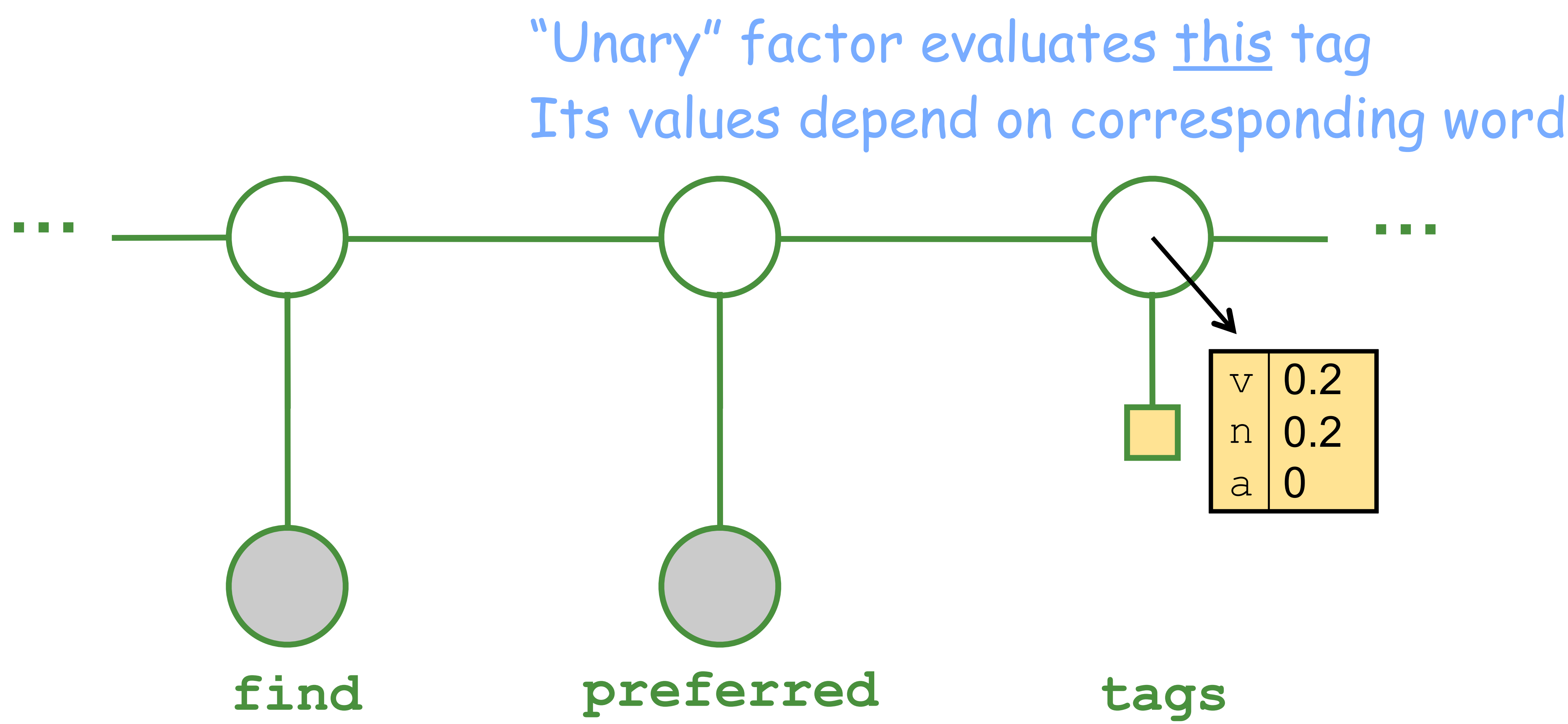

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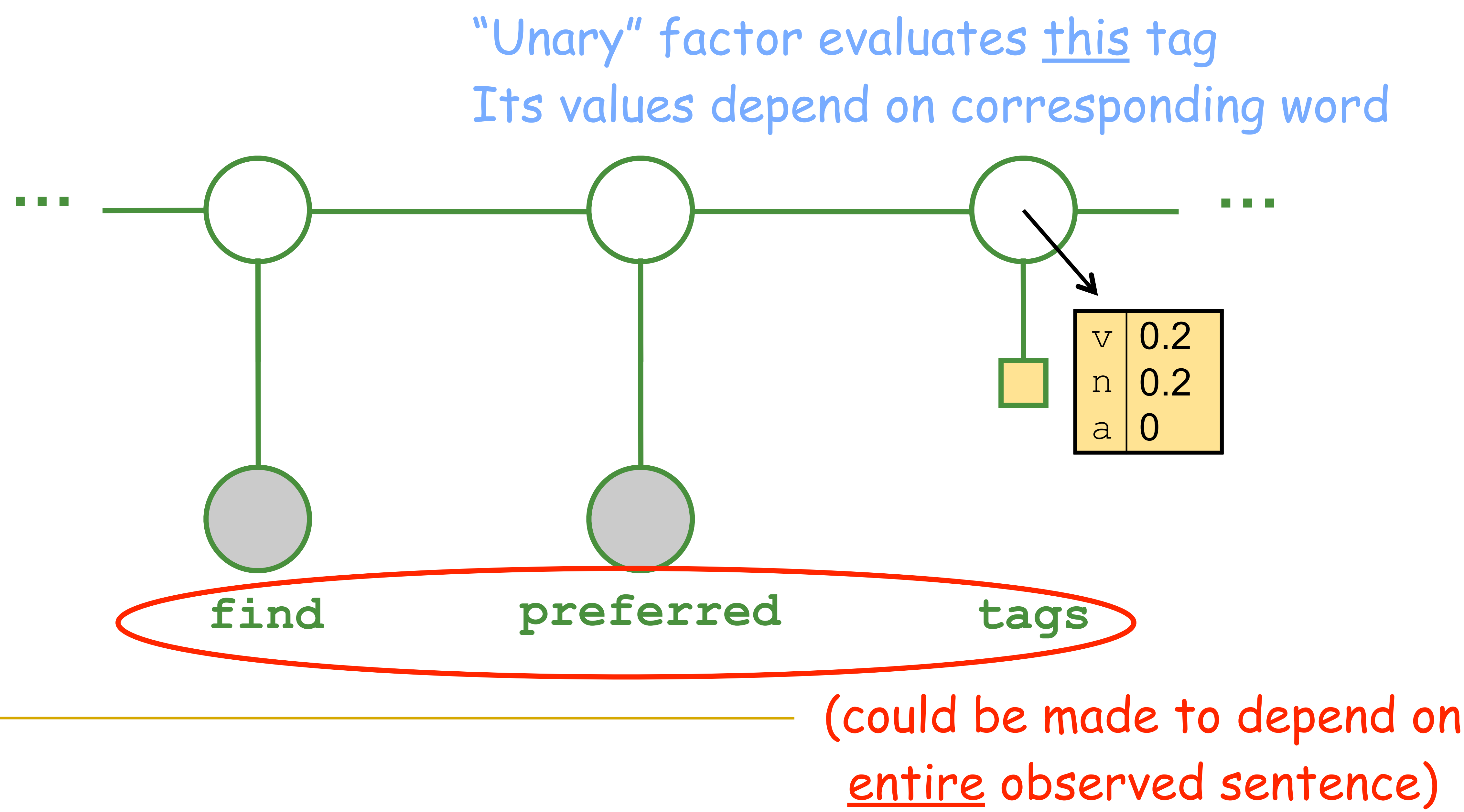

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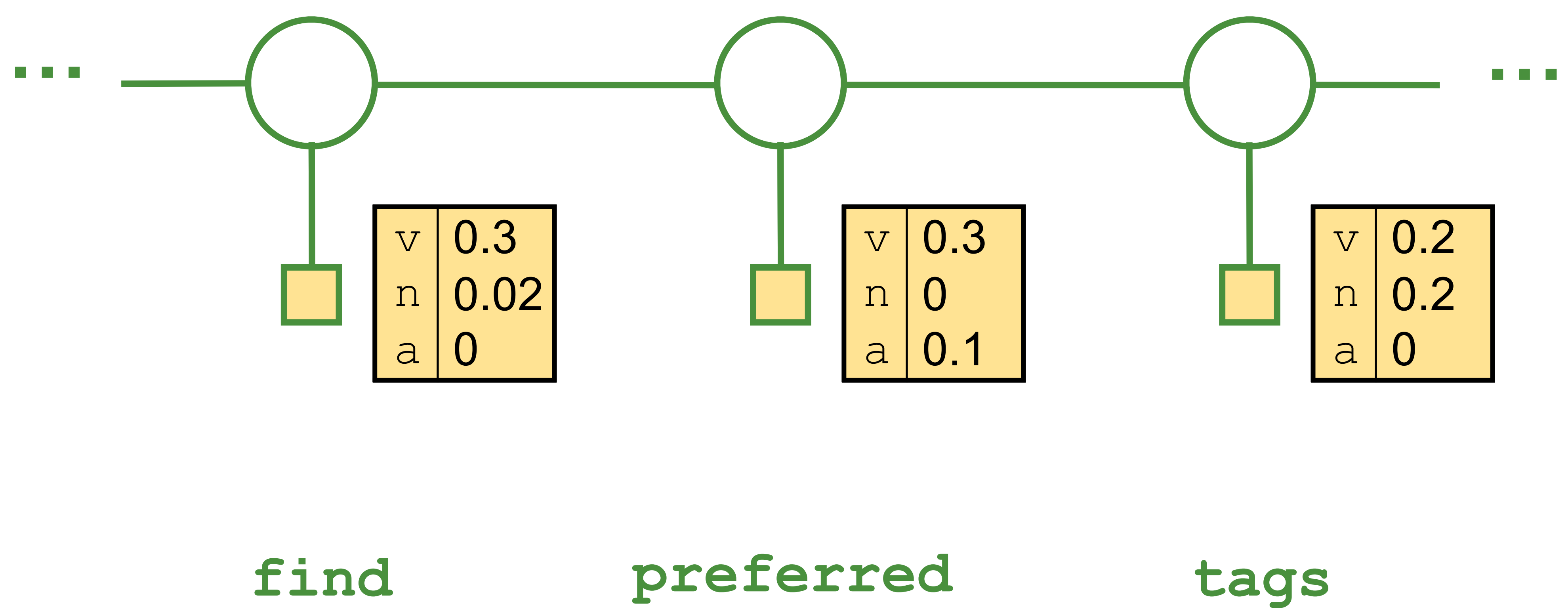

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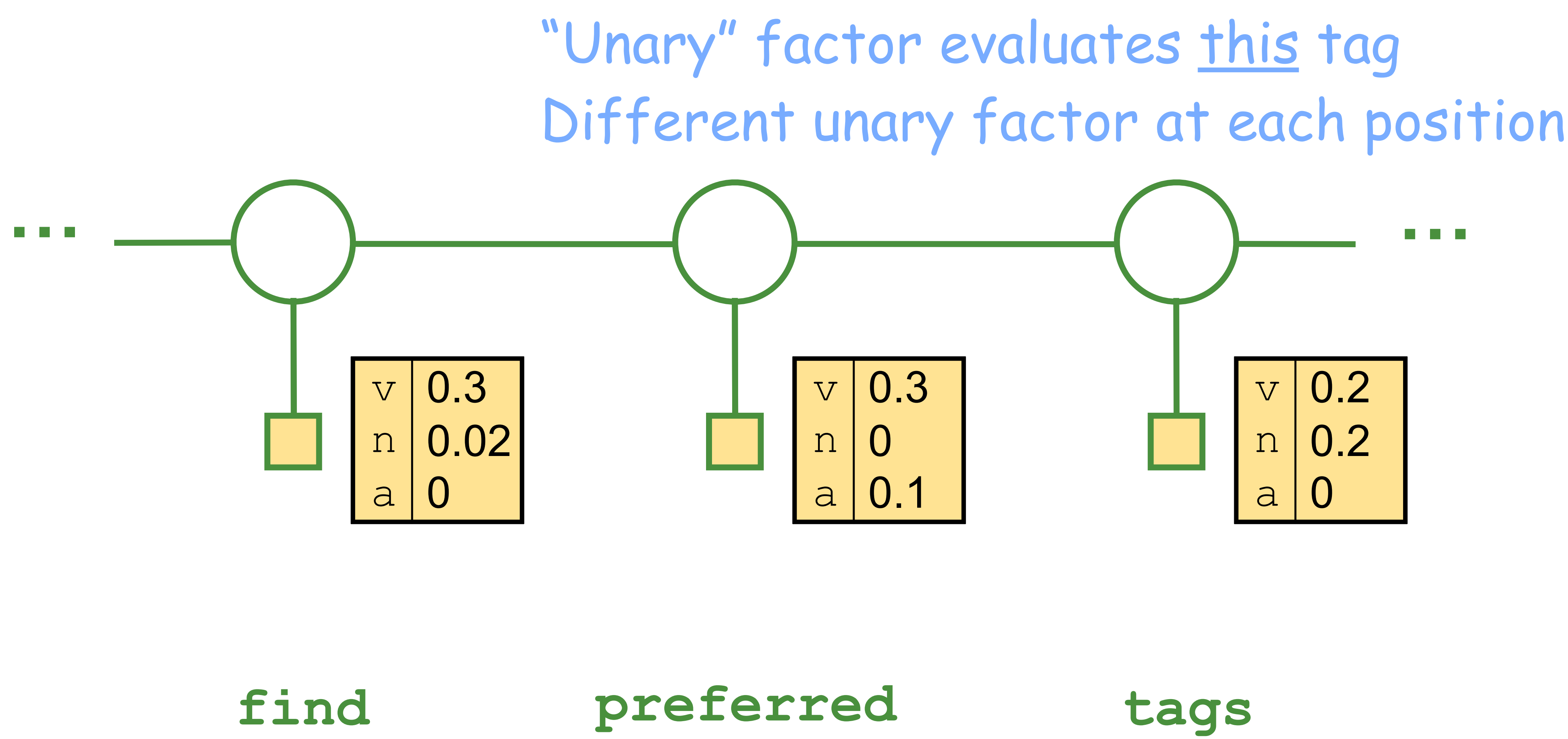

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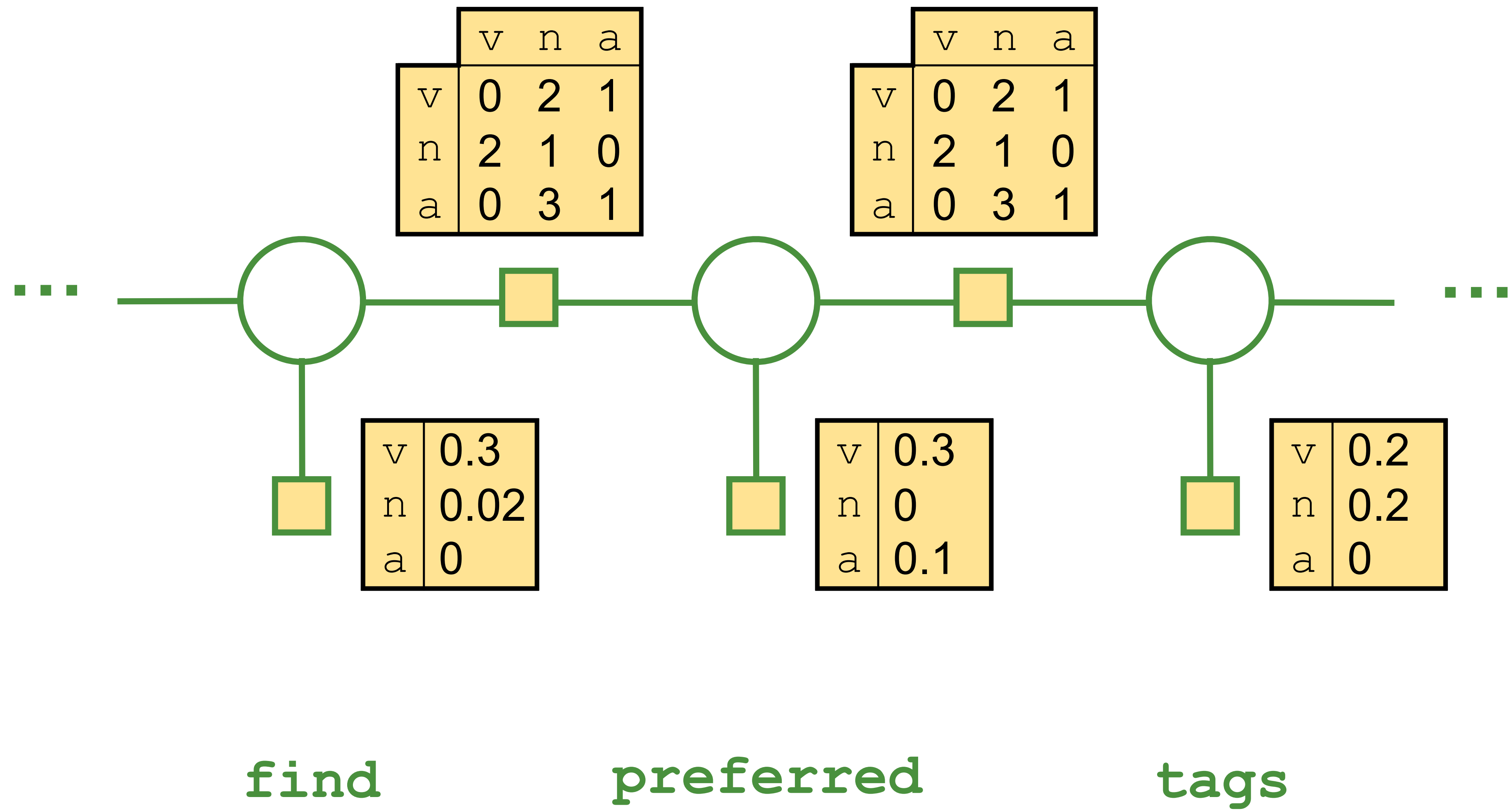
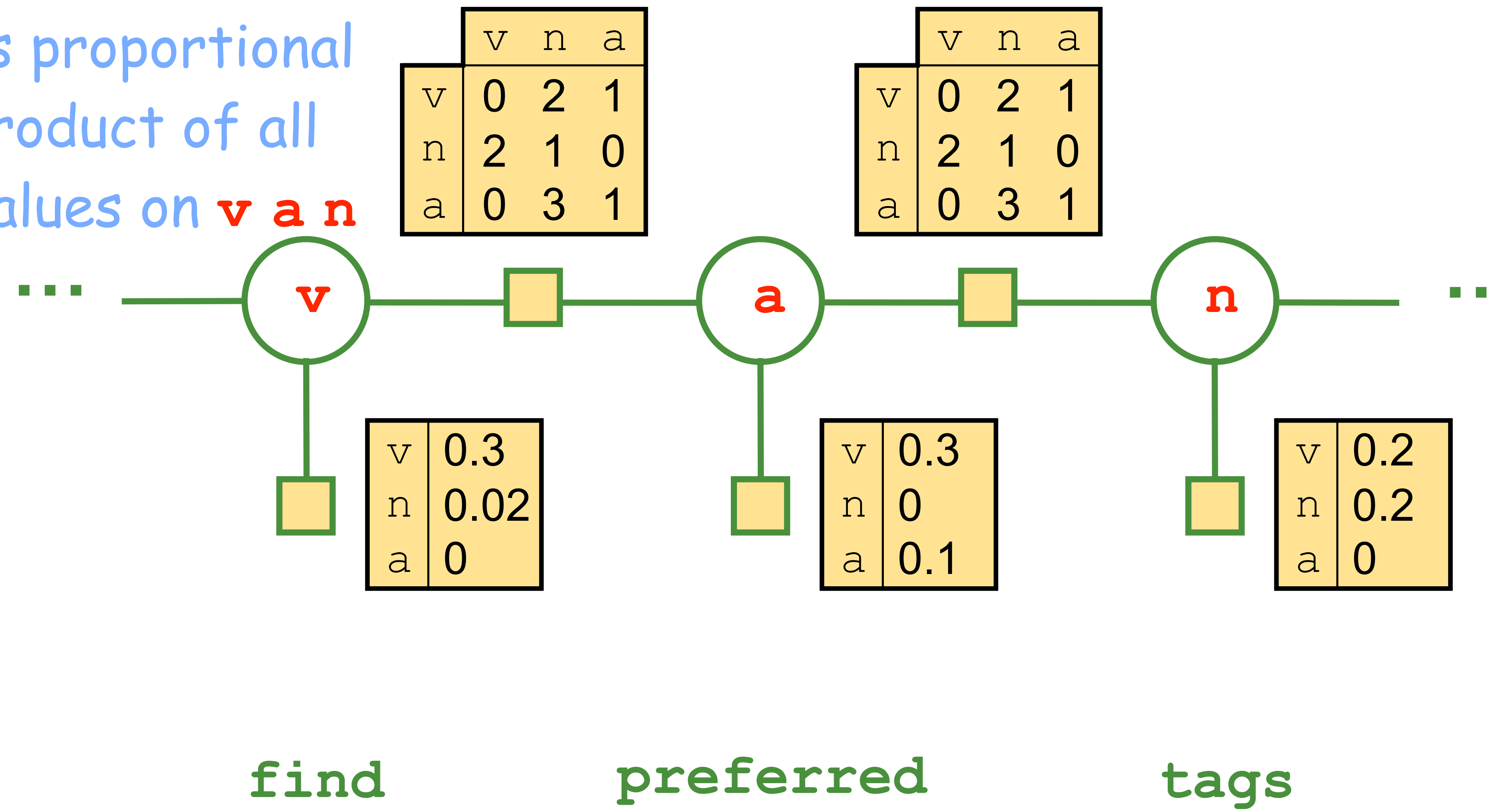


Local factors in a graphical model

- First, a familiar example
  - Conditional Random Field (CRF) for POS tagging

p(**v a n**) is proportional to the product of all factors' values on **v a n**



Local factors in a graphical model

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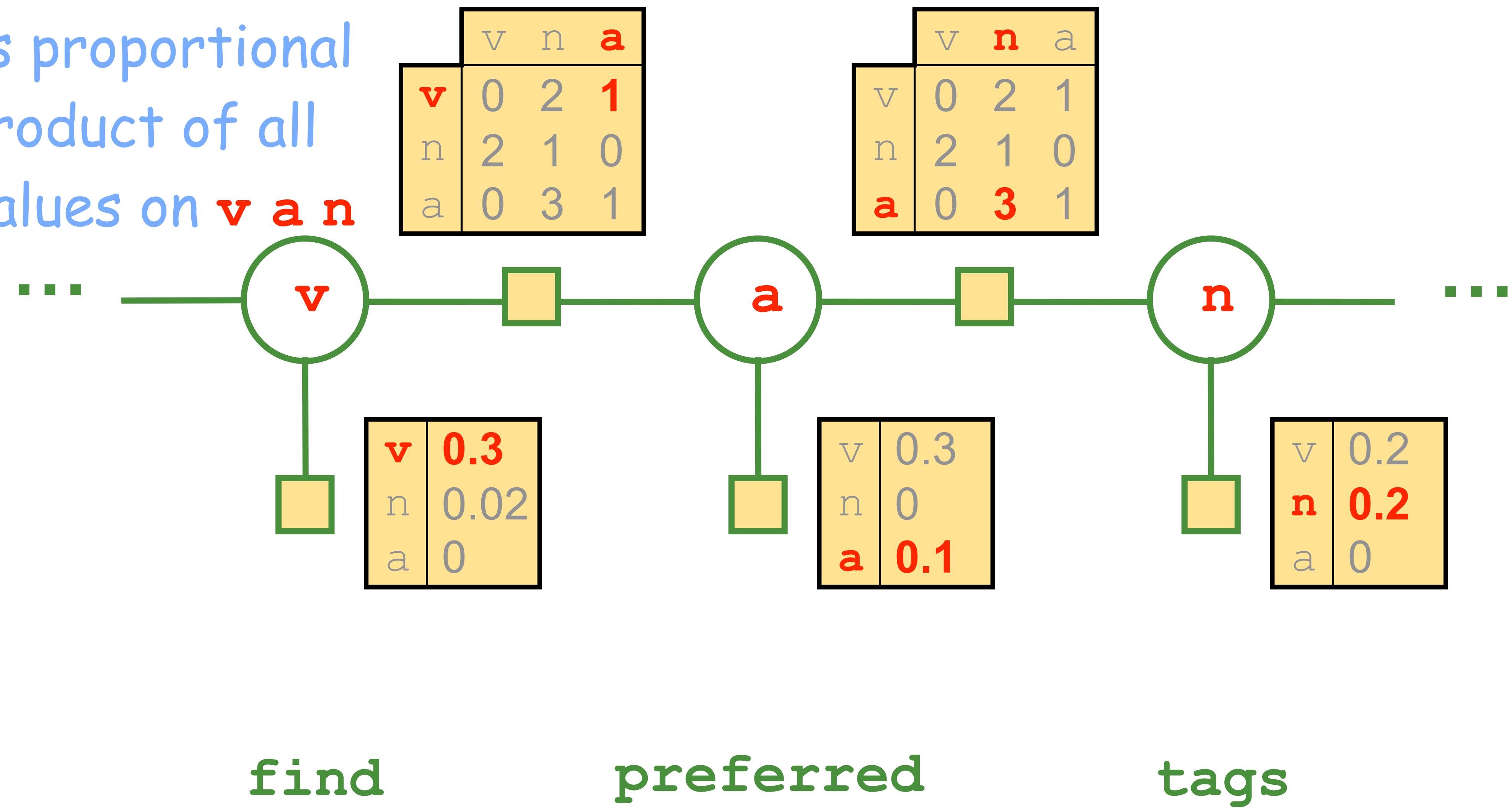
p(**v a n**) is proportional to the product of all factors' values on **v a n**



	v	n	a
v	0	2	1
n	2	1	0
a	0	3	1

	v	n	a
v	0	2	1
n	2	1	0
a	0	3	1

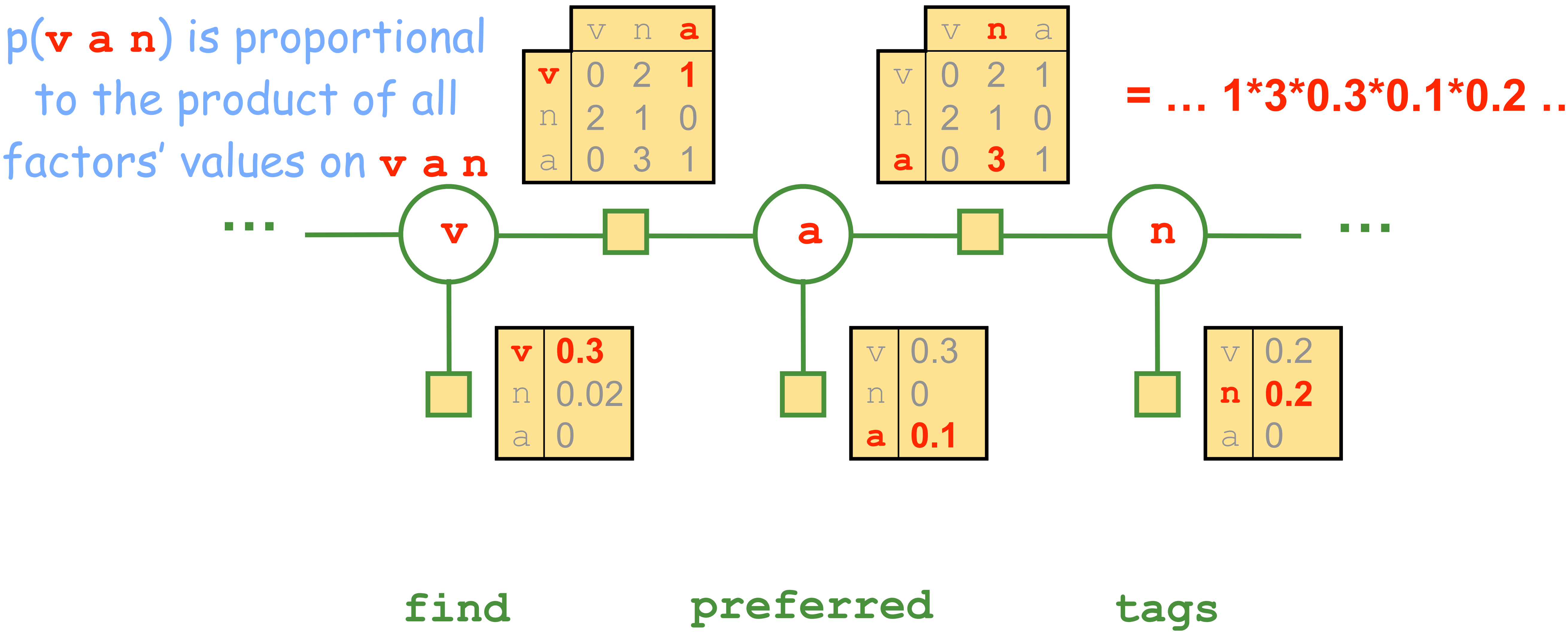
v	0.3
n	0.02
a	0

v	0.3
n	0
a	0.1

v	0.2
n	0.2
a	0

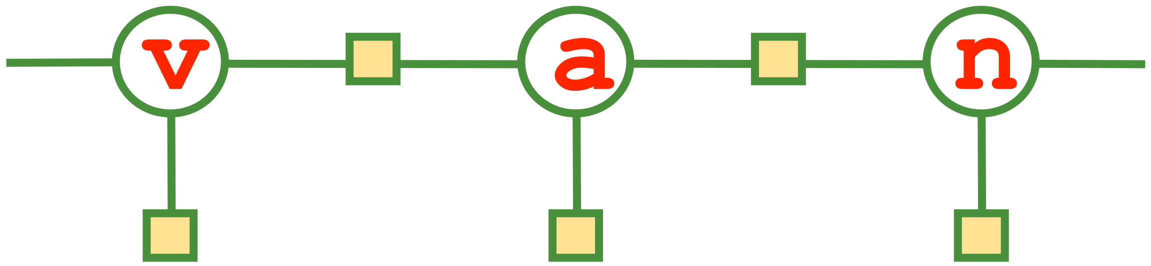
Local factors in a graphical model

- First, a familiar example
  - Conditional Random Field (CRF) for POS tagging



Graphical Models for Parsing

- First, a labeling example
  - CRF for POS tagging

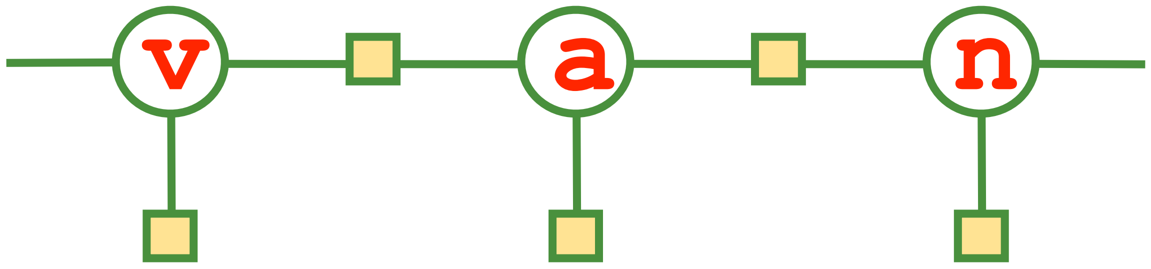


- Now let's do dependency parsing!
  - $O(n^2)$ boolean variables for the possible links

… find preferred links …

Graphical Models for Parsing

- First, a labeling example
  - CRF for POS tagging



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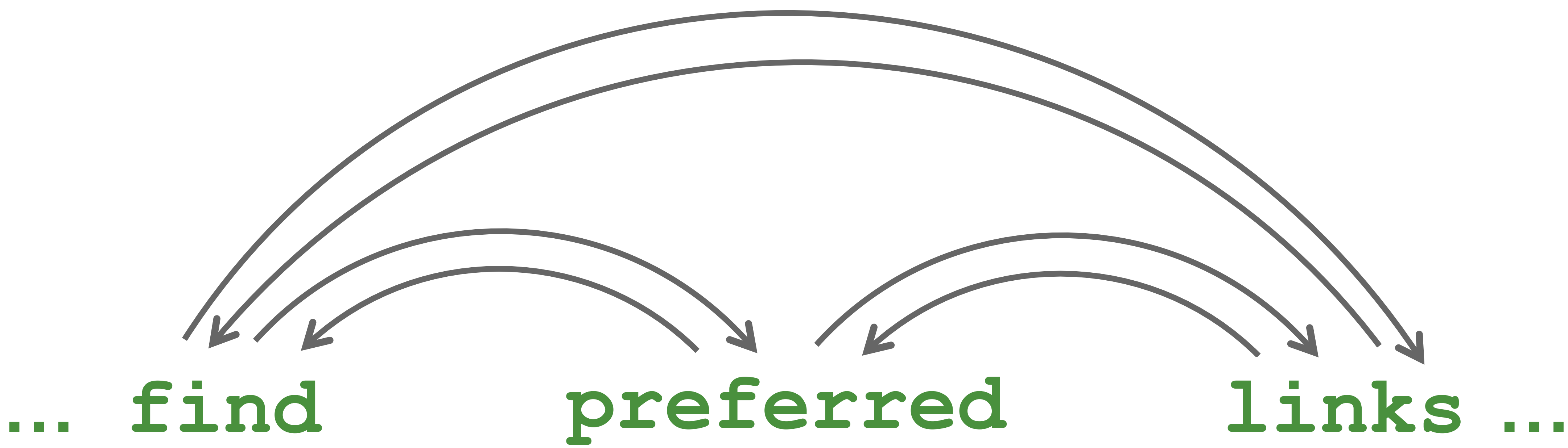
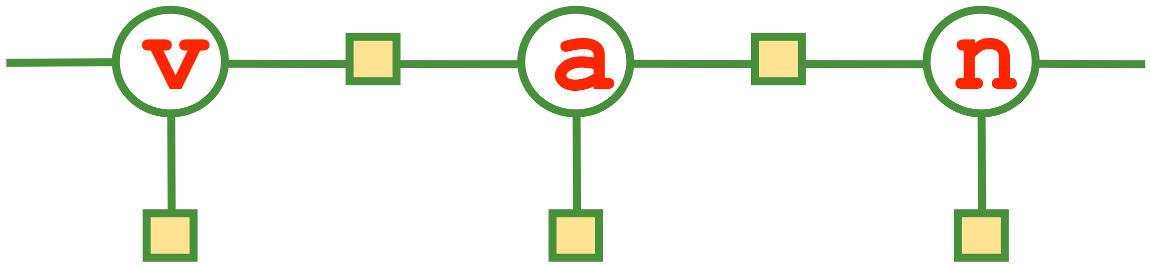


Graphical Models for Parsing

- First, a labeling example
  - CRF for POS tagging



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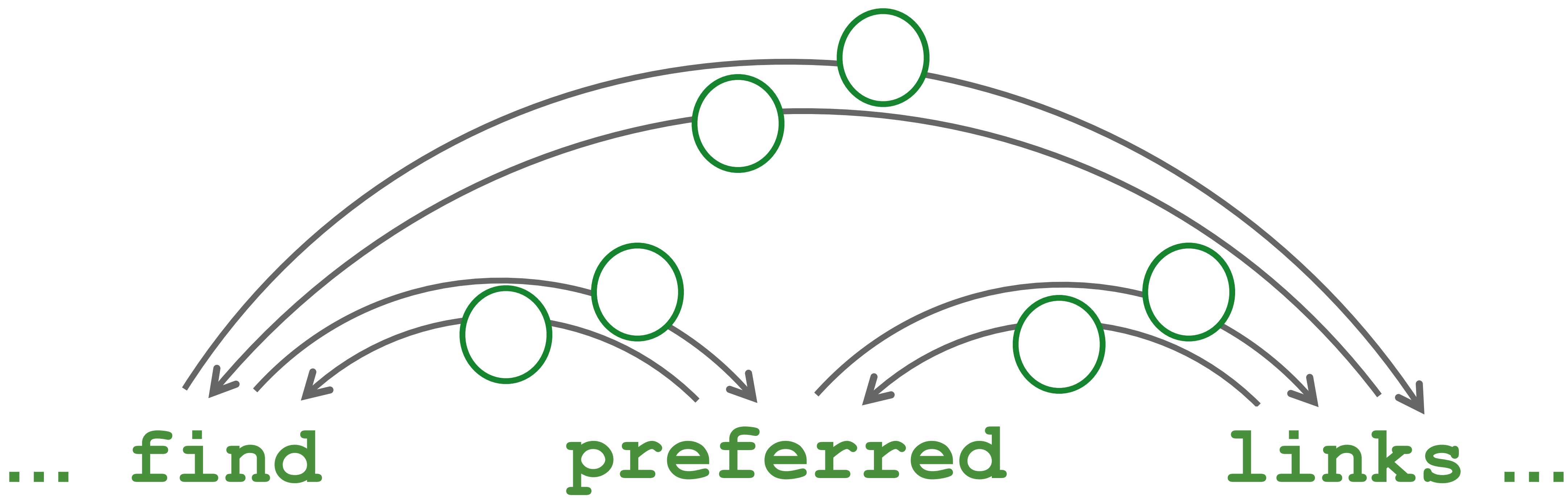
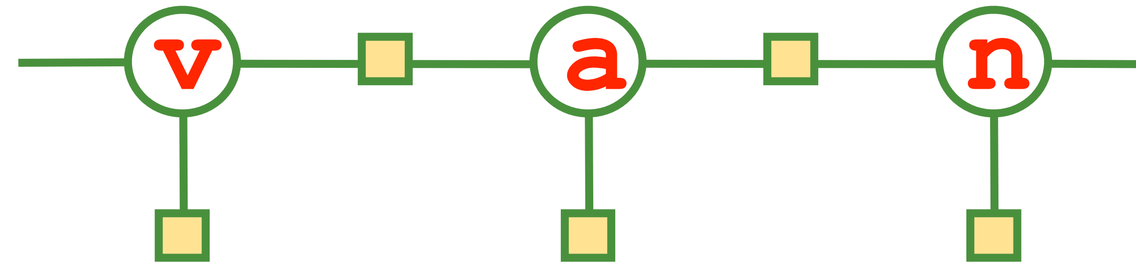


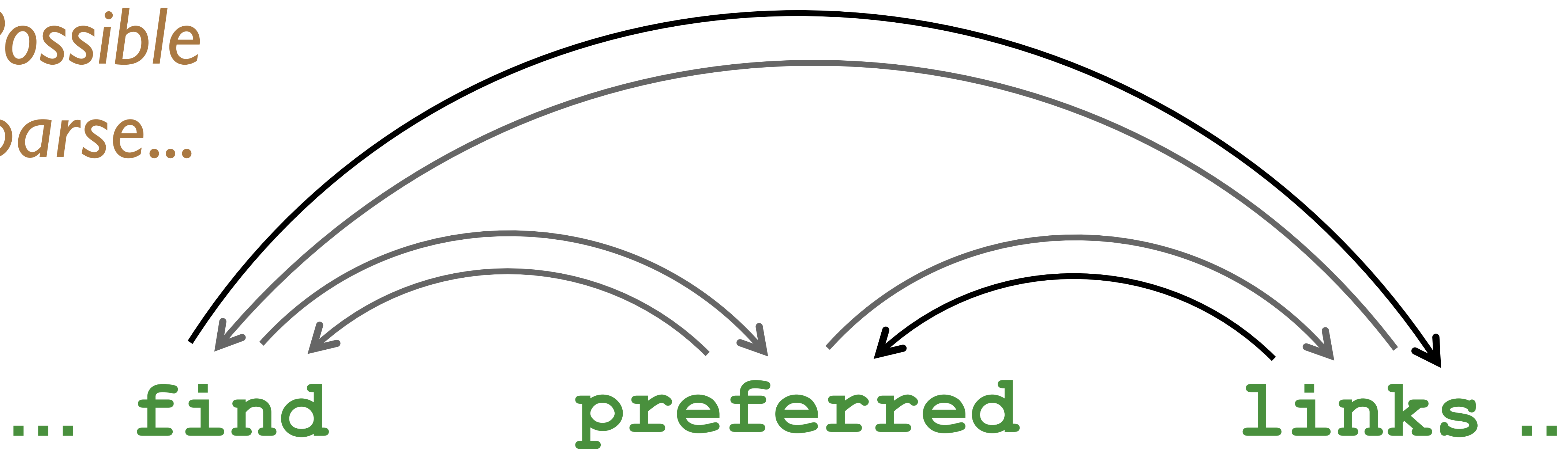
Graphical Models for Parsing

- First, a labeling example
  - ✤ CRF for POS tagging



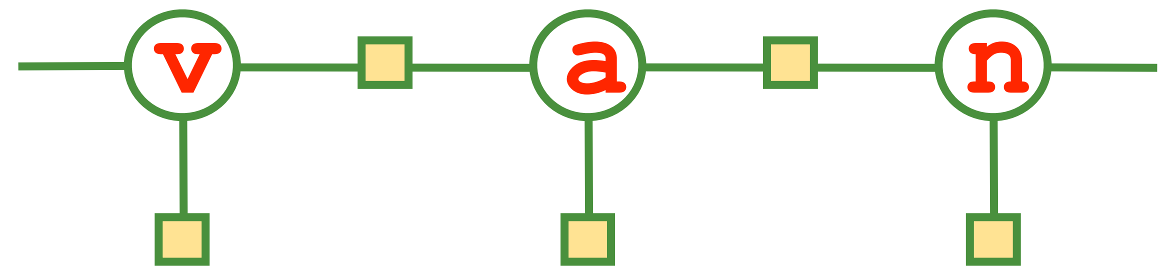
- Now let's do dependency parsing!
  - ✤ $O(n^2)$ boolean variables for the possible links

Possible parse...



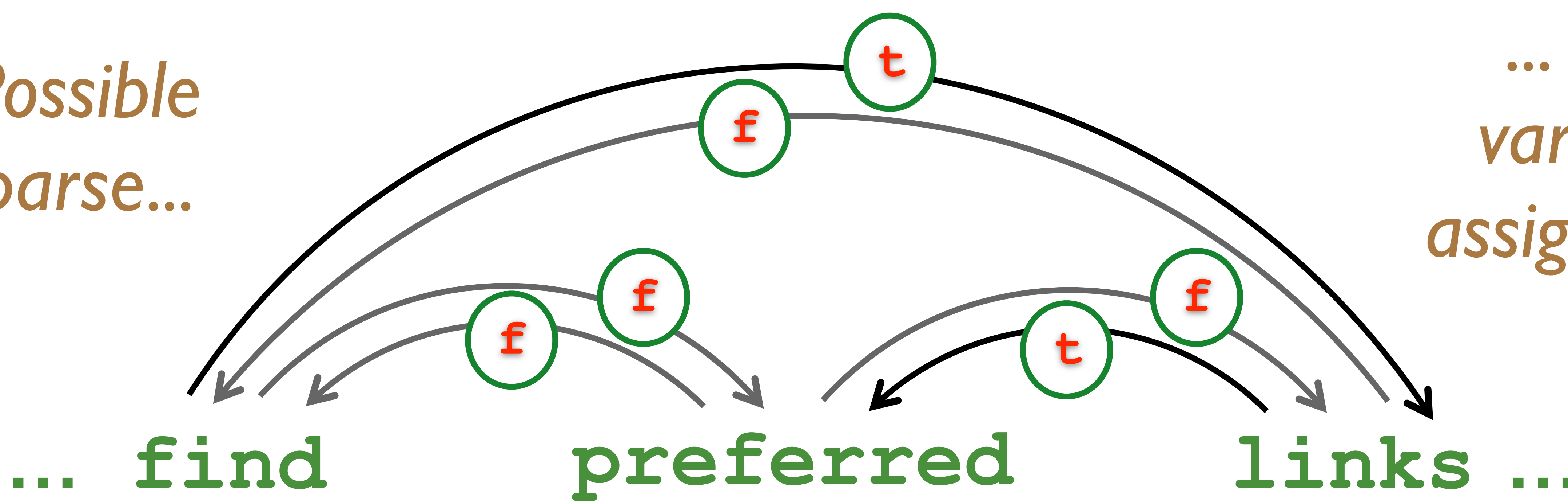
Graphical Models for Parsing

- First, a labeling example
  - CRF for POS tagging



- Now let's do dependency parsing!
  - $O(n^2)$ boolean variables for the possible links

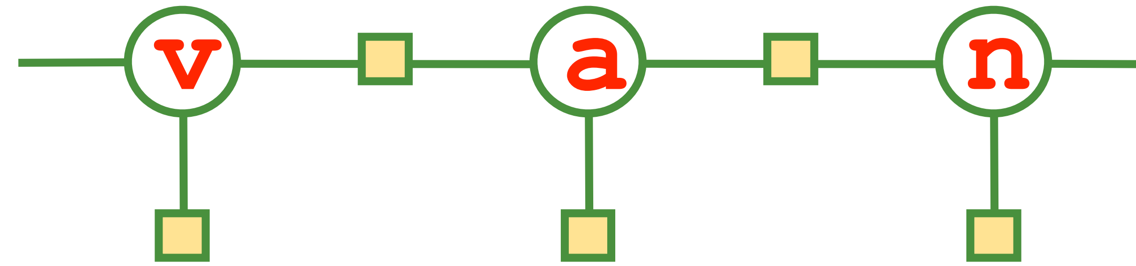
Possible parse…



… and variable assignment

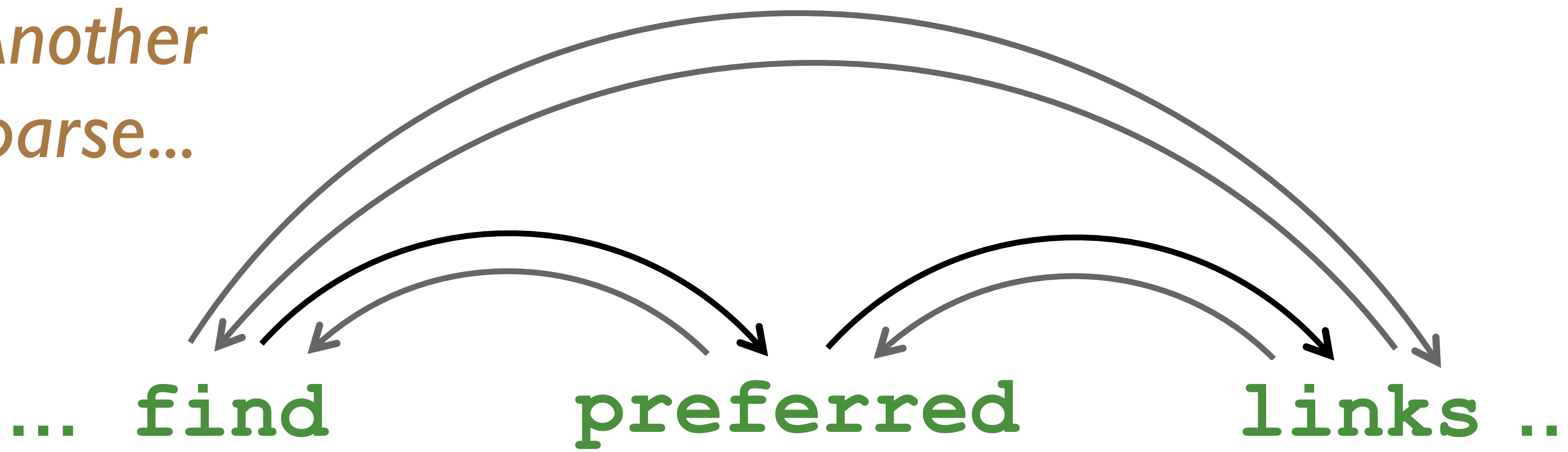
Graphical Models for Parsing

- First, a labeling example
  - CRF for POS tagging



- Now let's do dependency parsing!
  - O(n^2) boolean variables for the possible links

Another parse...



Graphical Models for Parsing

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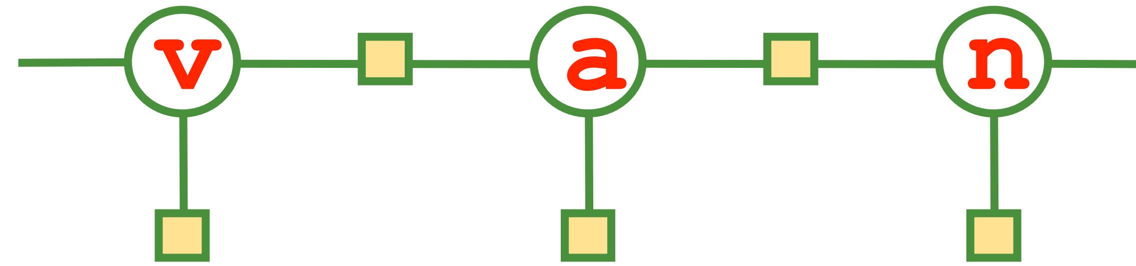
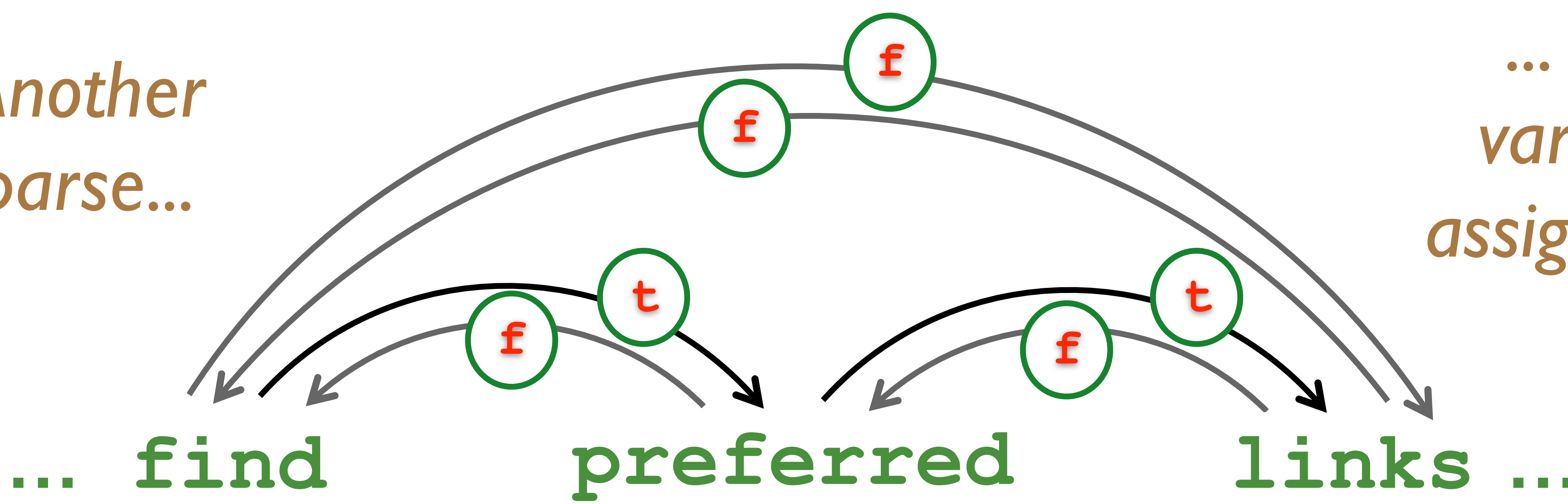
  - ✤ CRF for POS tagging



- Now let's do dependency parsing!
  - ✤ $O(n^2)$ boolean variables for the possible links

Another parse…



… and variable assignment

Graphical Models for Parsing

- First, a labeling example
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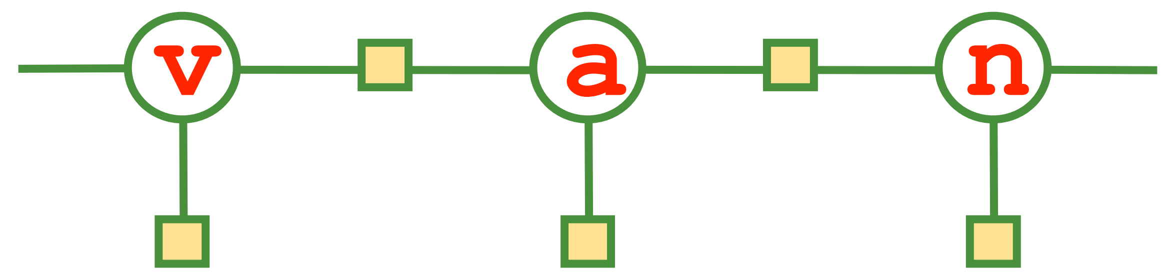
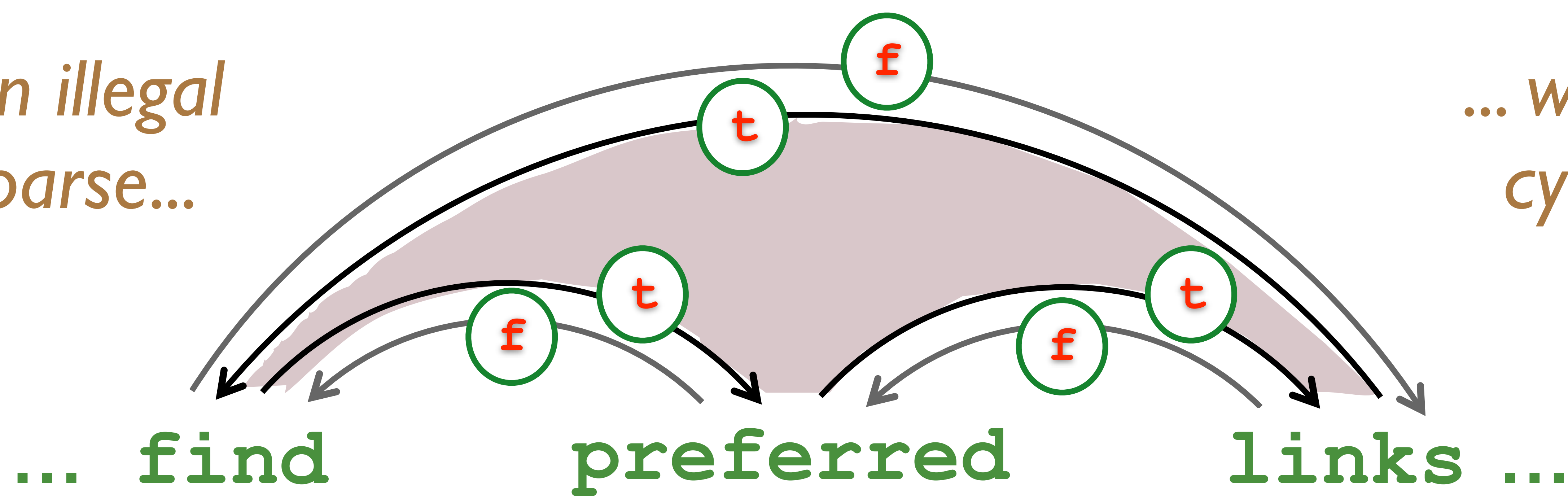


- Now let's do dependency parsing!
  - $O(n^2)$ boolean variables for the possible links

An illegal parse...



... with a cycle!

Graphical Models for Parsing

- First, a labeling example
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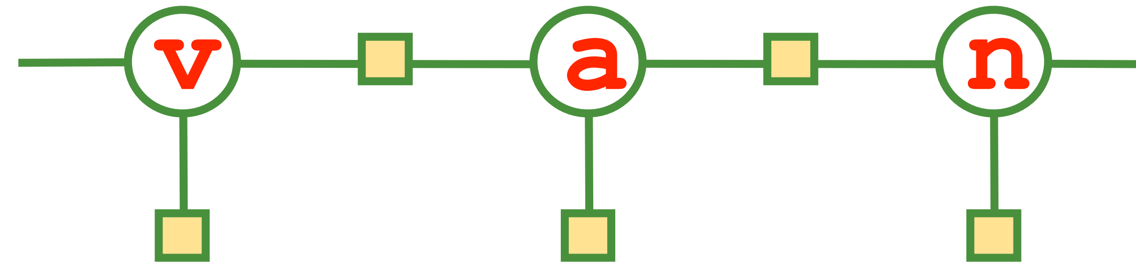

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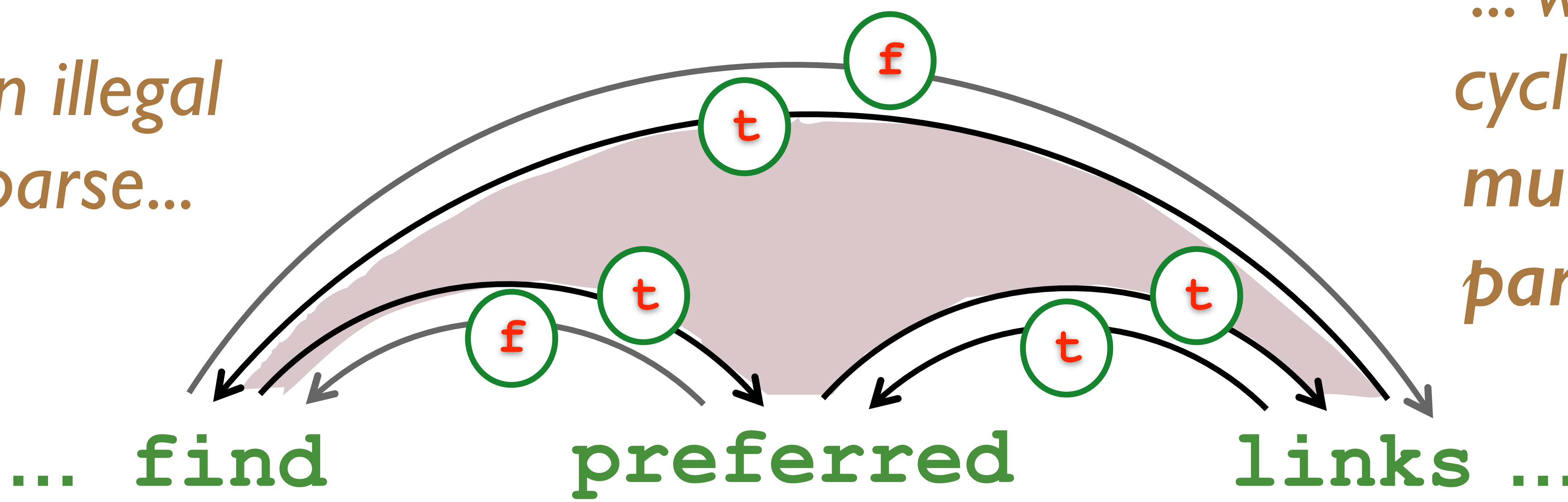
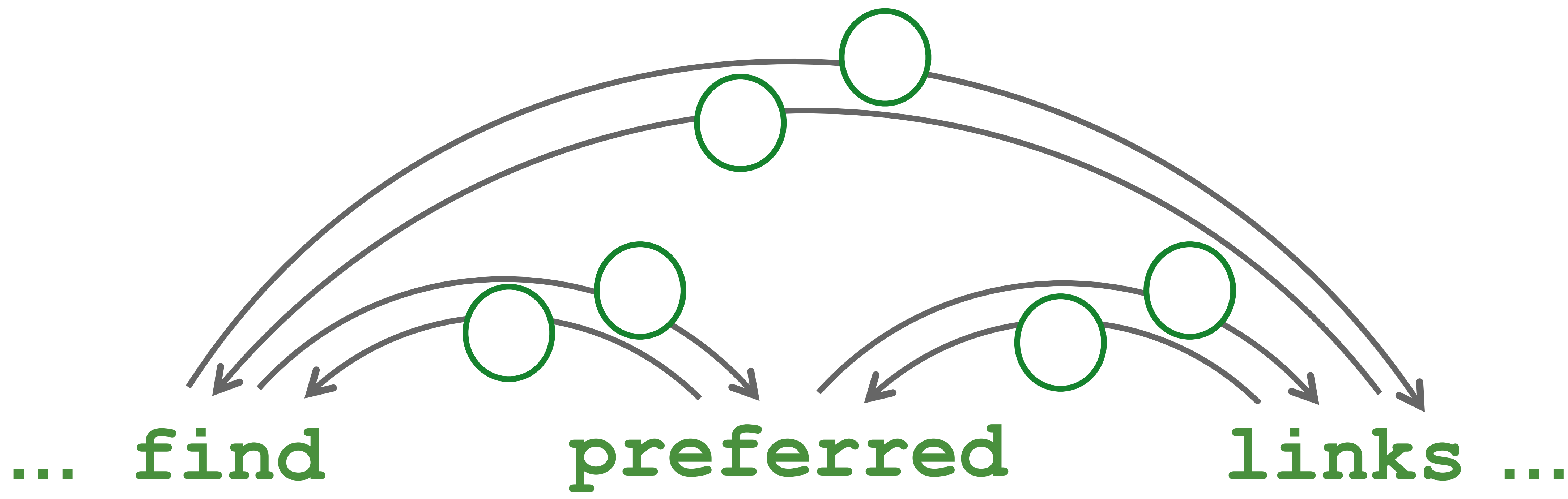


*... with a cycle and **multiple parents!***

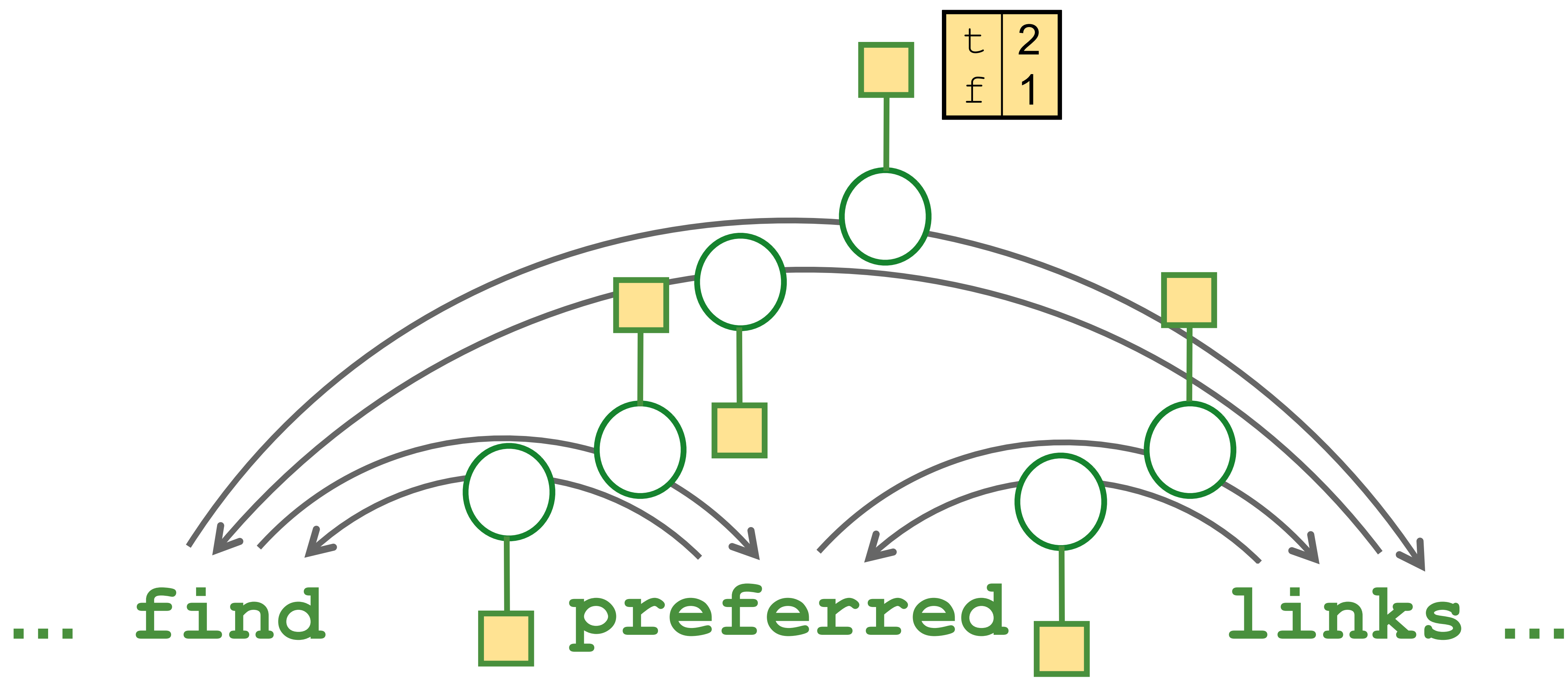
Local Factors for Parsing

- What factors determine parse probability?



Local Factors for Parsing

- What factors determine parse probability?
  - ✤ Unary factors to score each link in isolation



Local Factors for Parsing

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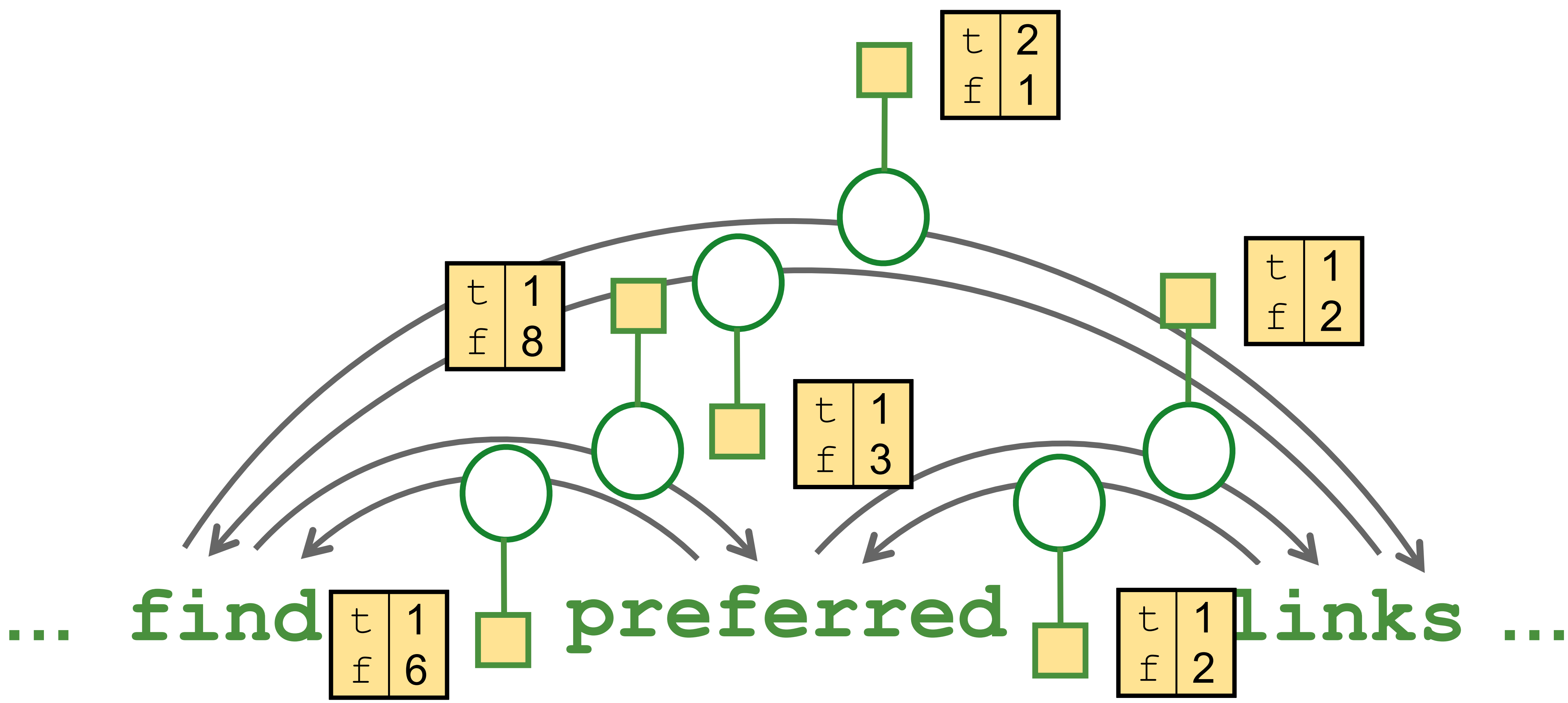
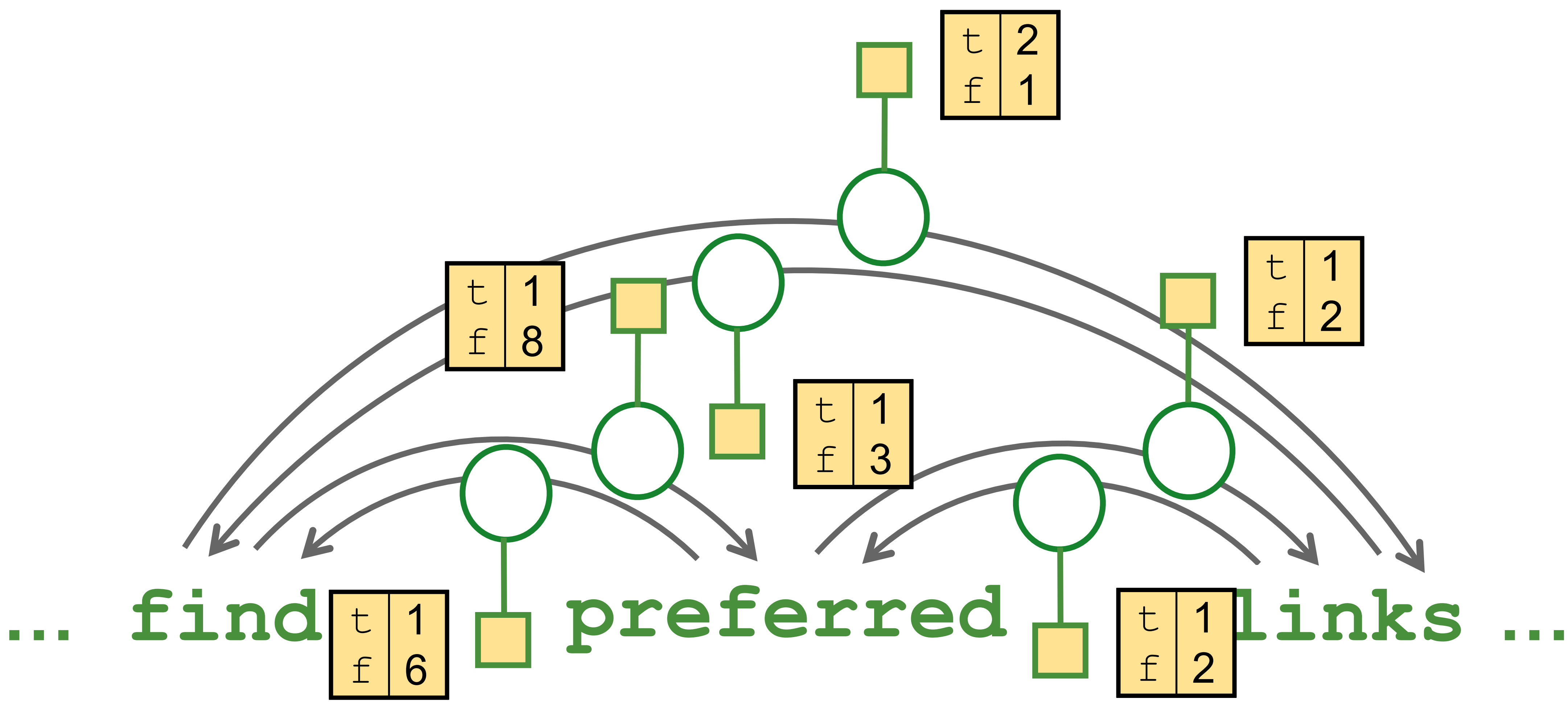


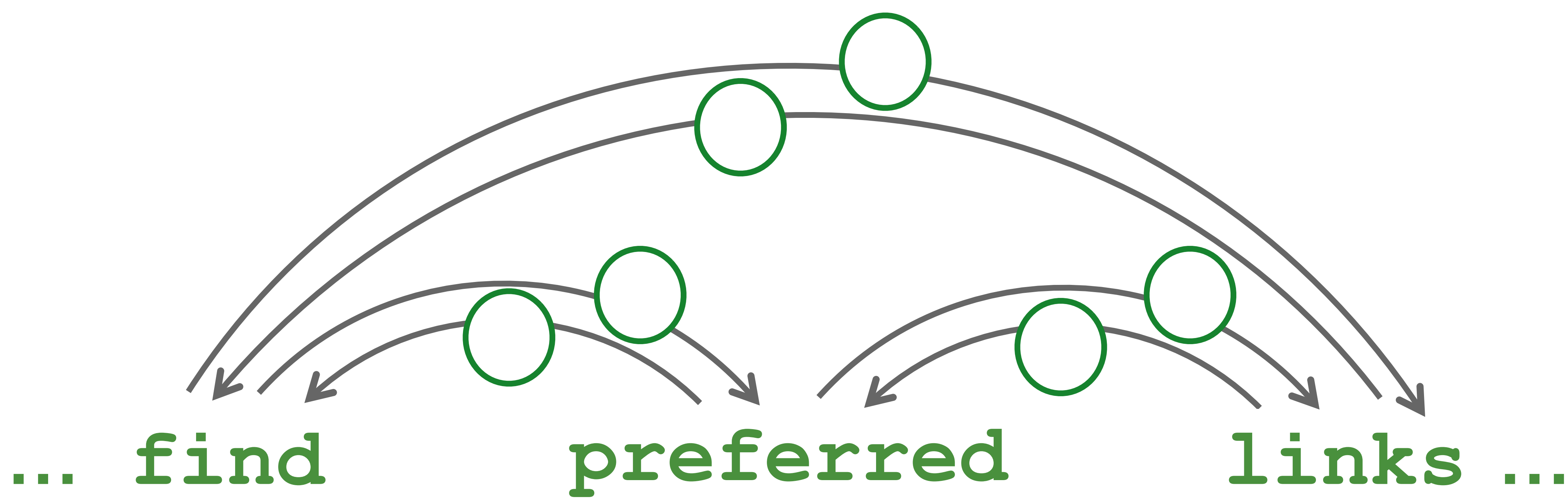
Local Factors for Parsing

- What factors determine parse probability?
  - Unary factors to score each link in isolation
- But what if the best assignment isn't a tree?



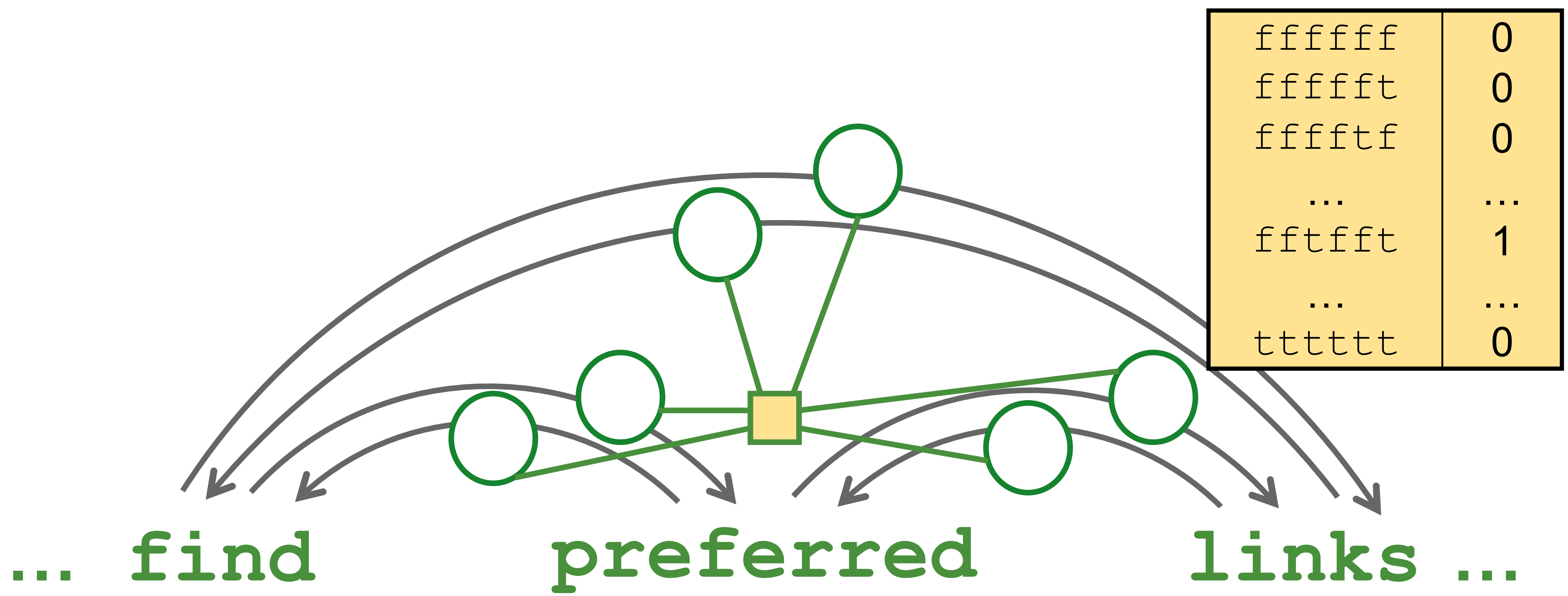
Global Factors for Parsing

- What factors determine parse probability?
  - Unary factors to score each link in isolation



Global Factors for Parsing

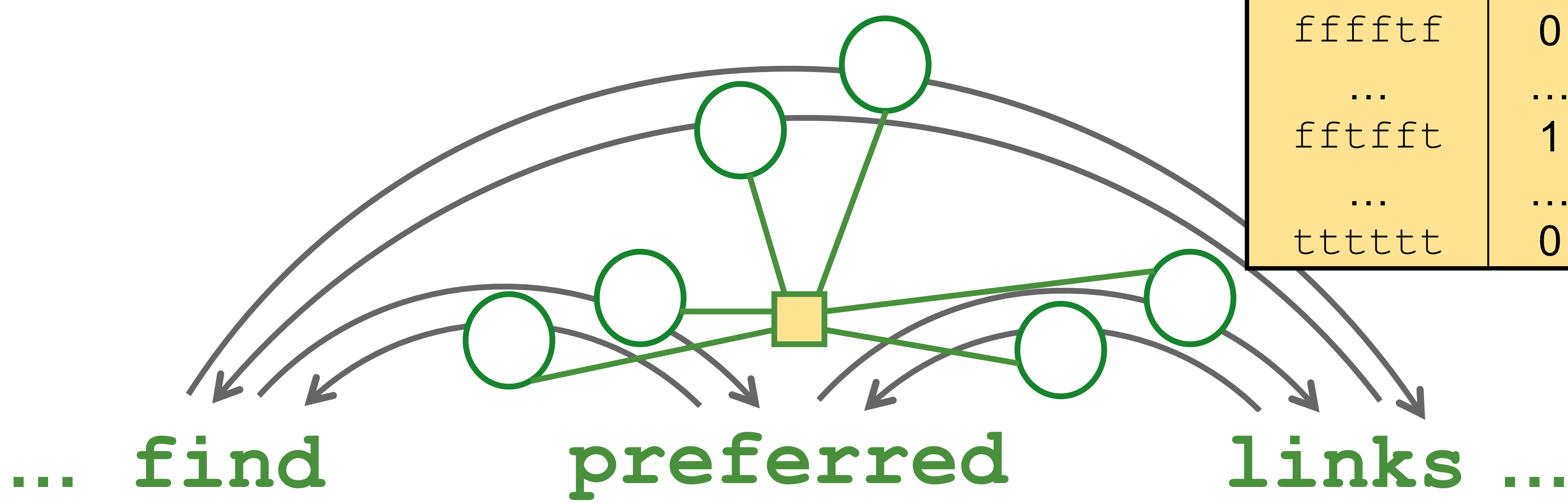
- What factors determine parse probability?
  - ✤ Unary factors to score each link in isolation
  - ✤ Global TREE factor to *require* links to form a legal tree



Global Factors for Parsing

- What factors determine parse probability?
  - ✤ Unary factors to score each link in isolation
  - ✤ Global TREE factor to *require* links to form a legal tree
    - A *hard constraint*: potential is either 0 or 1

ffffff	0
ffffft	0
fffftf	0
…	…
fftfft	1
…	…
tttttt	0



Global Factors for Parsing

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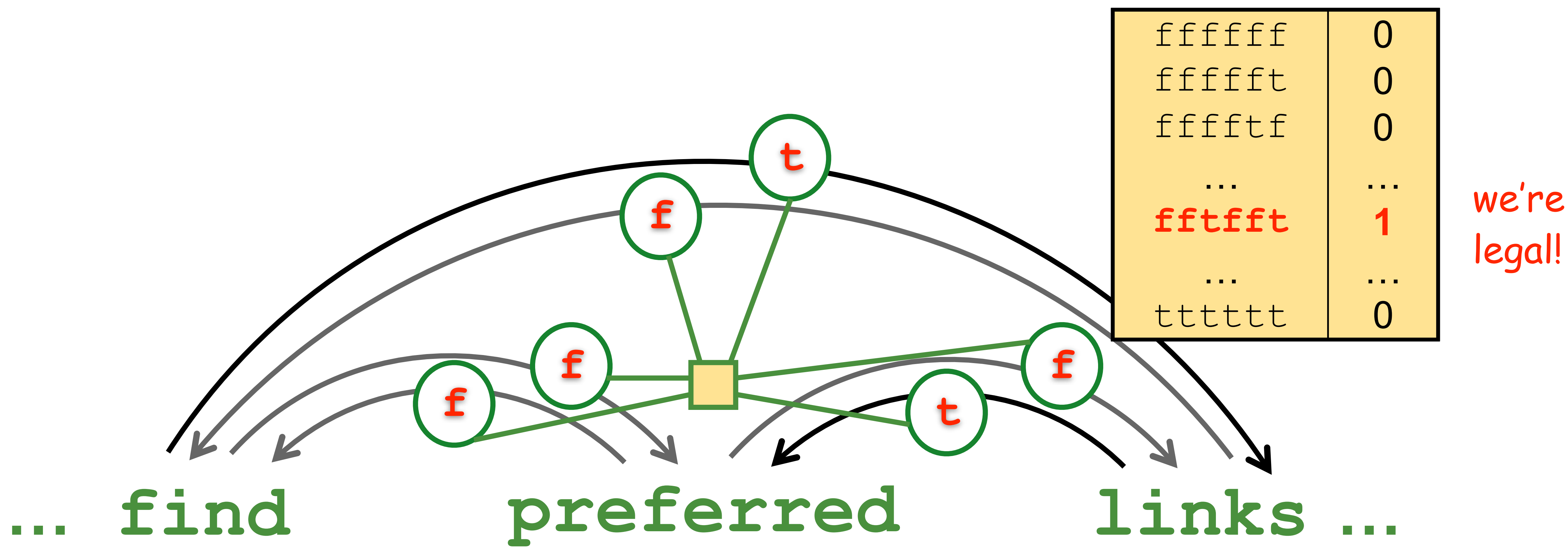


Global Factors for Parsing

optionally require the tree to be projective (no crossing links)

- What factors determine parse

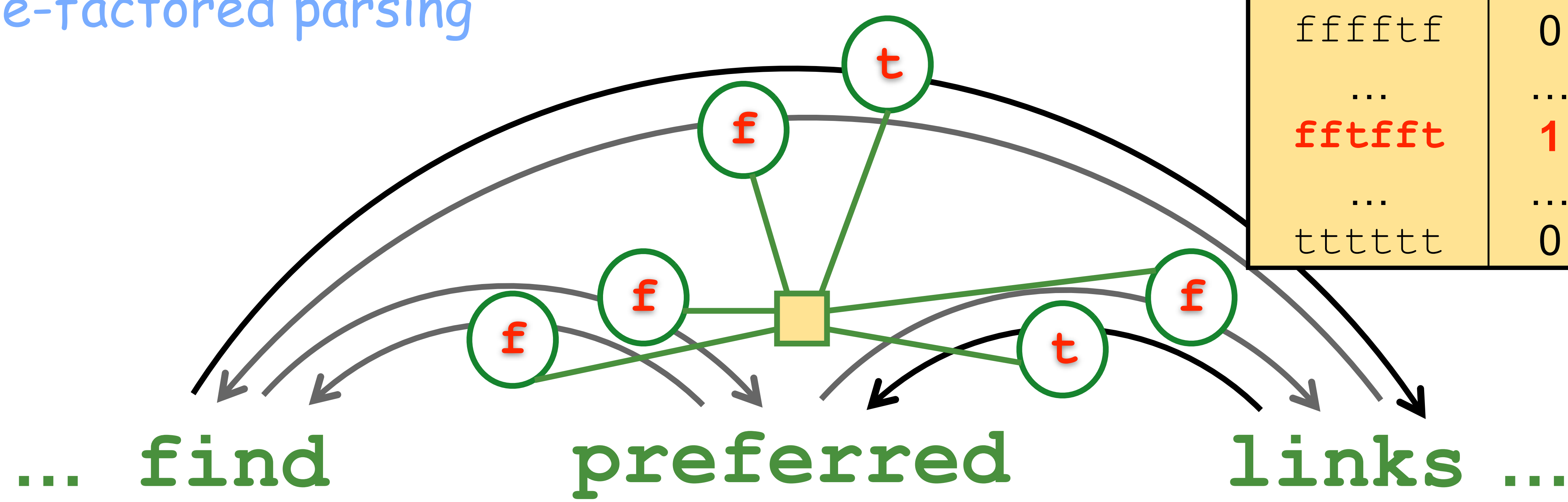
  - Unary factors to score each link in isolation
  - Global TREE factor to *require* links to form a legal tree
    - A *hard constraint*: potential is either 0 or 1

So far, this is equivalent to edge-factored parsing

64 entries (0/1)

ffffff	0
ffffft	0
fffftf	0
…	…
fftfft	**1**
…	…
tttttt	0

we're legal!



Global Factors for Parsing

optionally require the tree to be projective (no crossing links)

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So far, this is equivalent to edge-factored parsing

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tttttt	0

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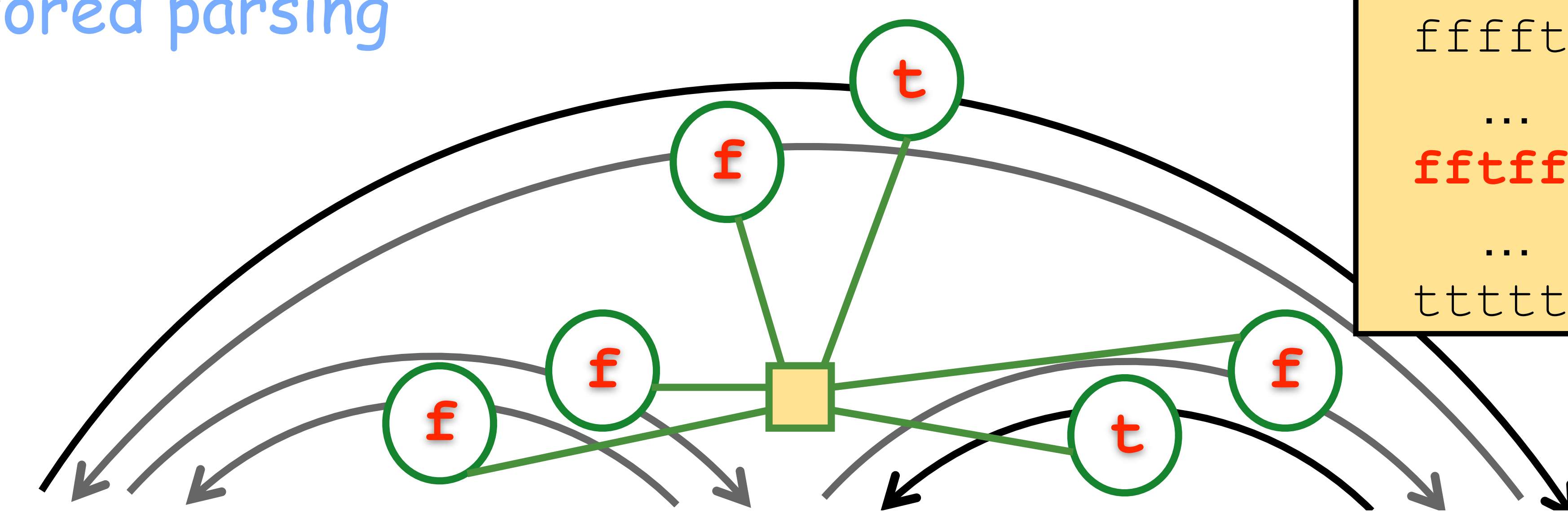
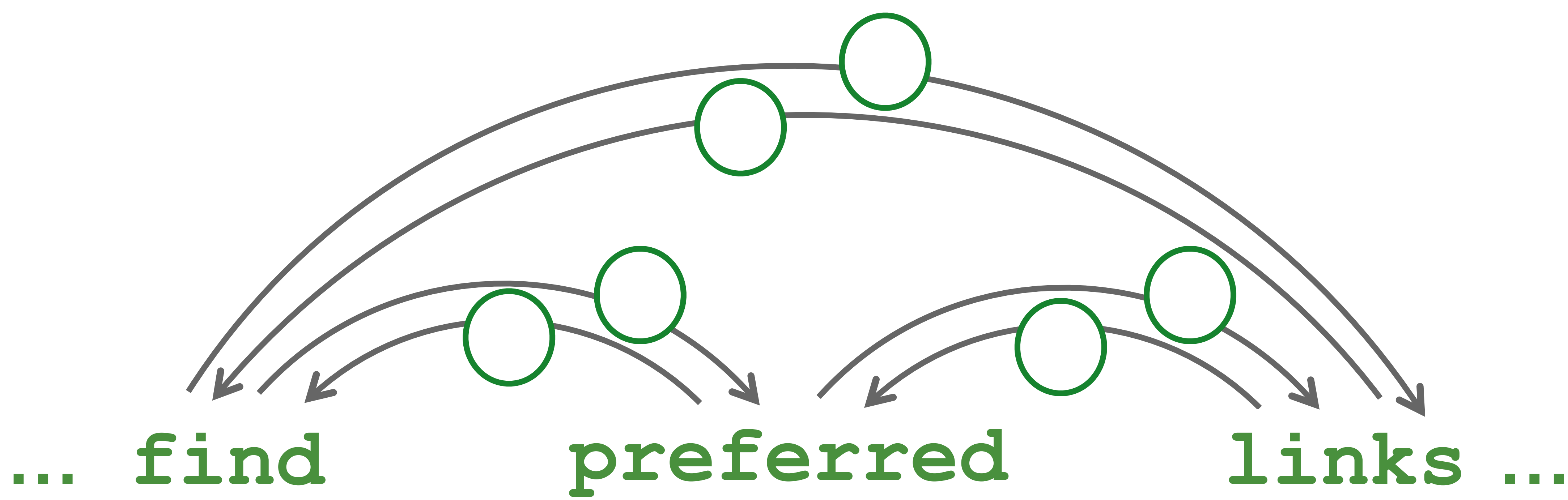


… fin

Note: traditional parsers don't loop through this table to consider exponentially many trees one at a time. They use combinatorial algorithms; so should we!

Local Factors for Parsing

- What factors determine parse probability?
  - Unary factors to score each link in isolation
  - Global TREE factor to *require* links to form a legal tree
    - A *hard constraint*: potential is either 0 or 1
  - Second order effects: factors on 2 variables
    - Grandparent–parent–child chains



Local Factors for Parsing

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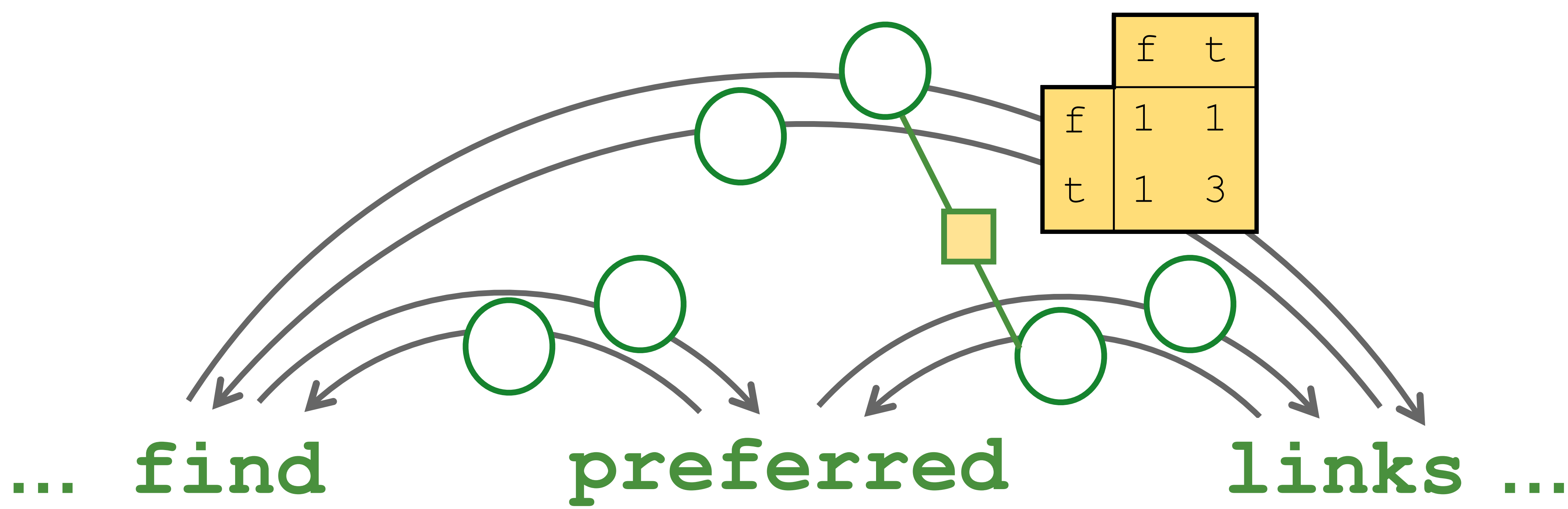
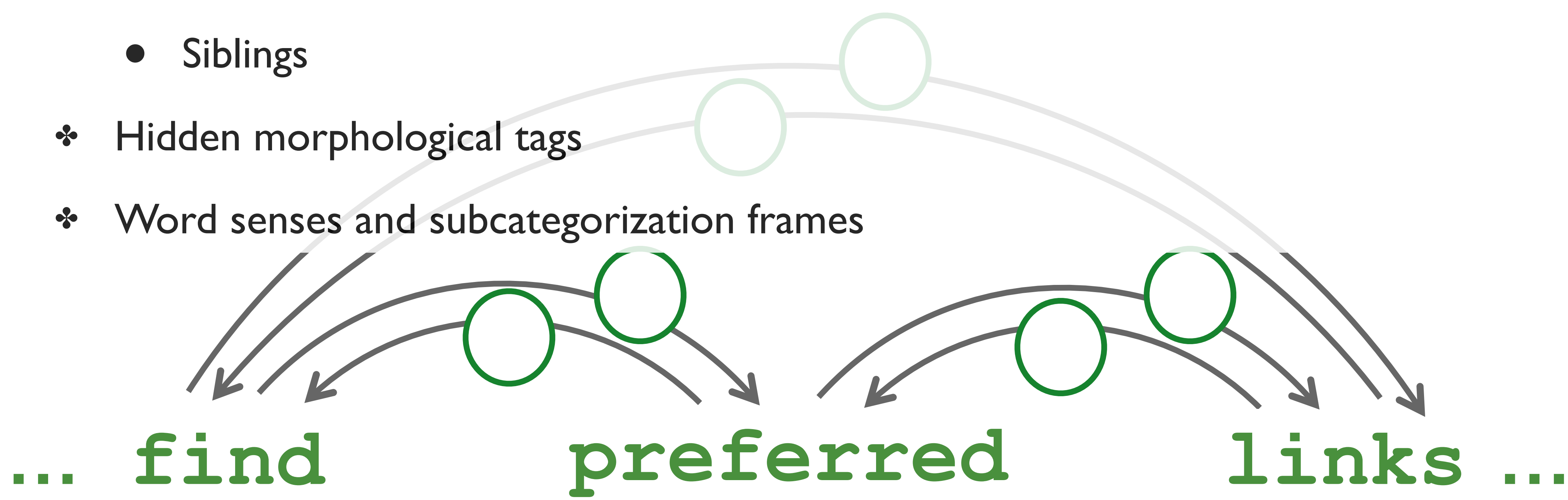


Local Factors for Parsing

- What factors determine parse probability?
  - Unary factors to score each link in isolation
  - Global TREE factor to *require* links to form a legal tree
    - A *hard constraint*: potential is either 0 or 1
  - Second order effects: factors on 2 variables
    - Grandparent–parent–child chains
    - No crossing links
    - Siblings
  - Hidden morphological tags
  - Word senses and subcategorization frames



Great Ideas in ML: Message Passing

adapted from MacKay (2003) textbook

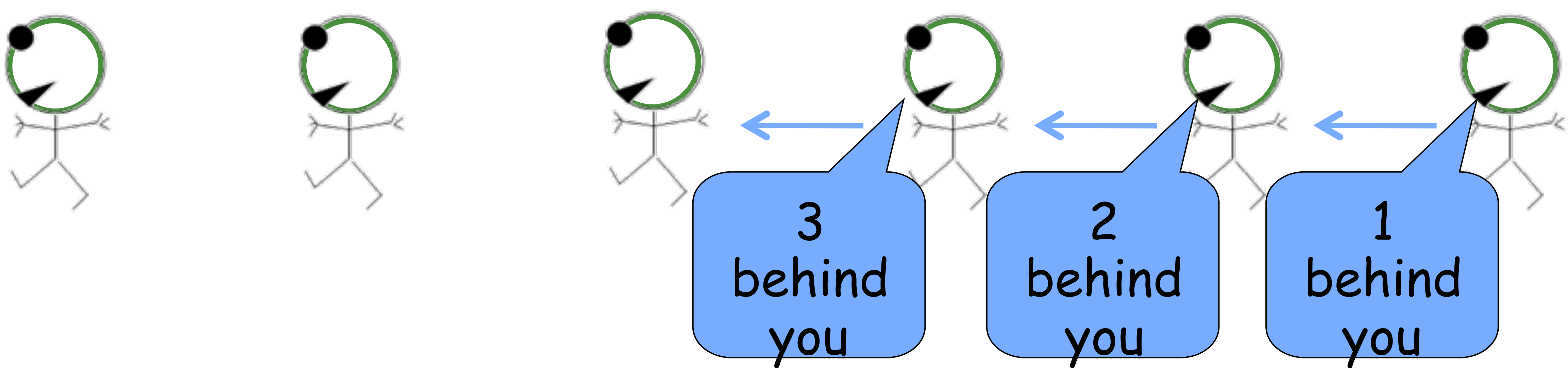
Great Ideas in ML: Message Passing

Count the soldiers

adapted from MacKay (2003) textbook

Great Ideas in ML: Message Passing

Count the soldiers



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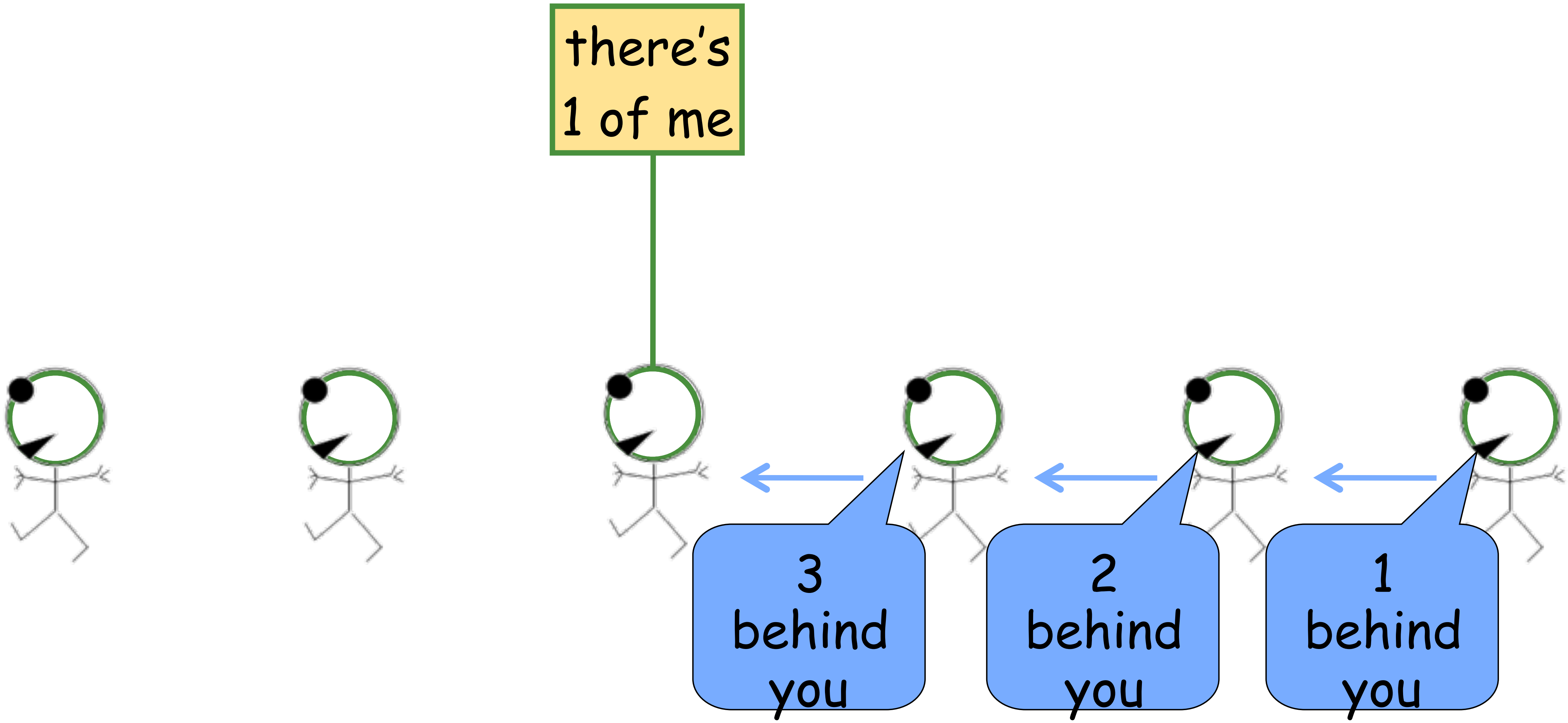
Great Ideas in ML: Message Passing

Count the soldiers



adapted from MacKay (2003) textbook

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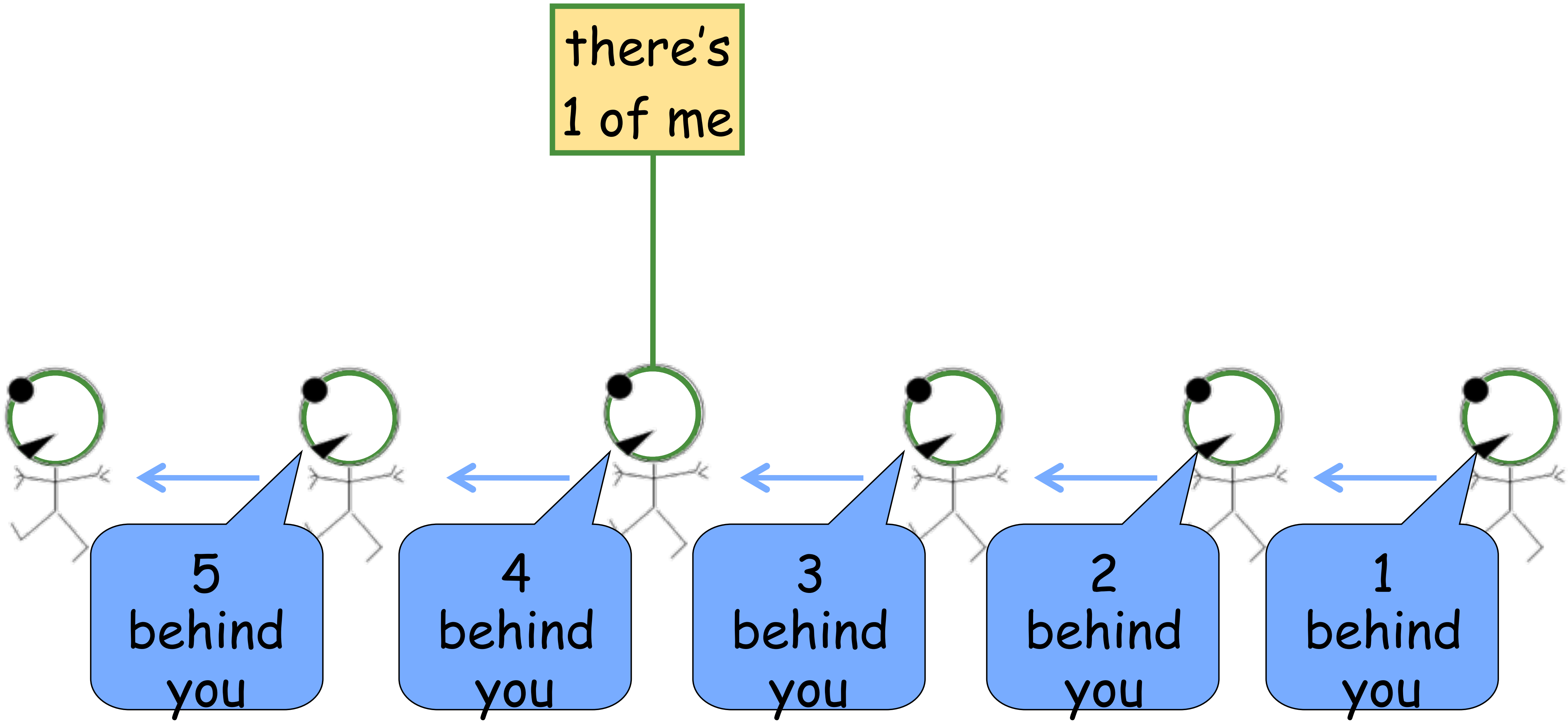

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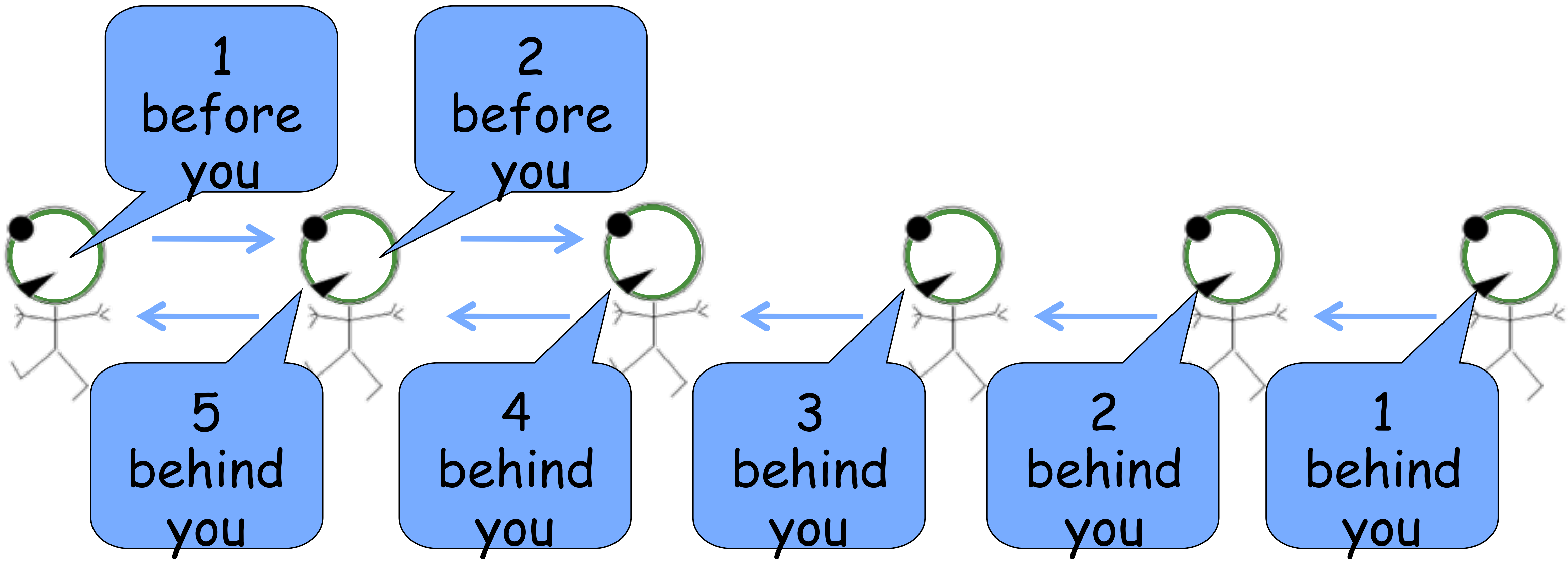

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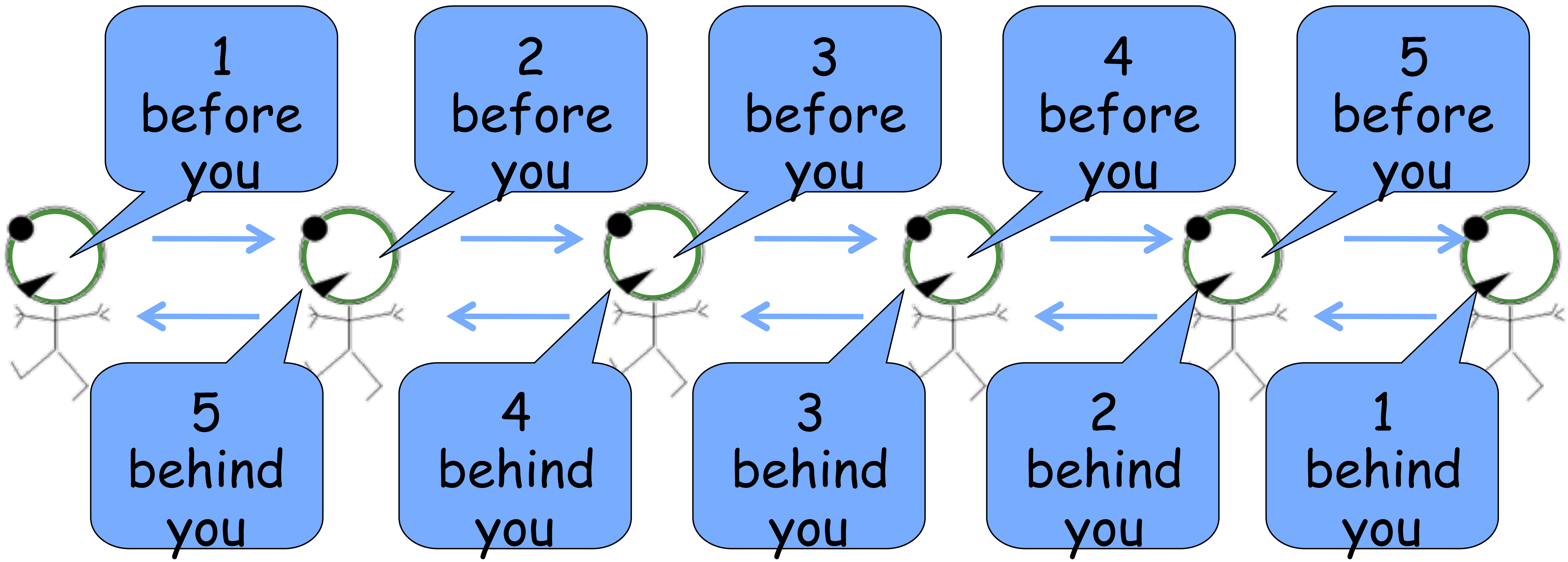

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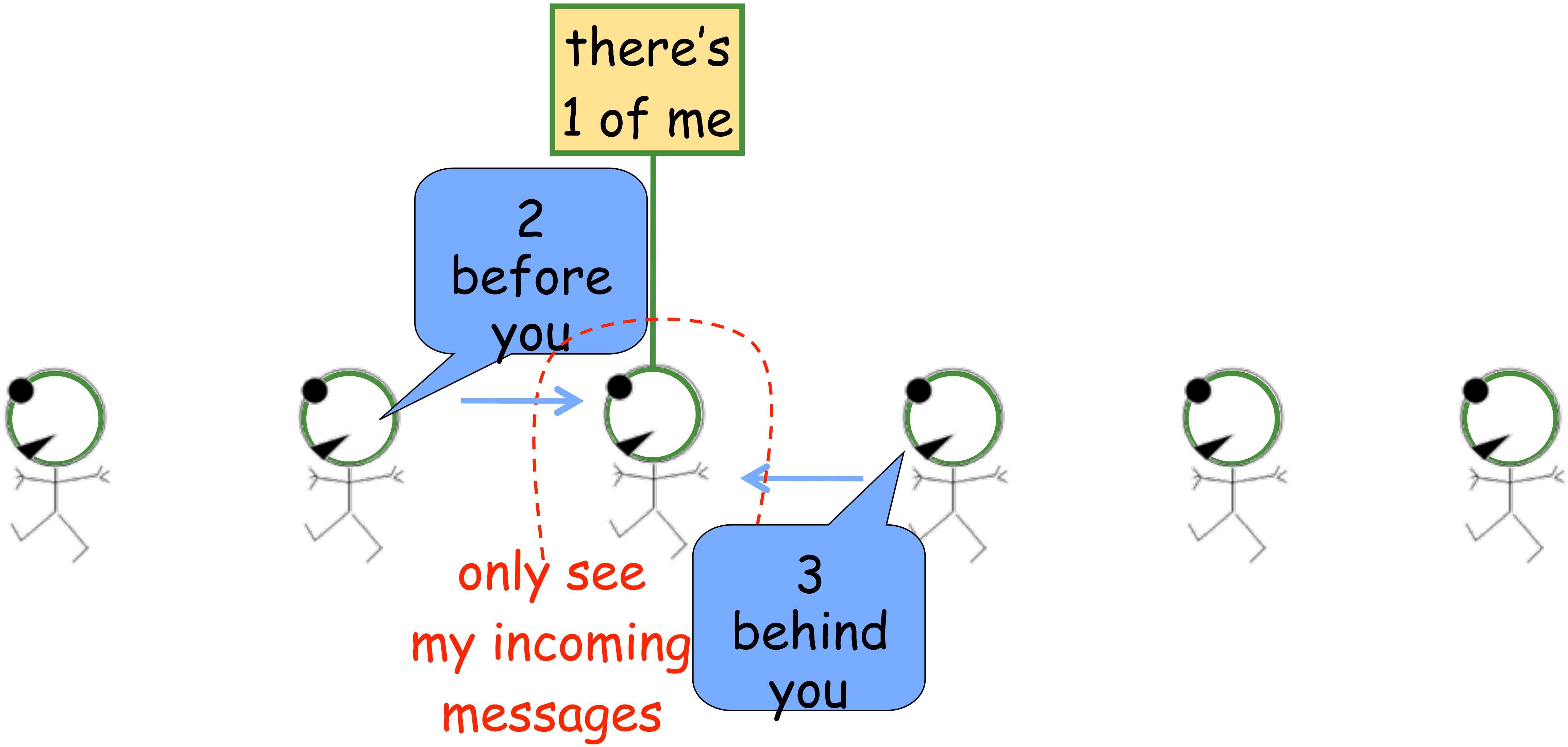

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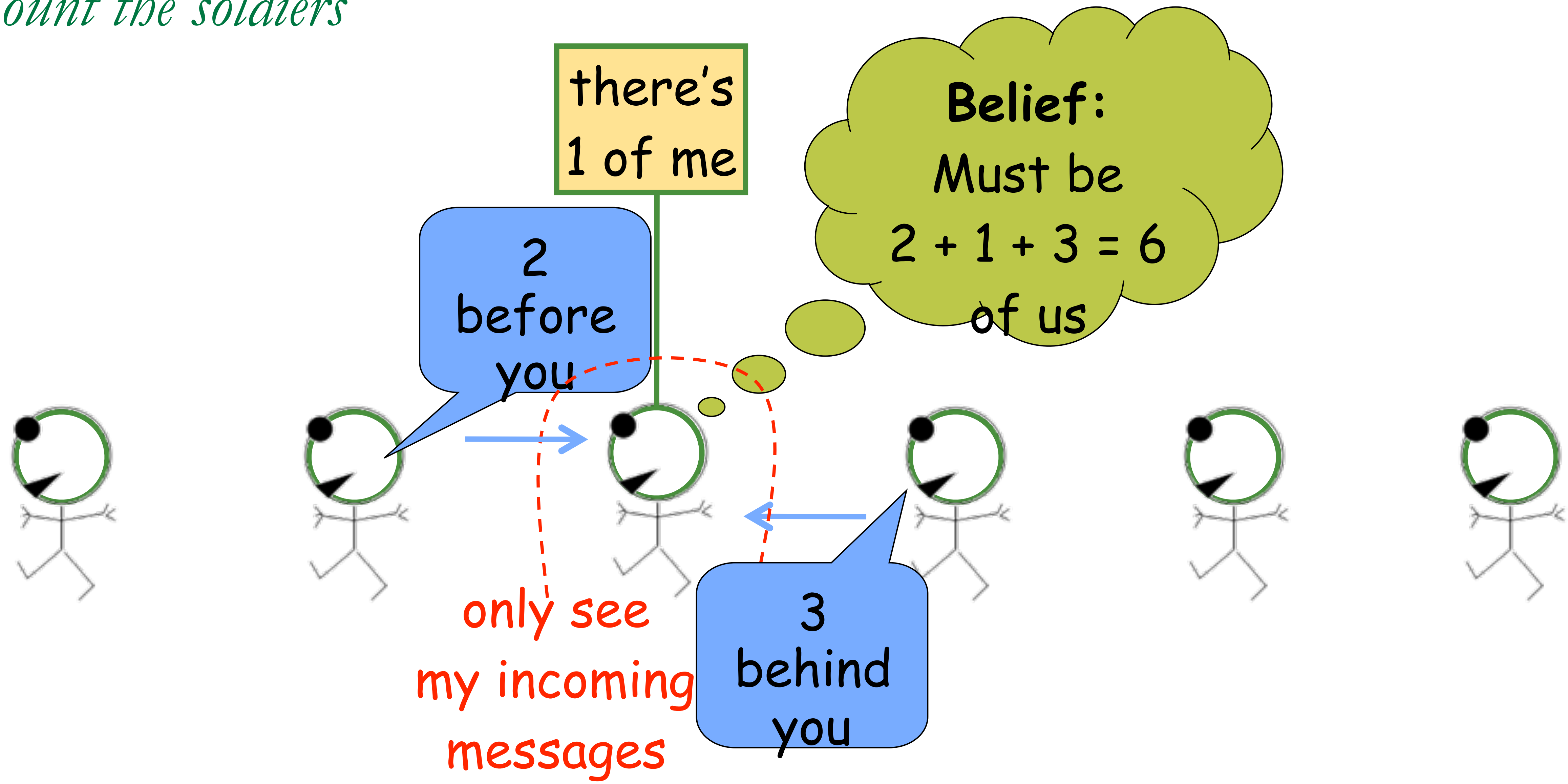

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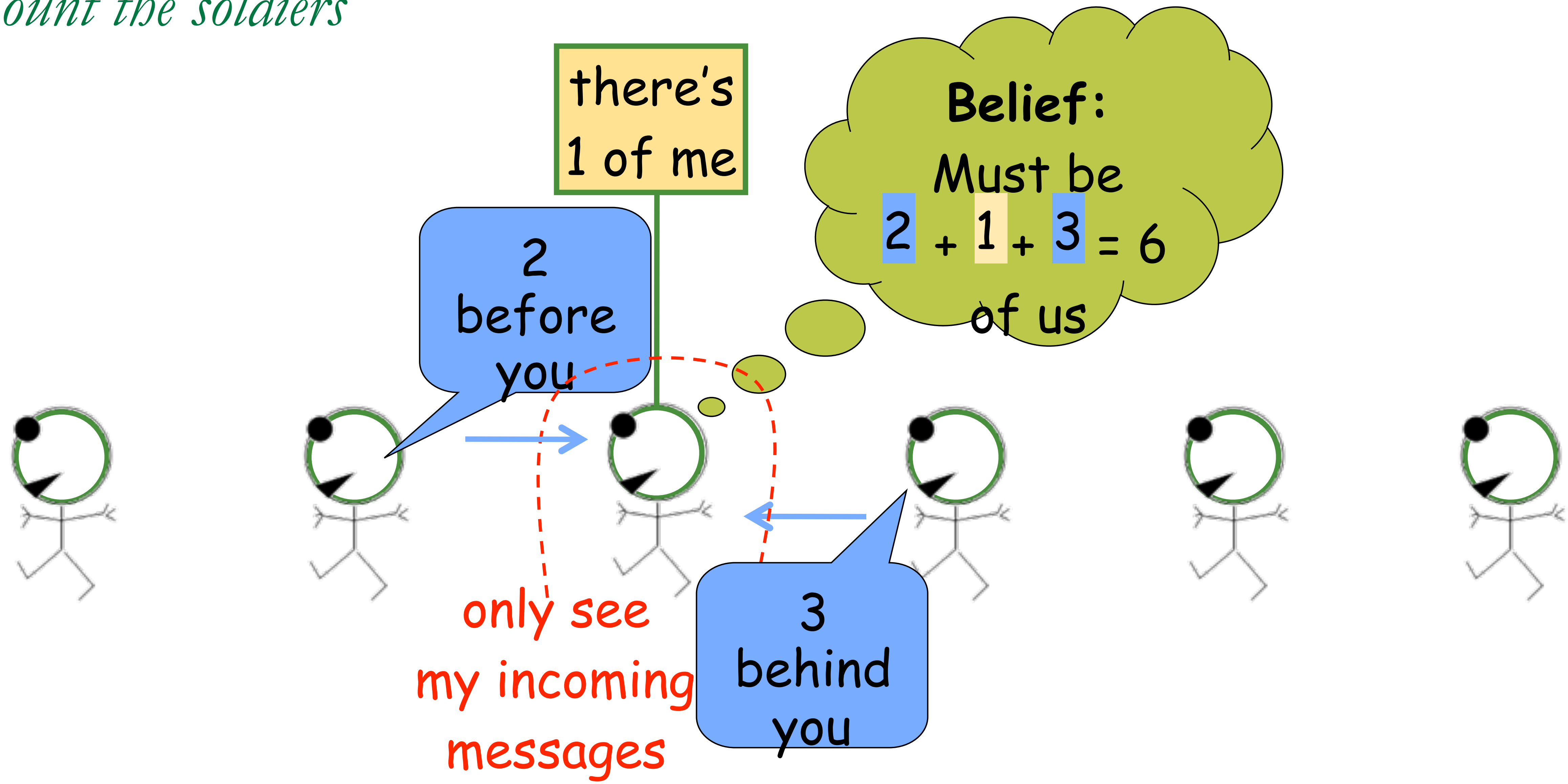

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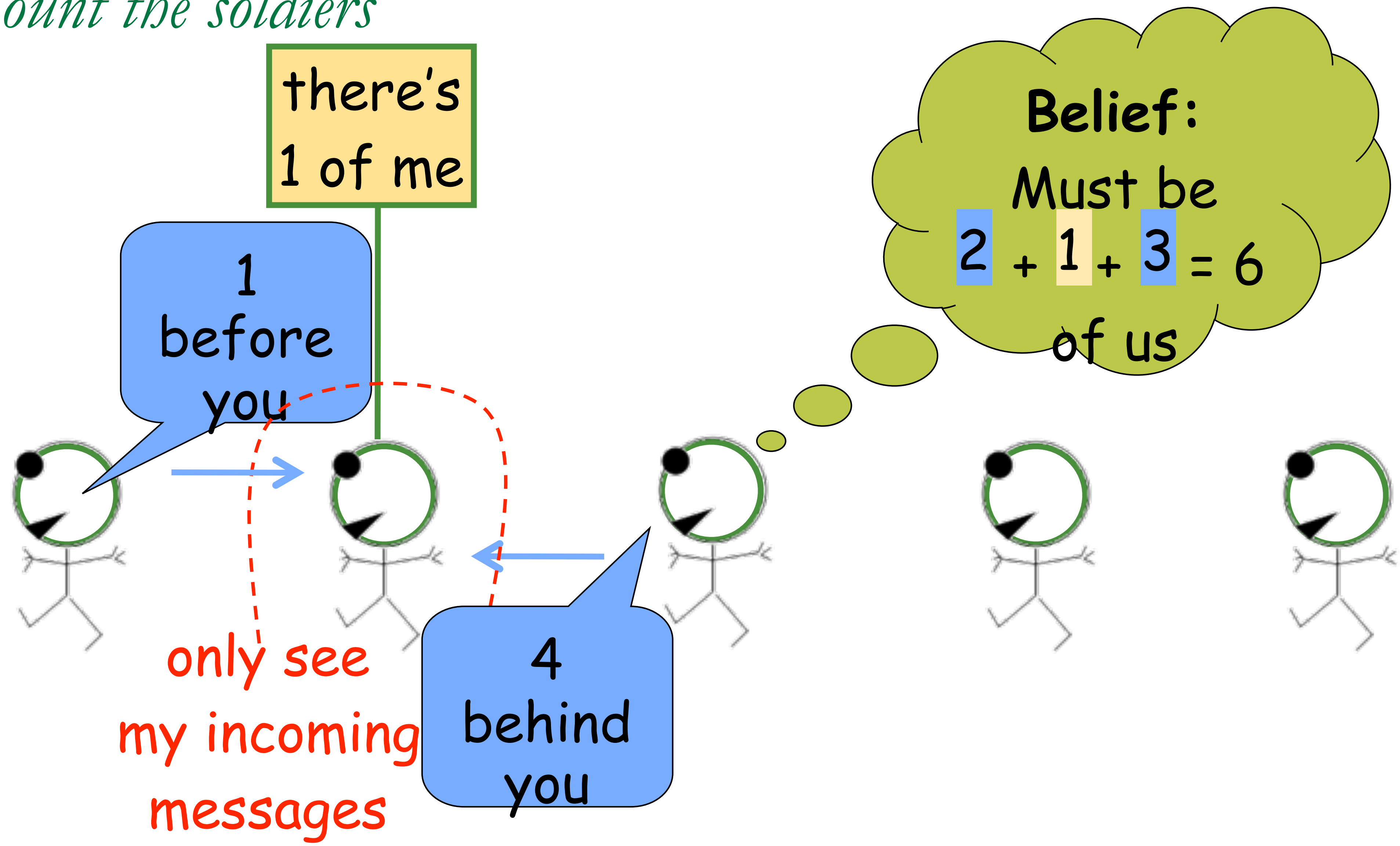

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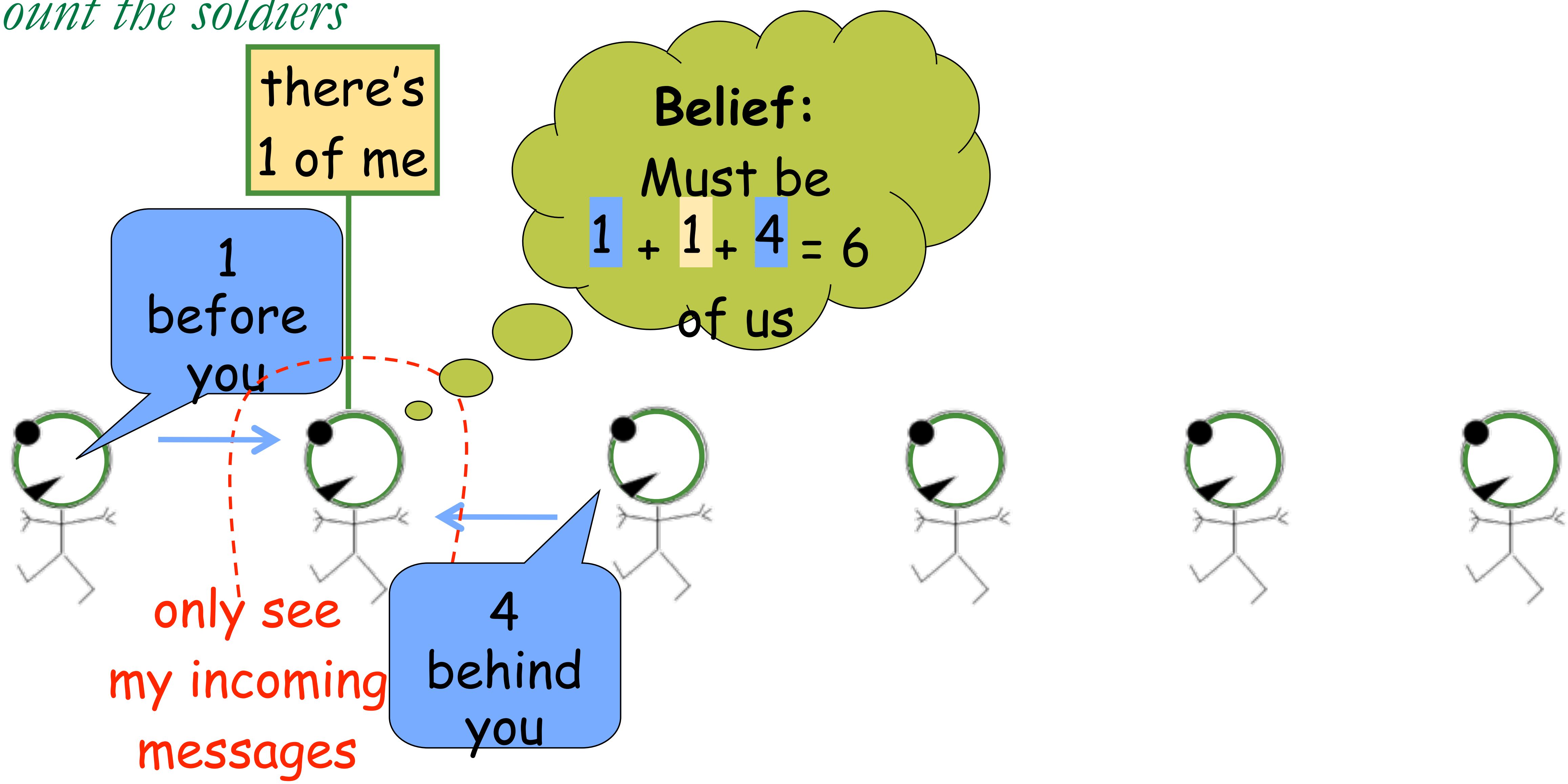

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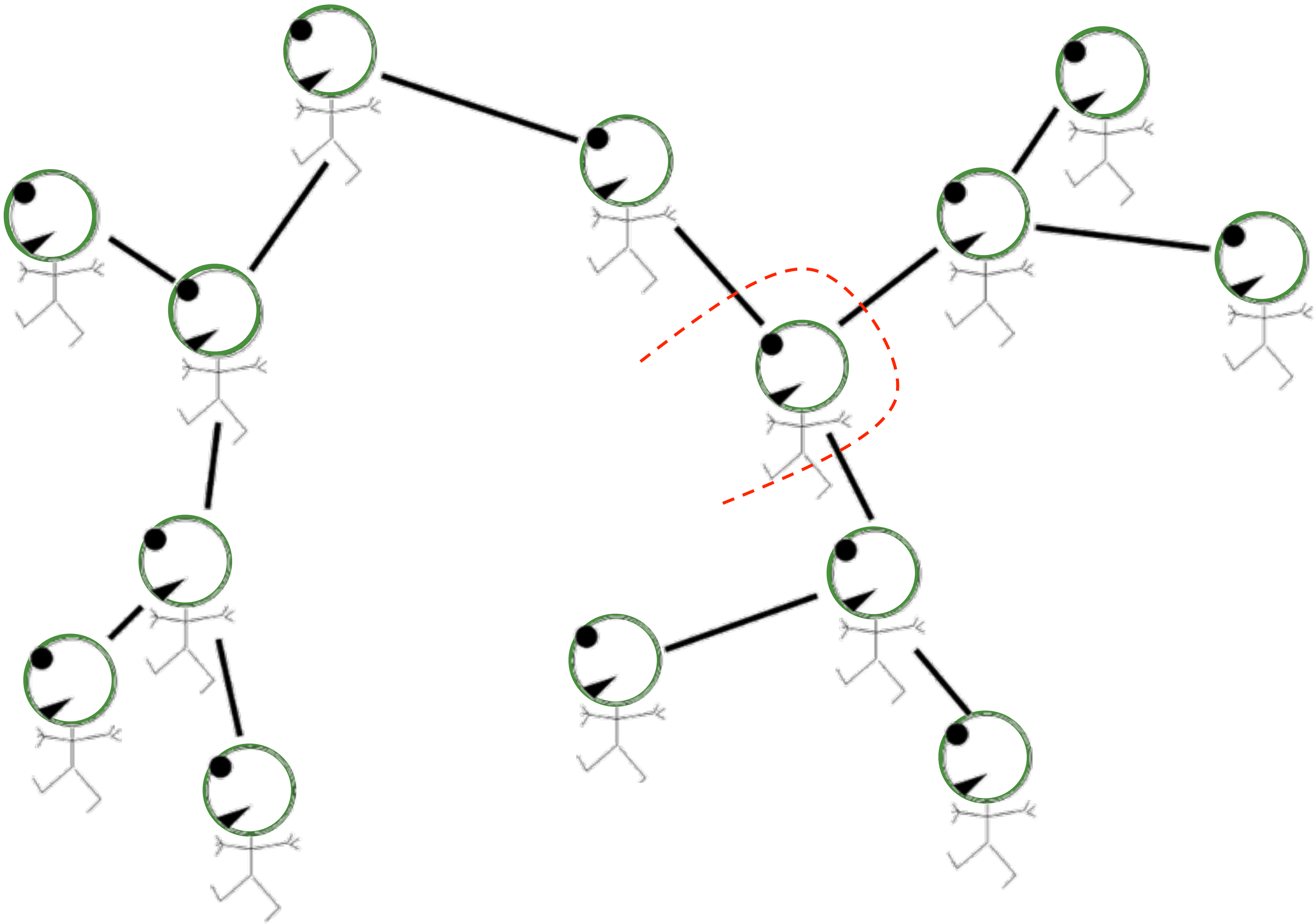
Great Ideas in ML: Message Passing

Each soldier receives reports from all branches of tree



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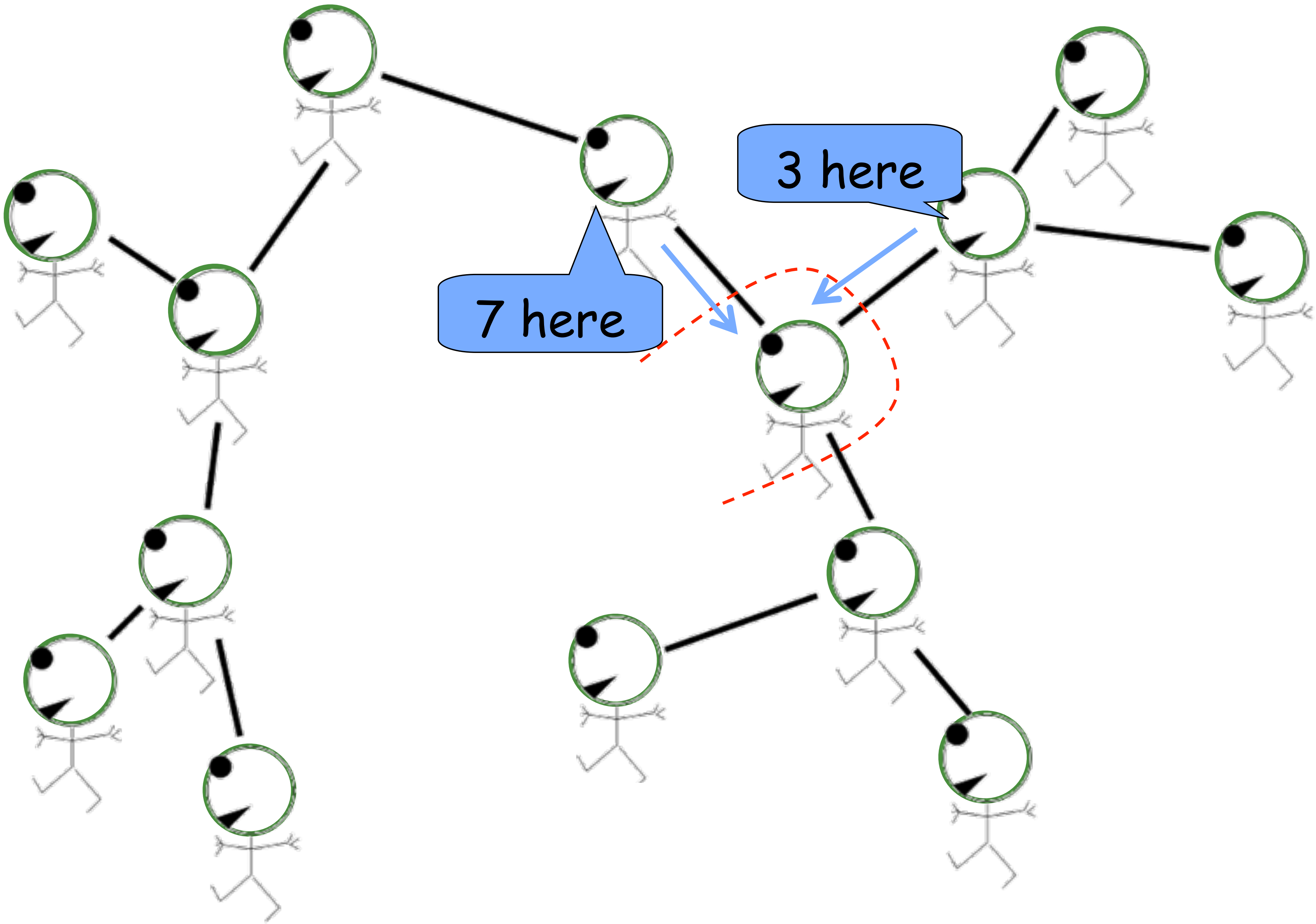

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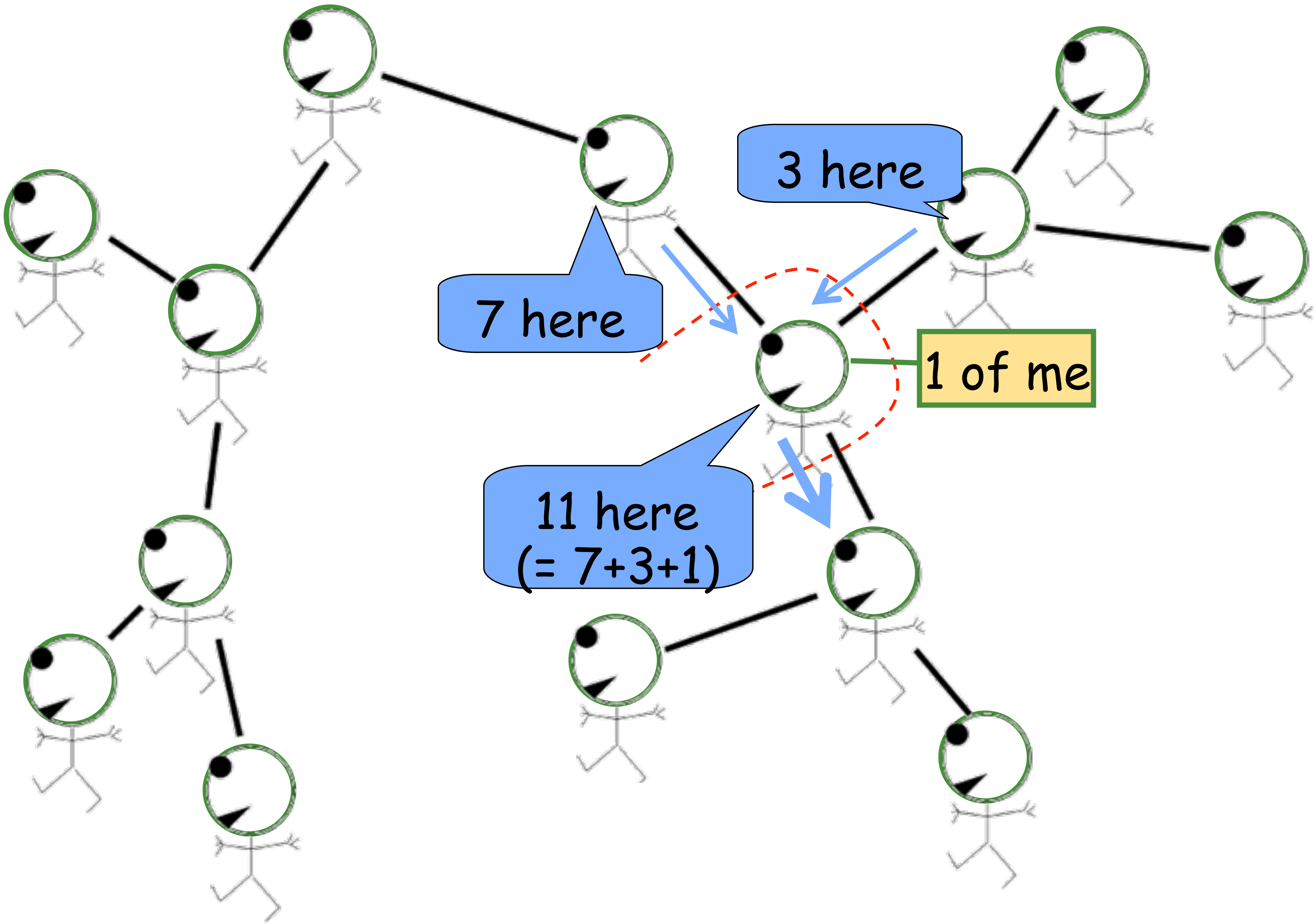

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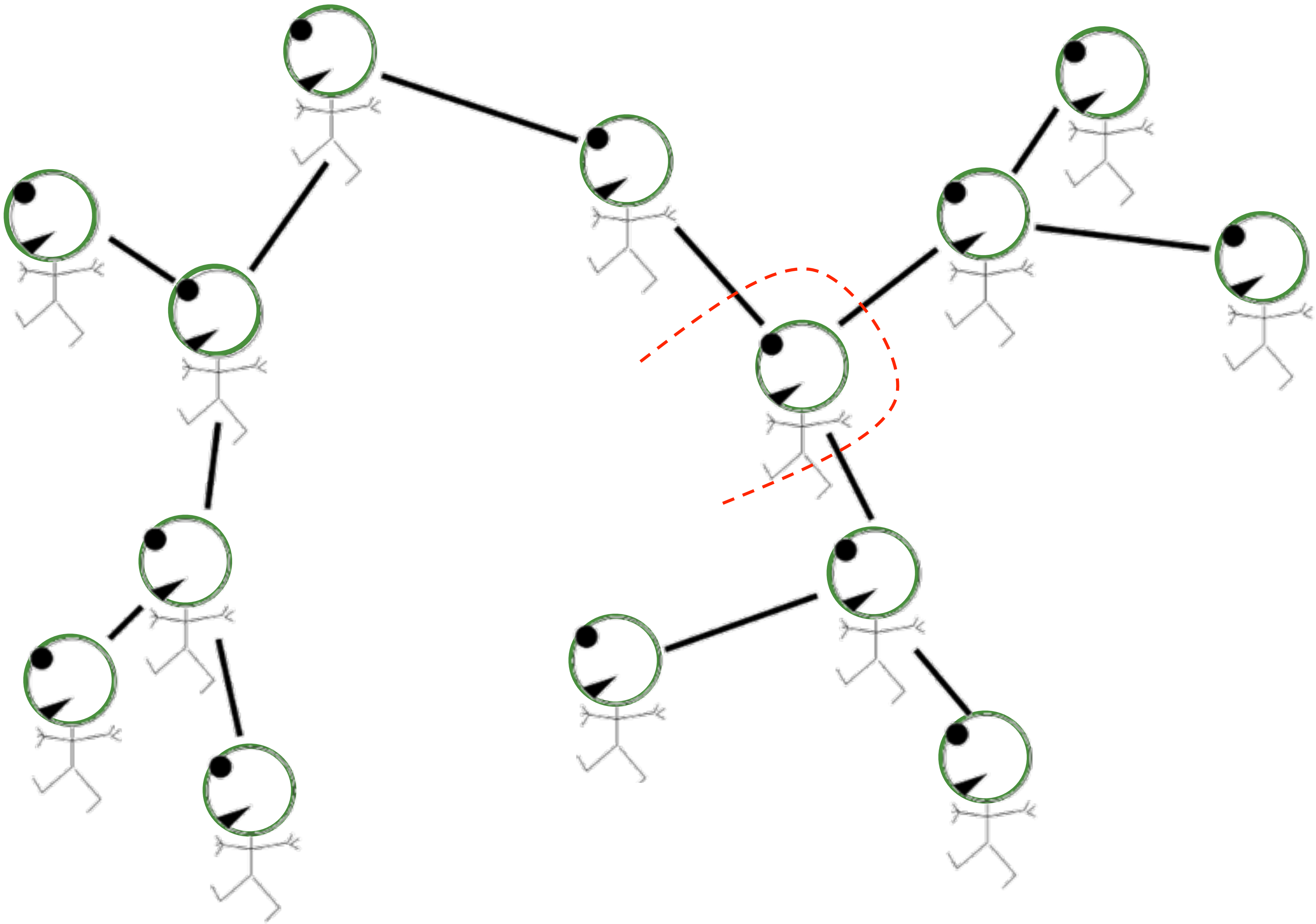

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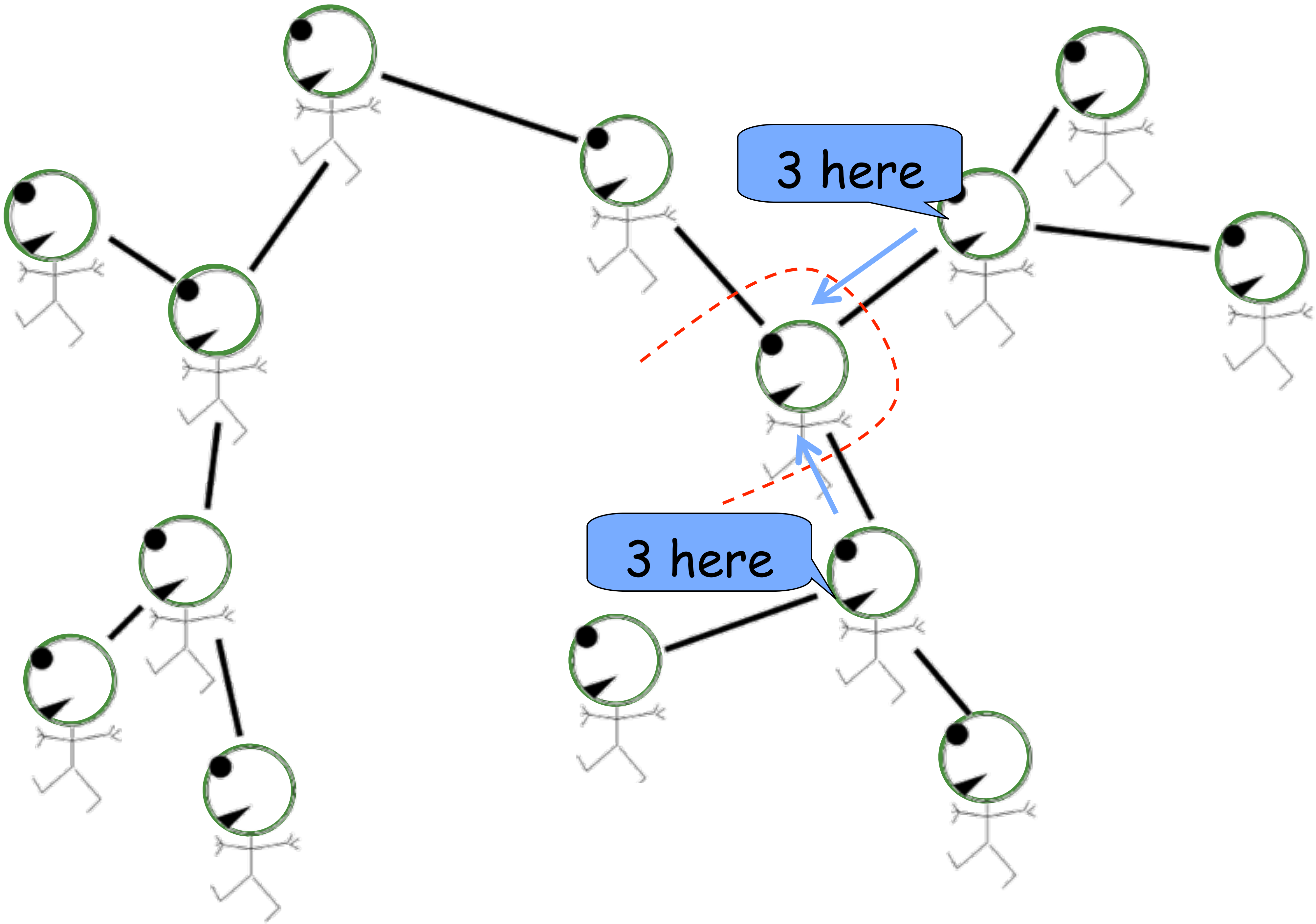

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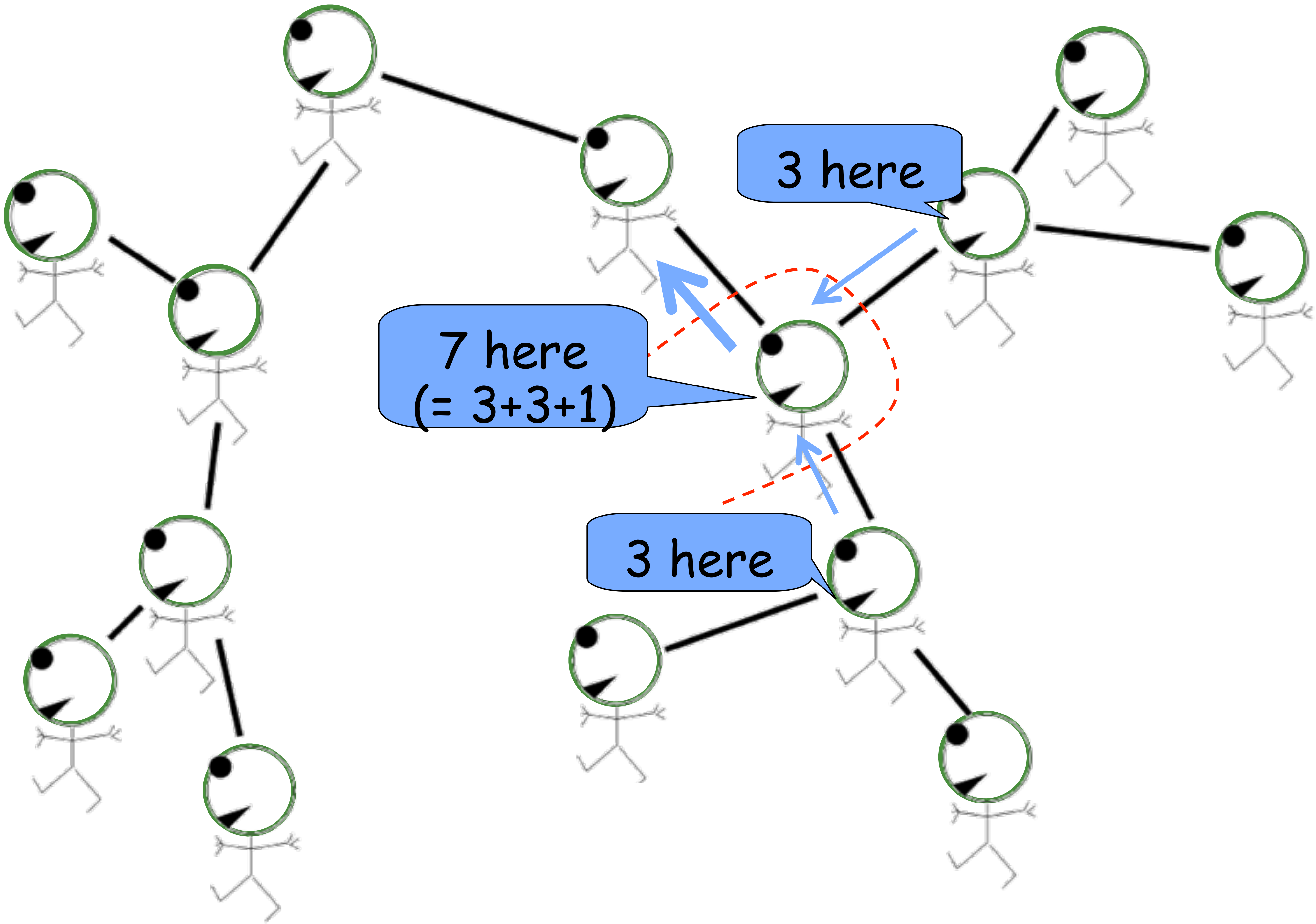

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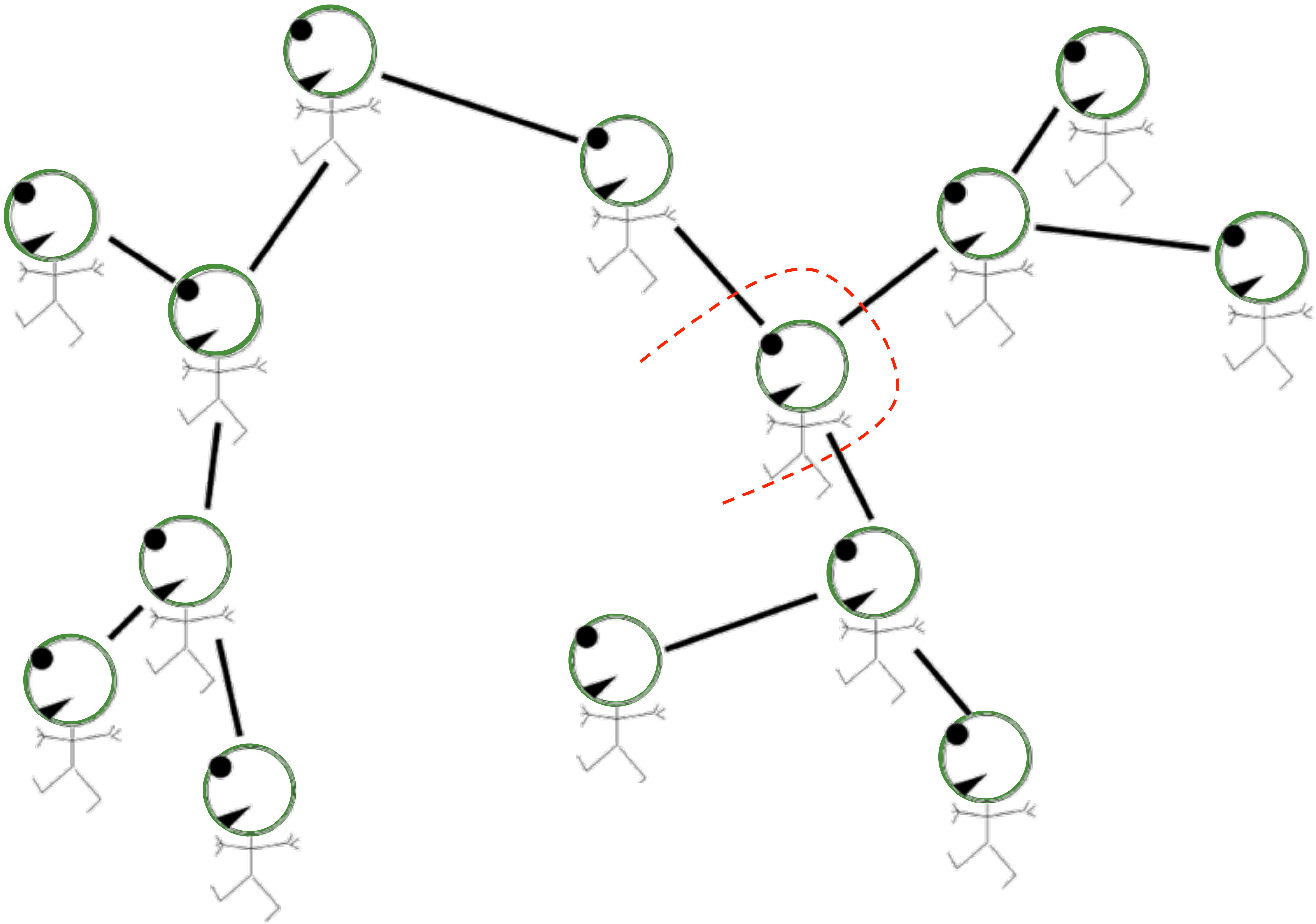

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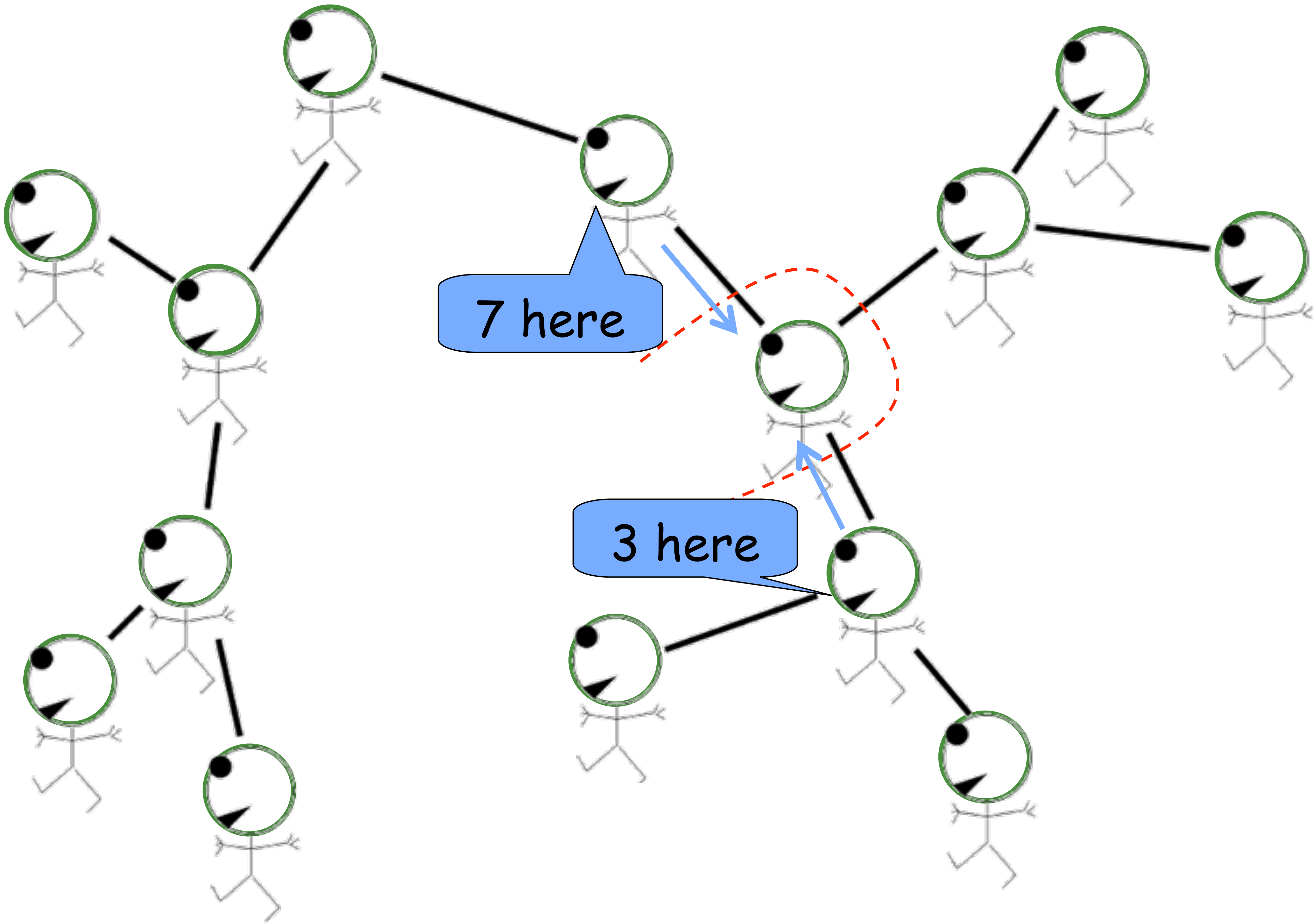

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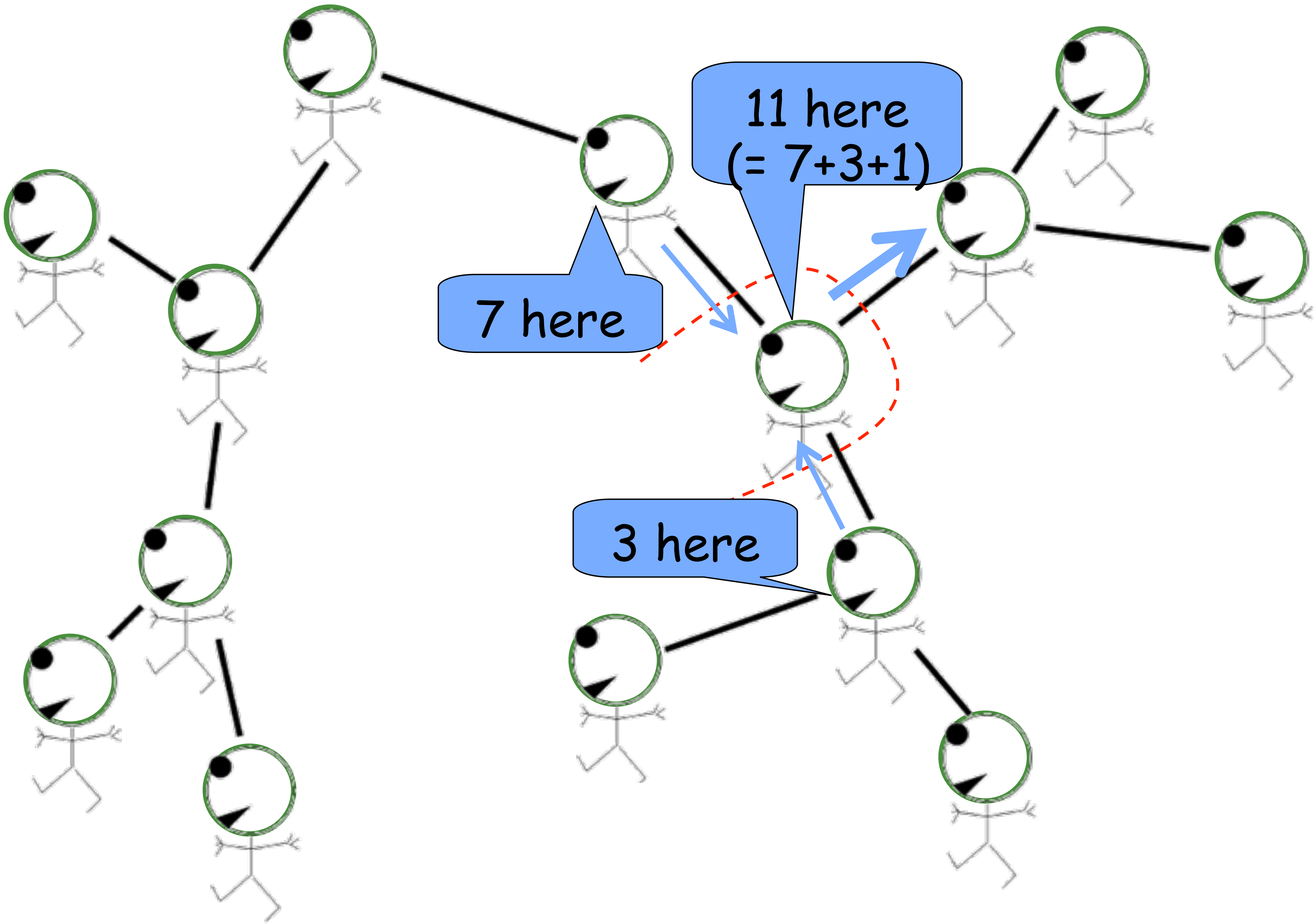

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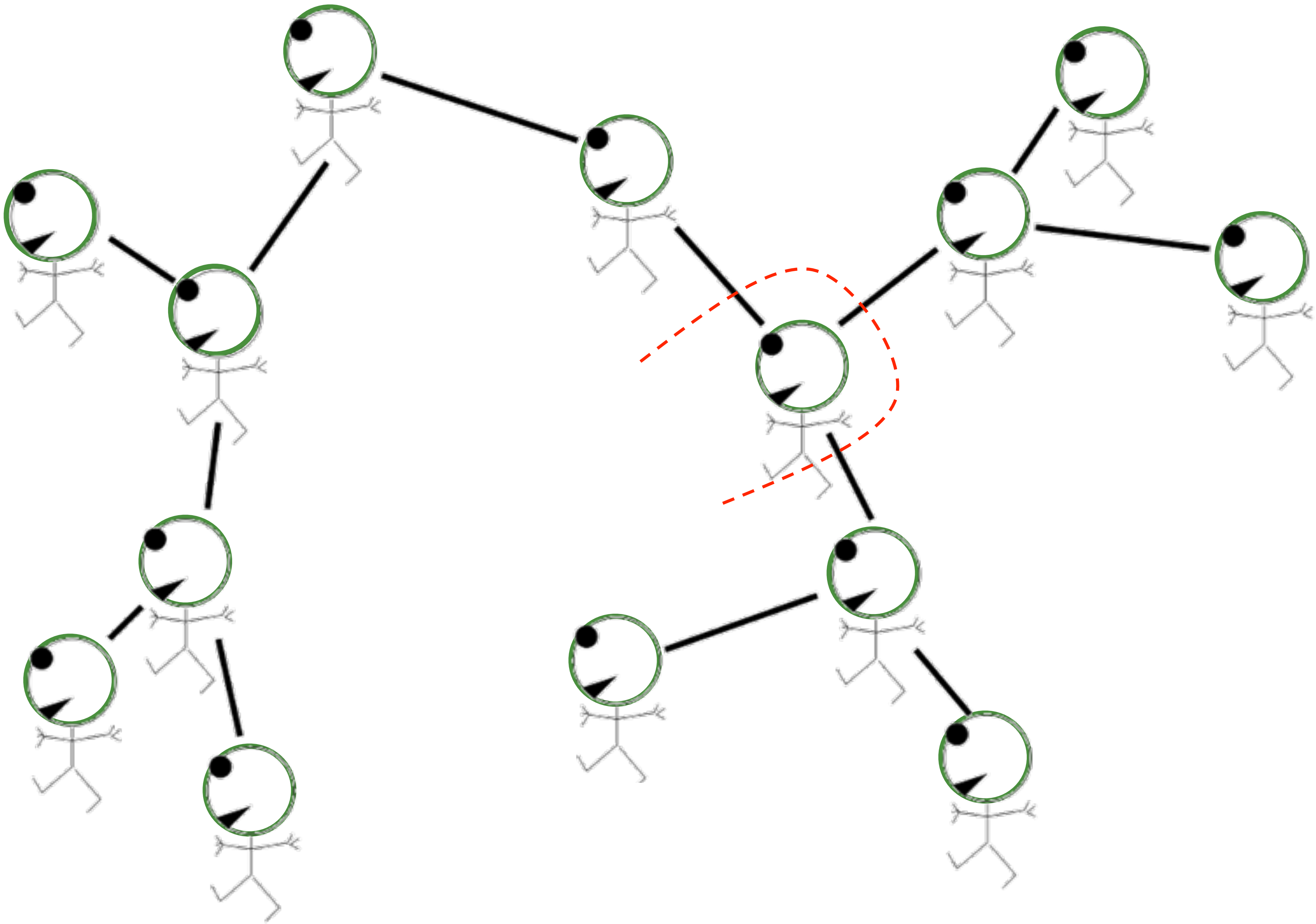

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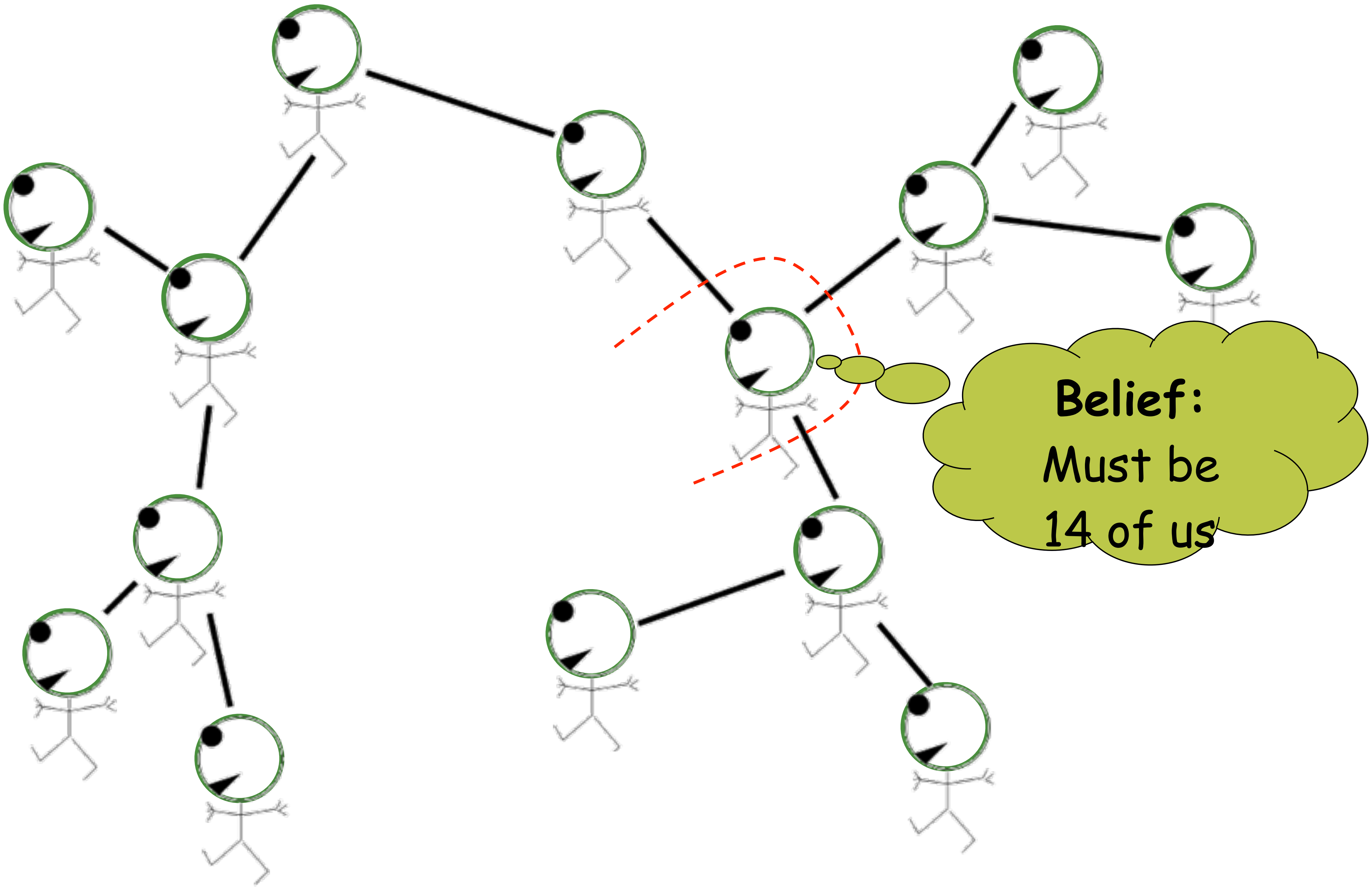

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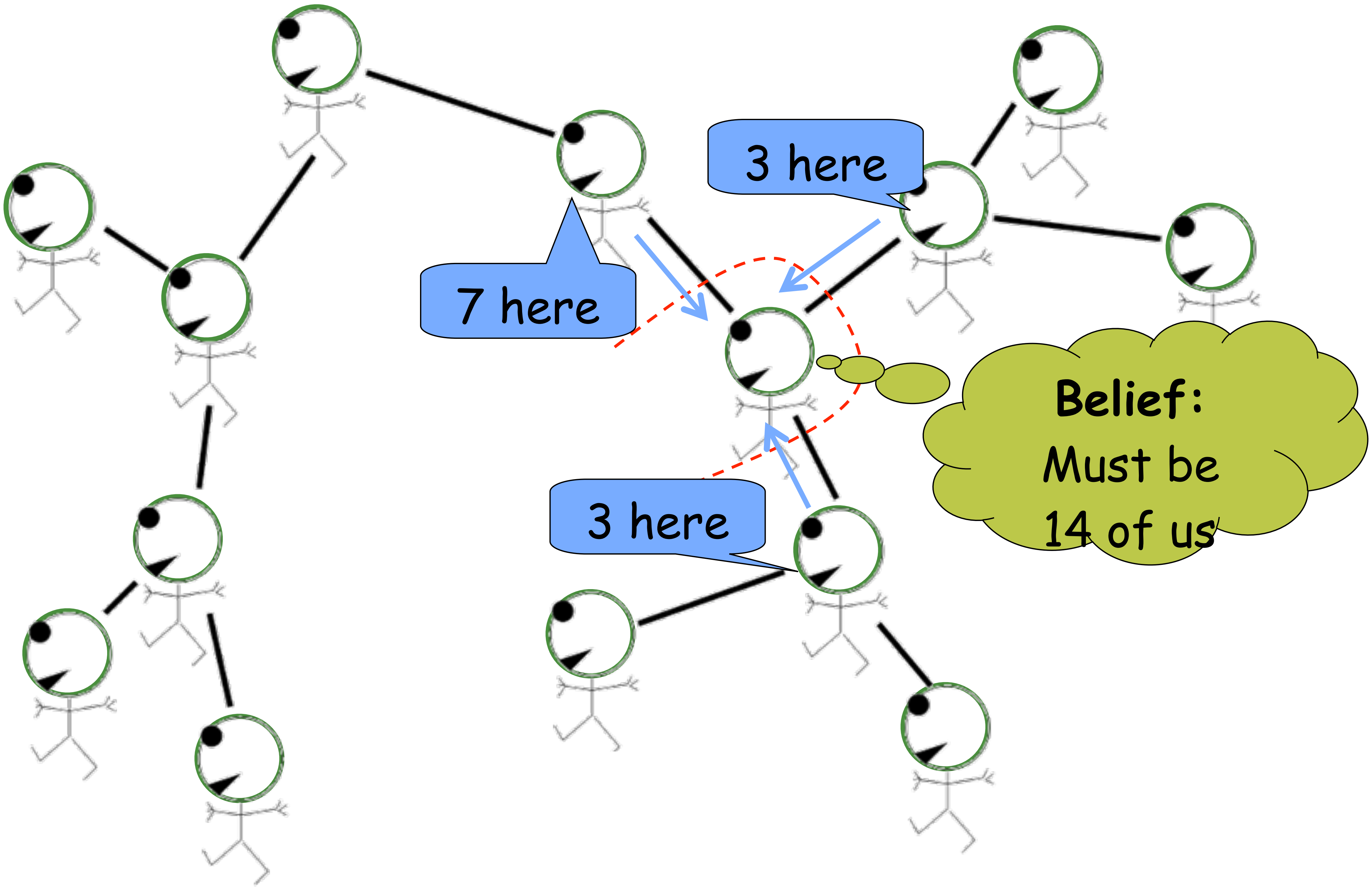

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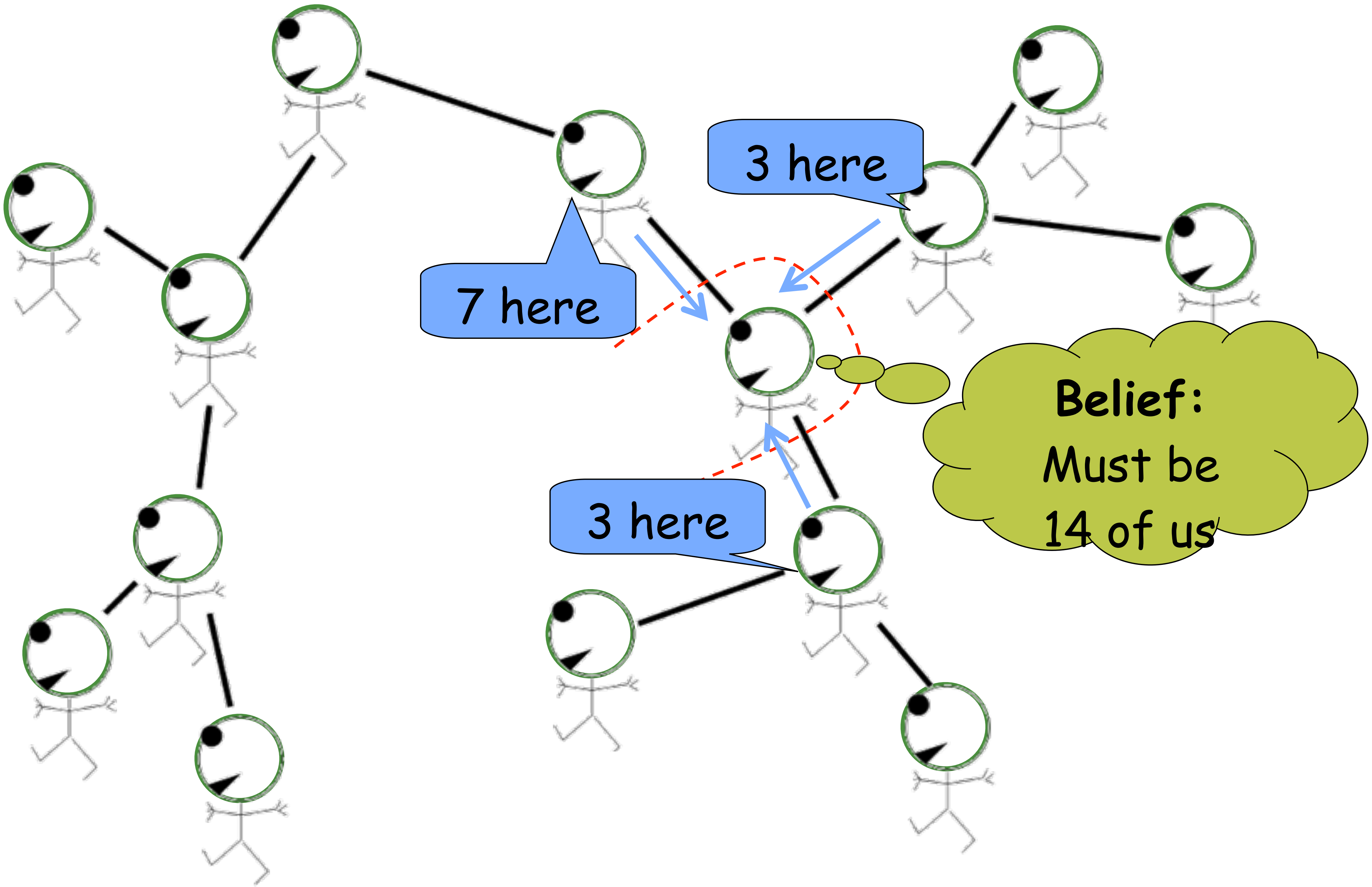

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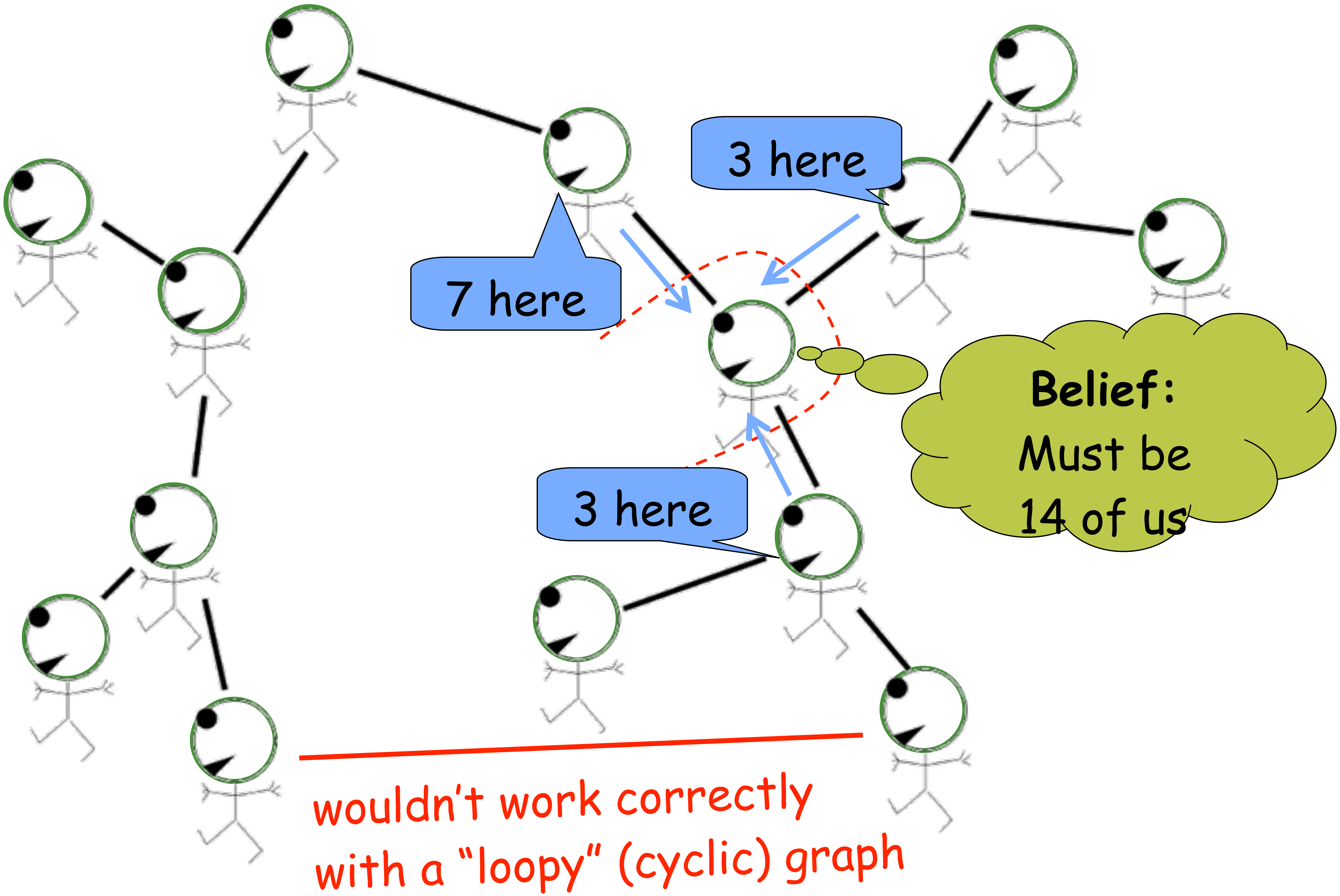

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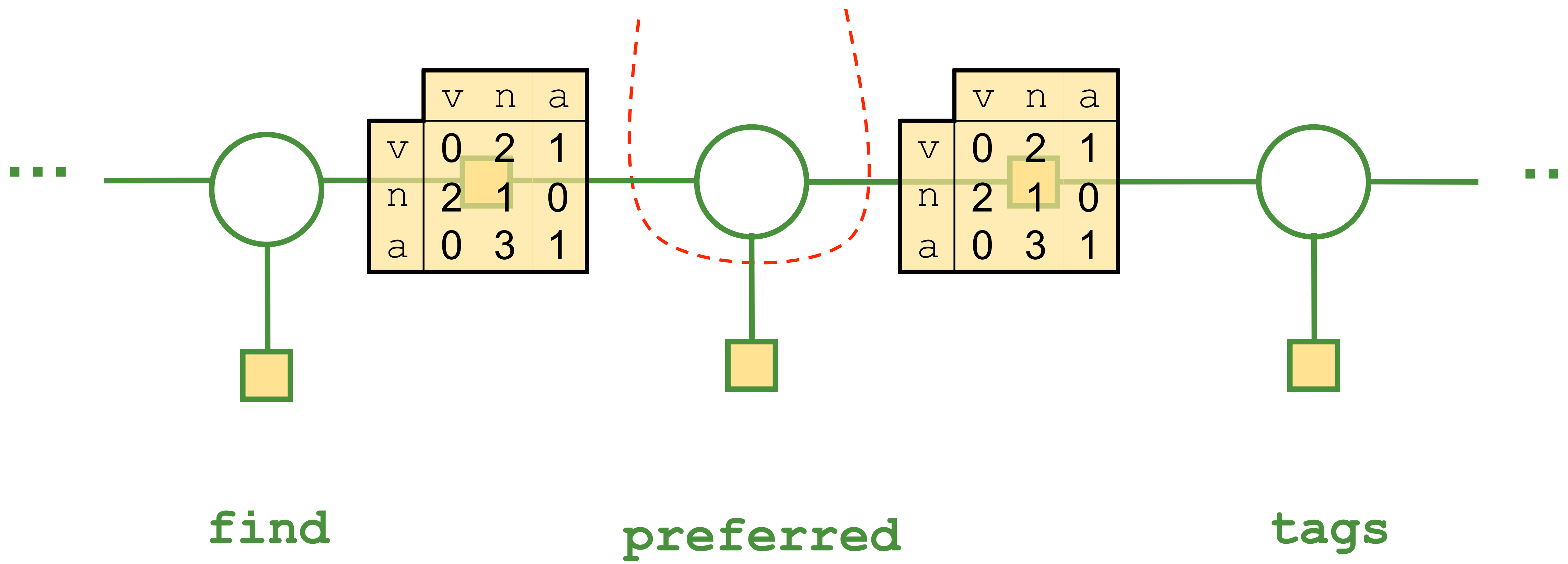
Great ideas in ML: Forward-Backward

- In the CRF, message passing = forward-backward= "sum-product algorithm"



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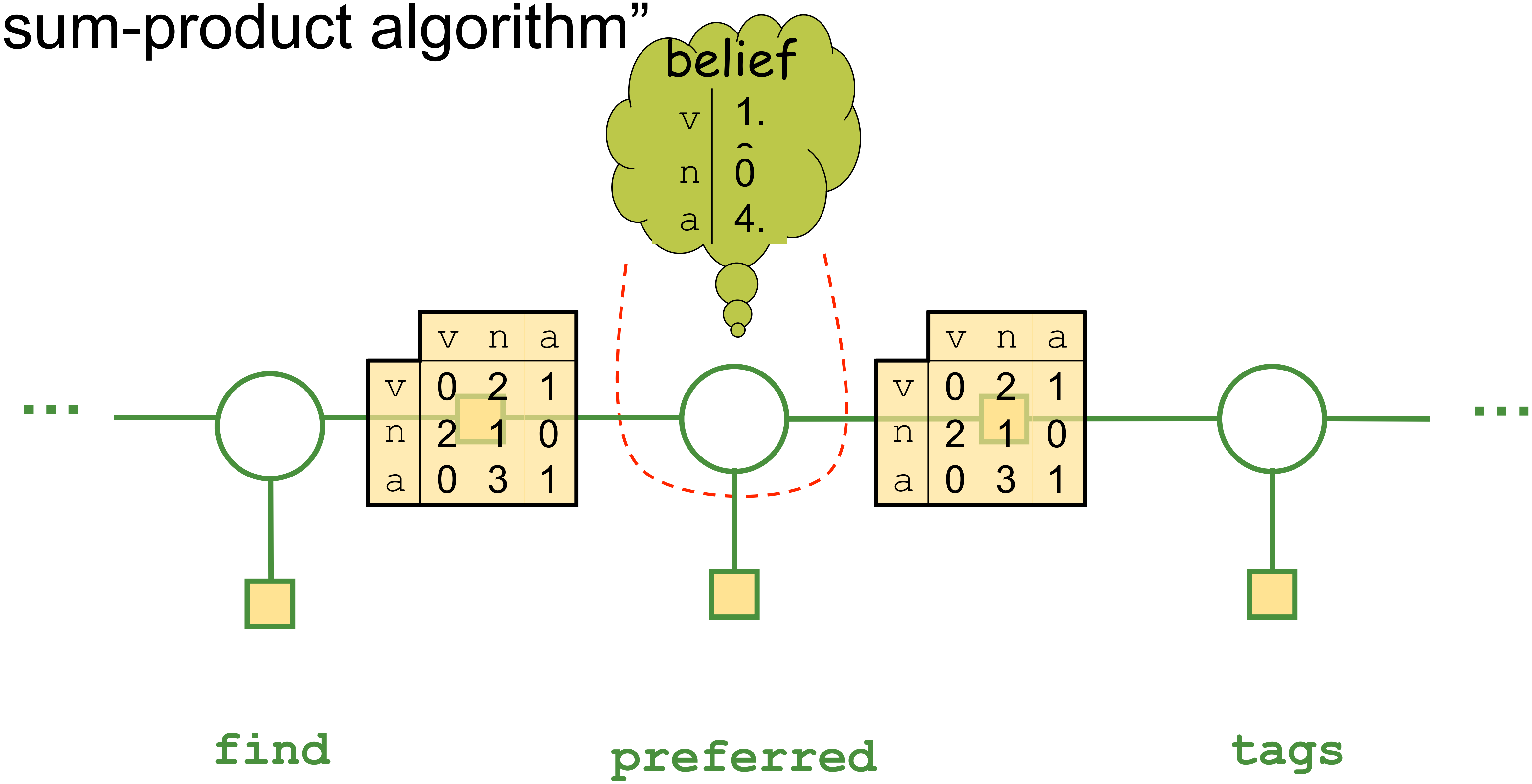

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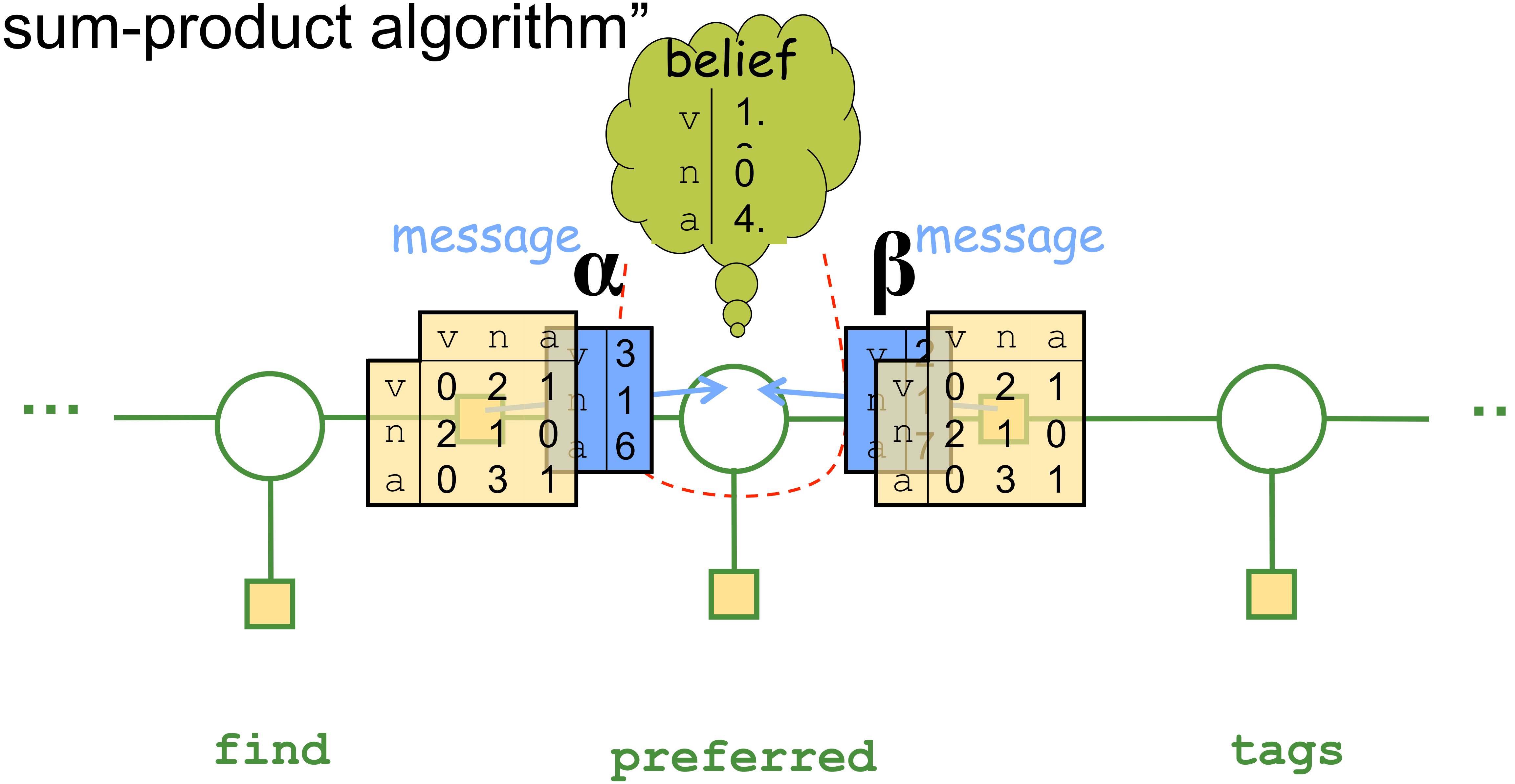

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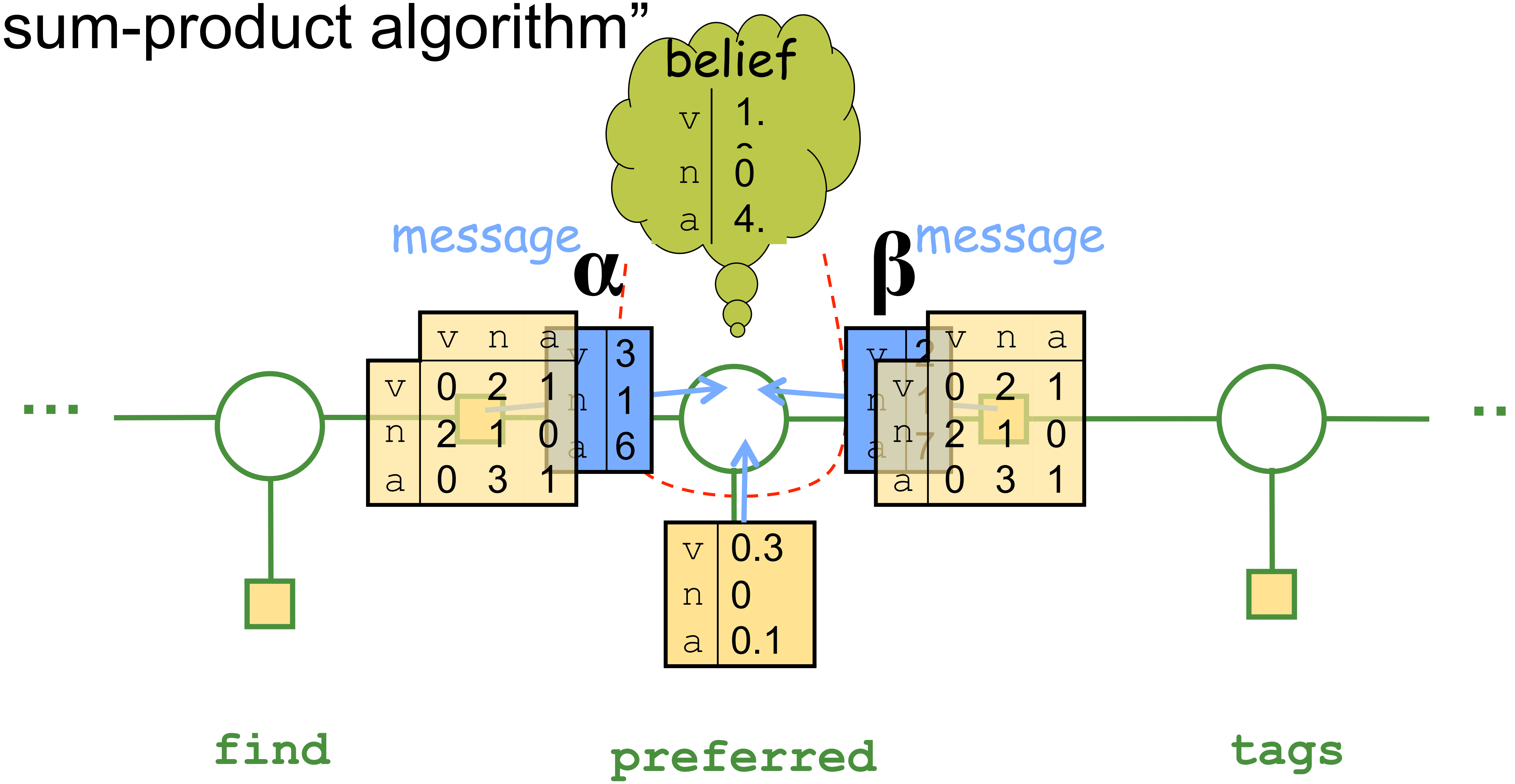

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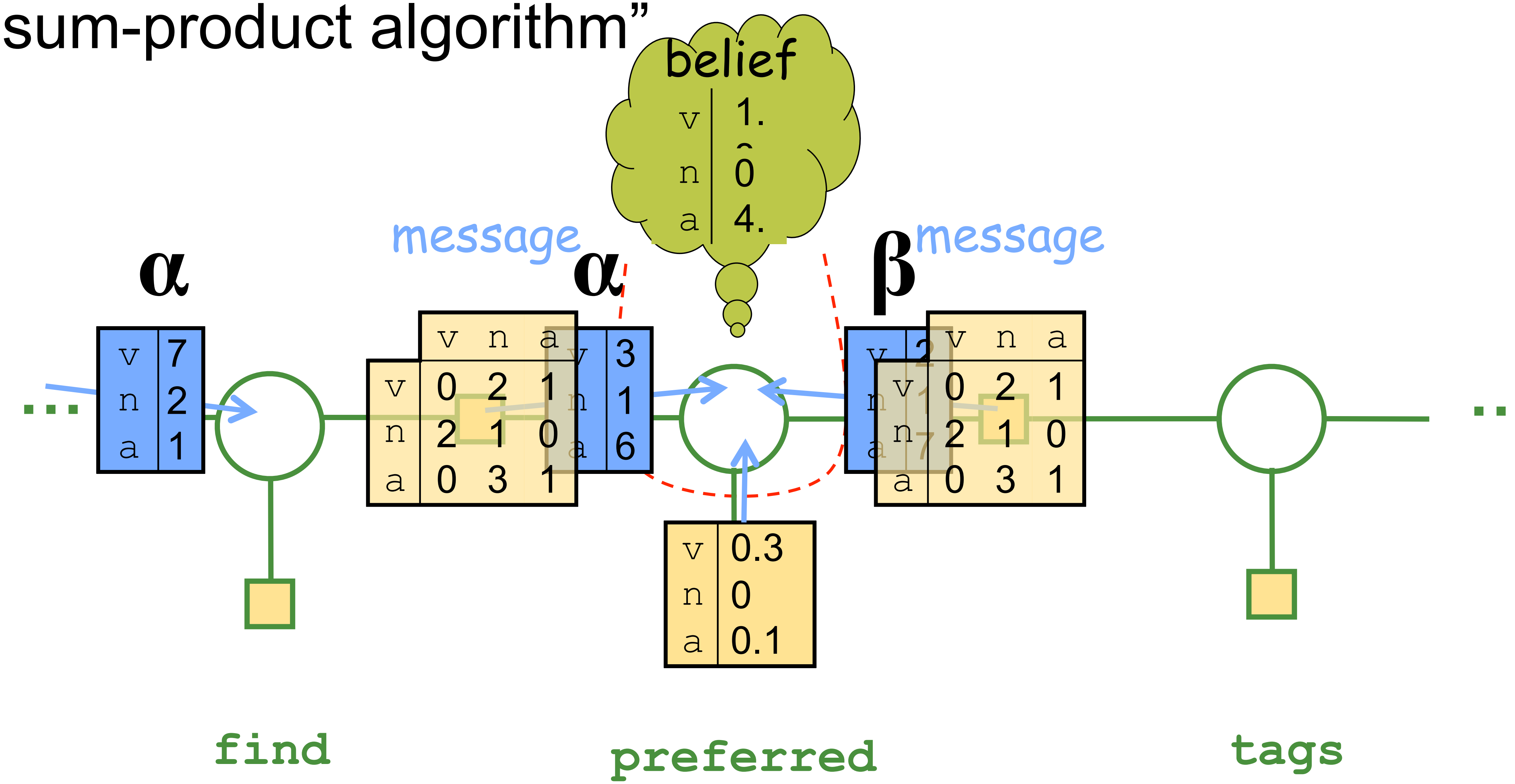

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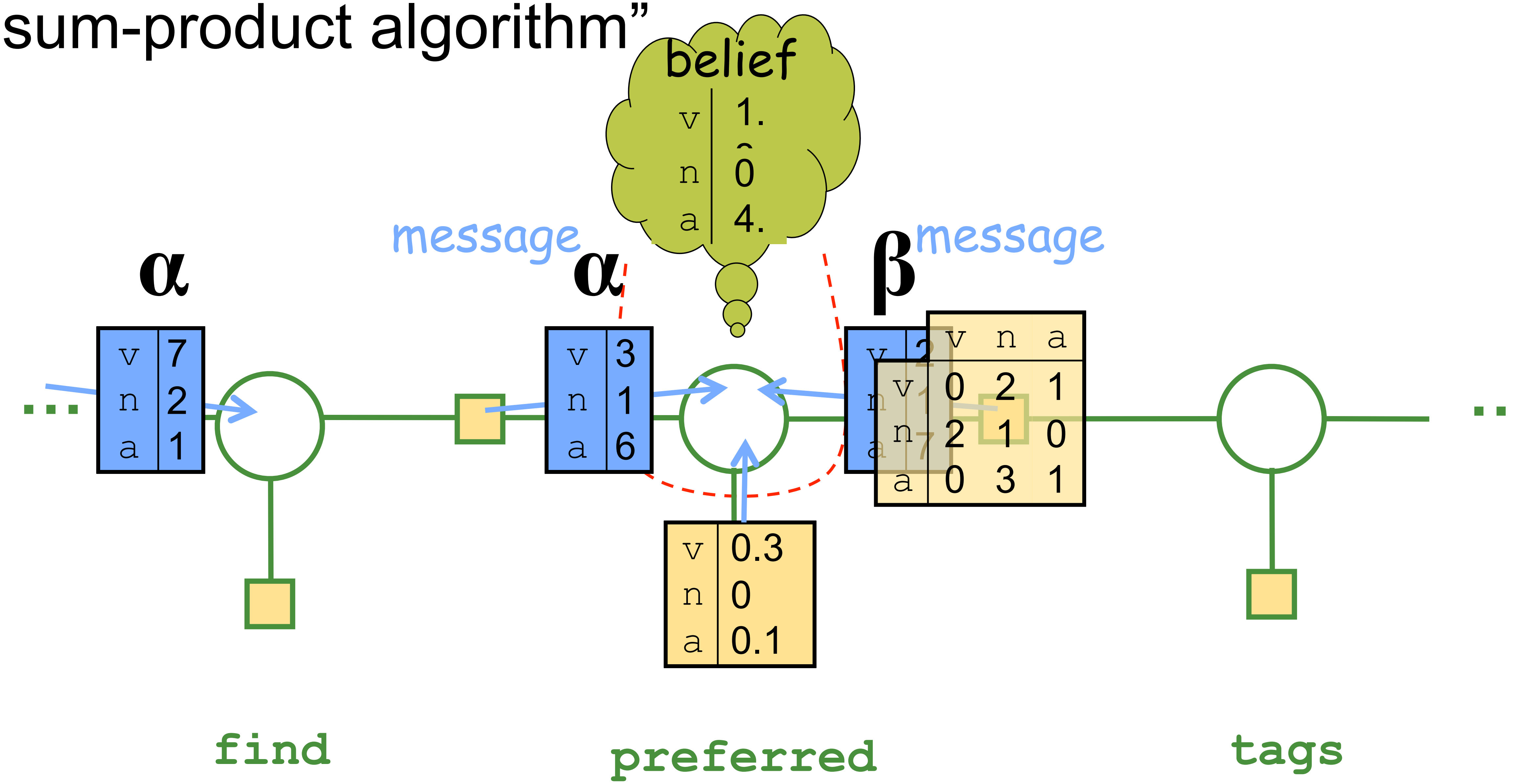

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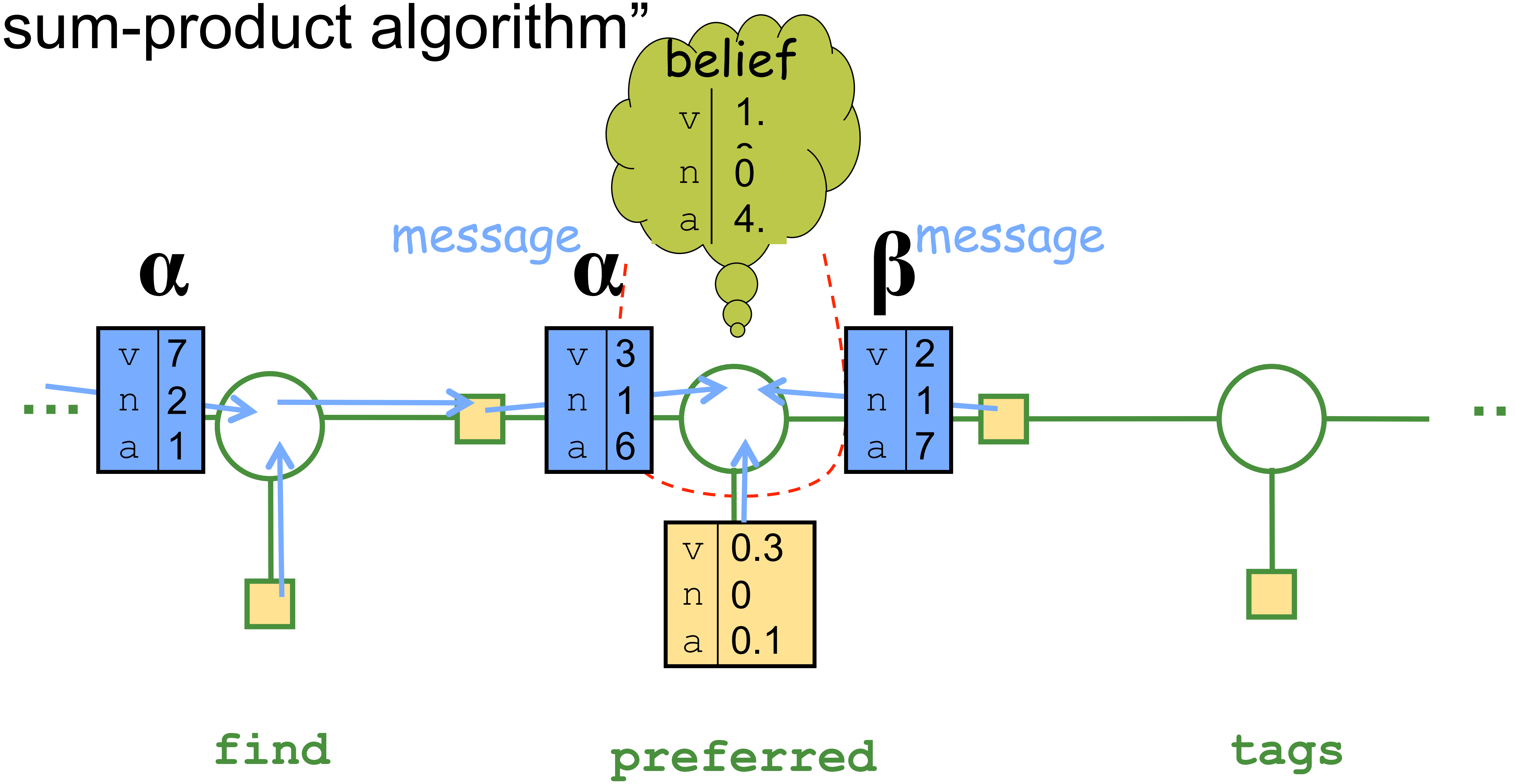

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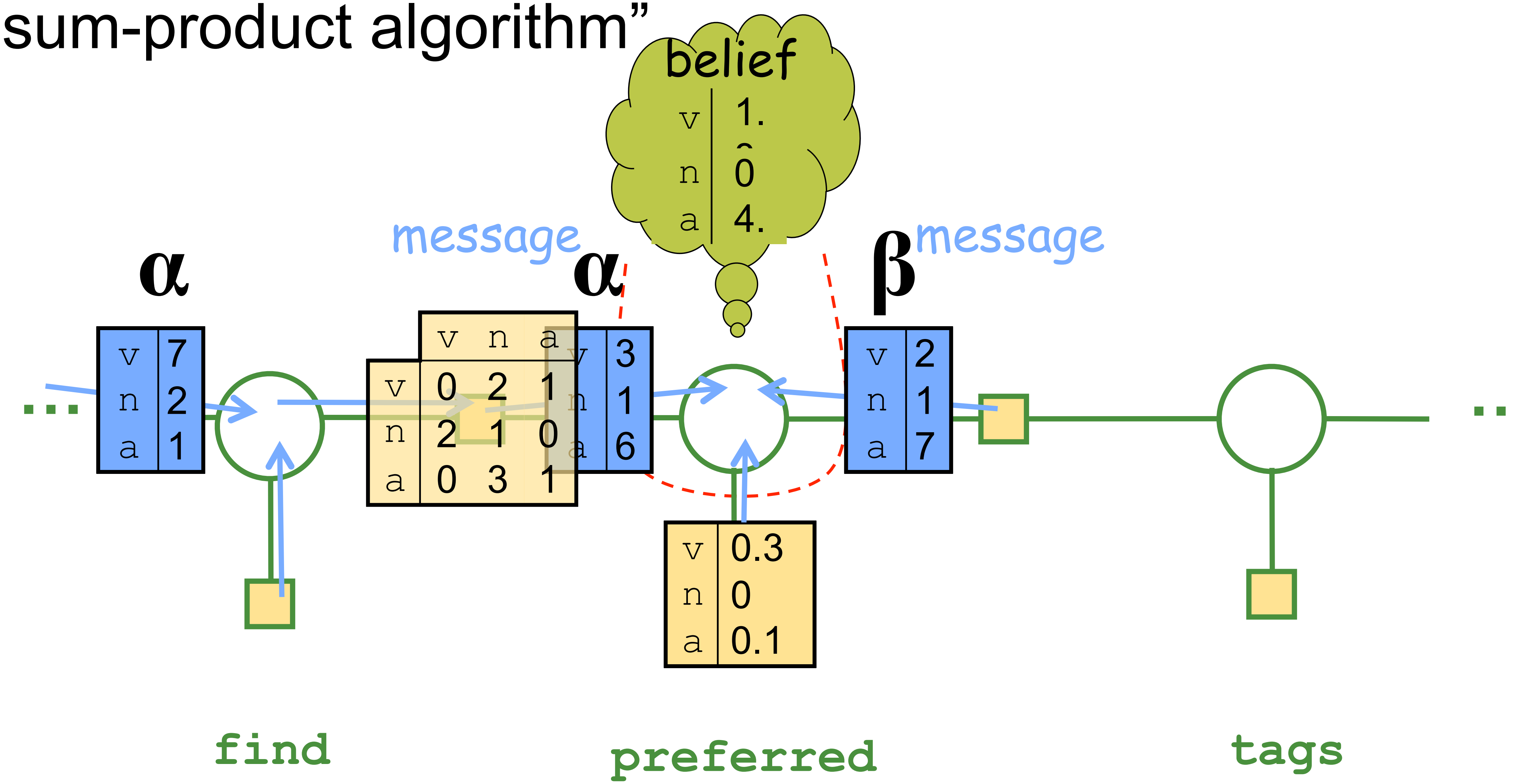

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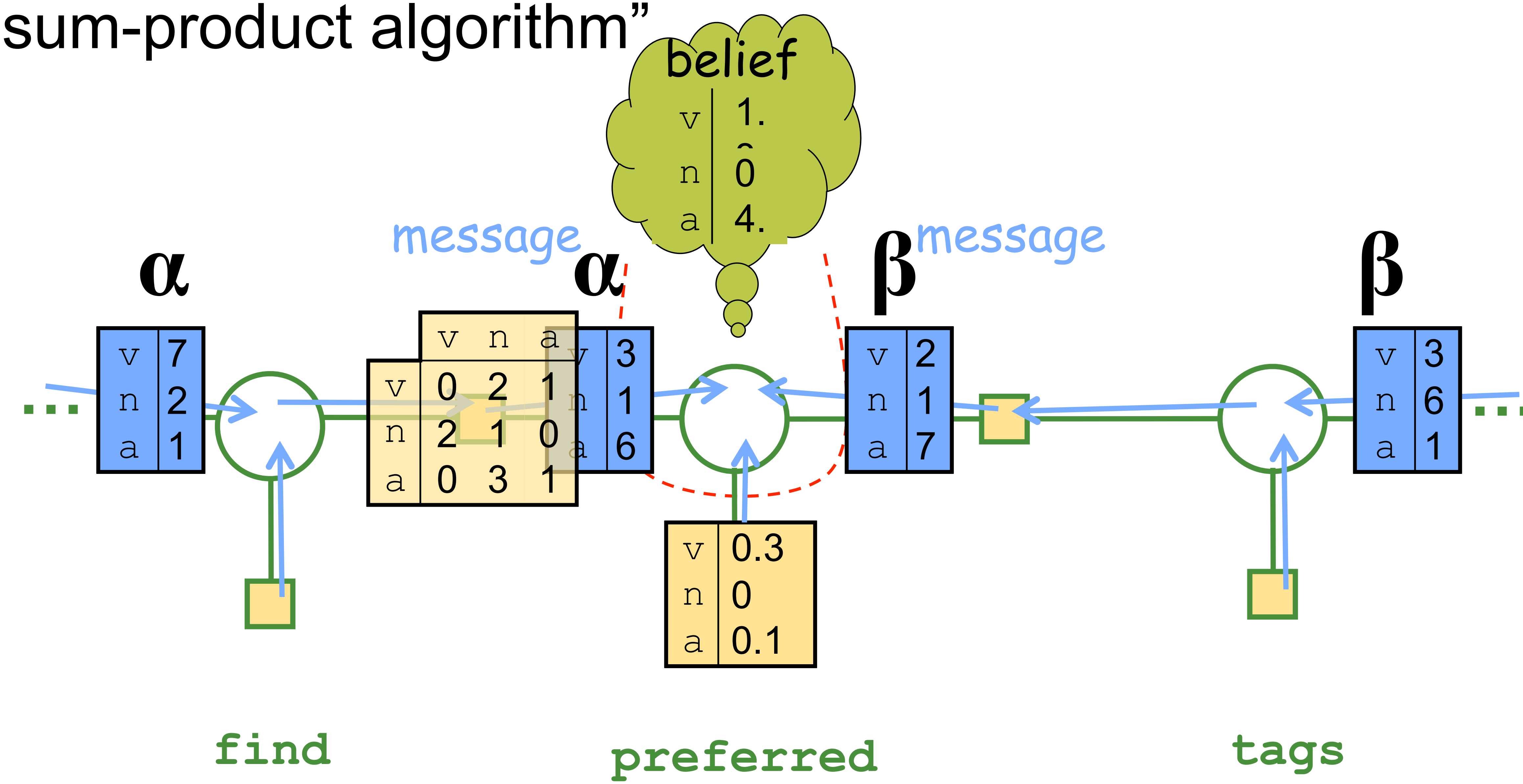

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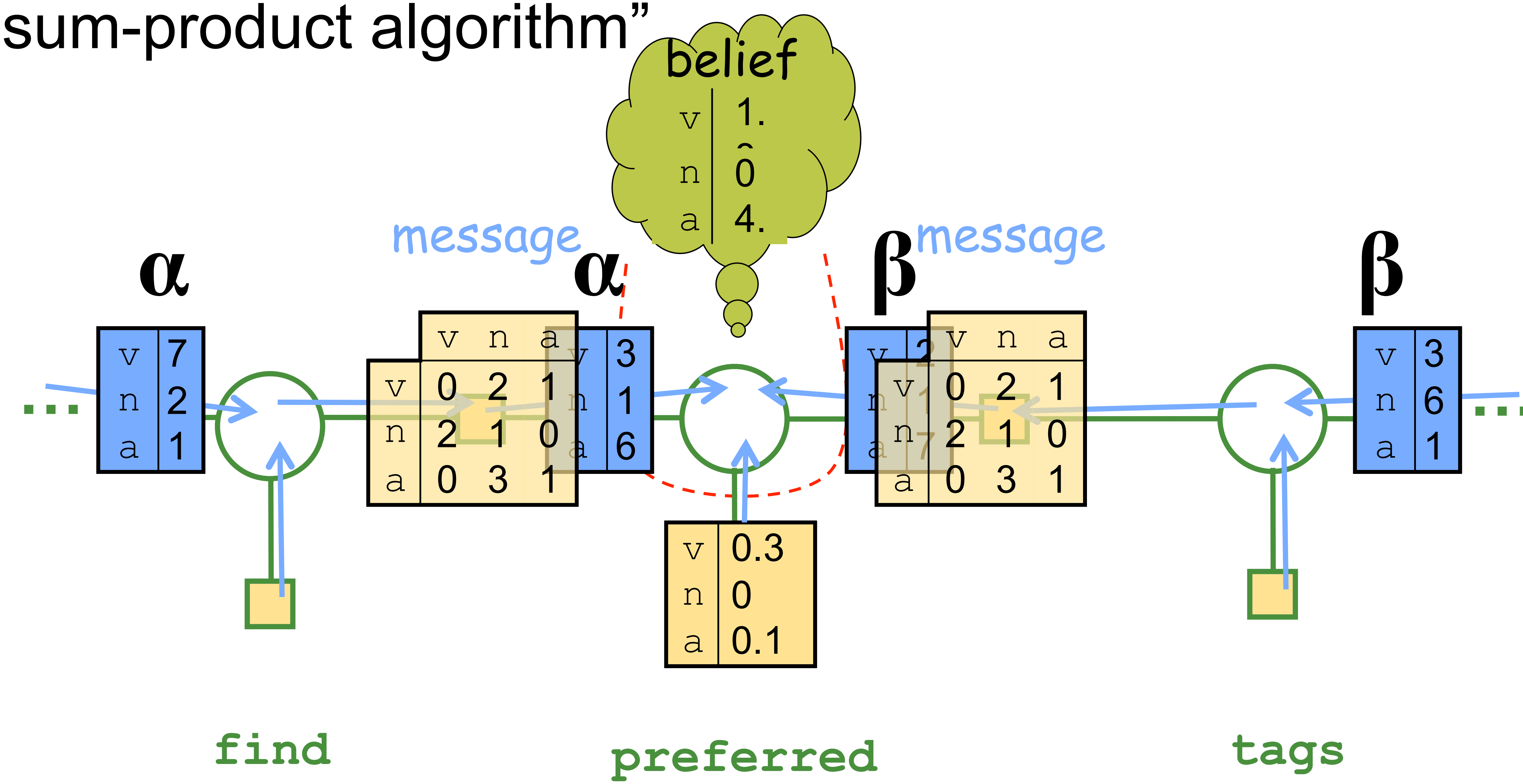


Sum-Product Equations

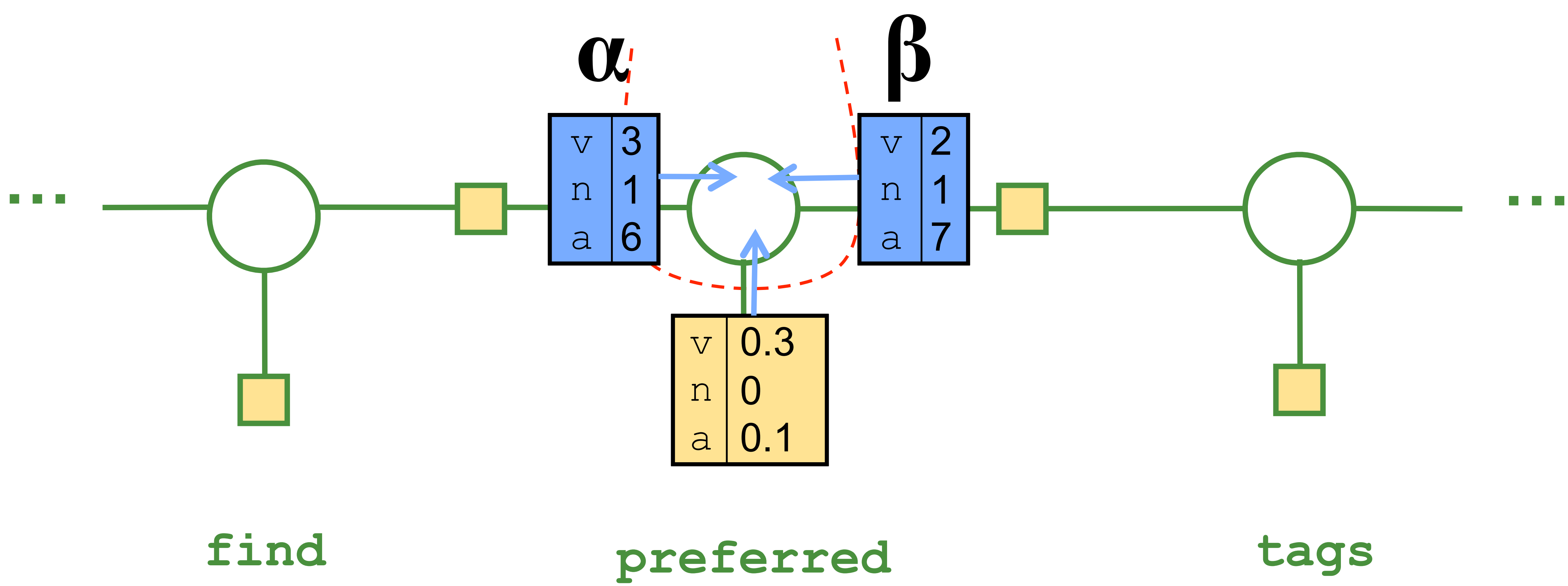
- Message from variable *v* to factor *f*

$$m_{v\to f}(x) = \prod_{f'\in N(v)\backslash\{f\}} m_{f'\to v}(x)$$

- Message from factor *f* to variable *v*

$$m_{f\to v}(x) = \sum_{N(f)\backslash\{v\}} \left[f(x_m) \prod_{v'\in N(f)\backslash\{v\}} m_{v'\to f}(y) \right]$$

Great ideas in ML: Forward-Backward



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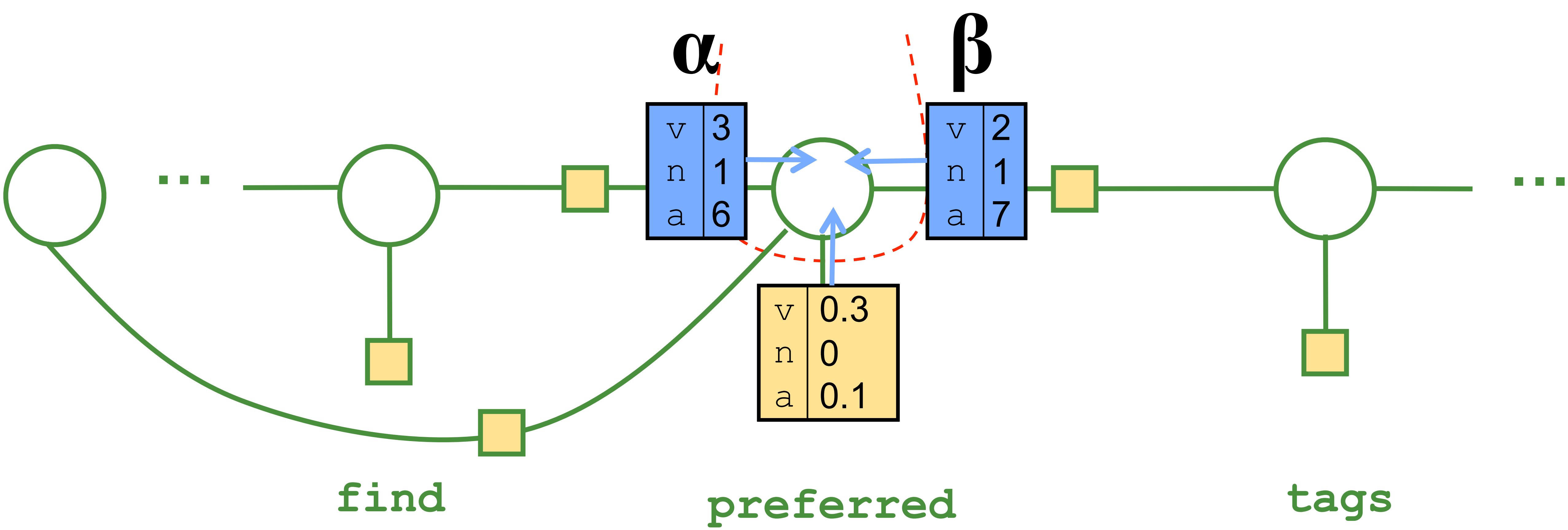

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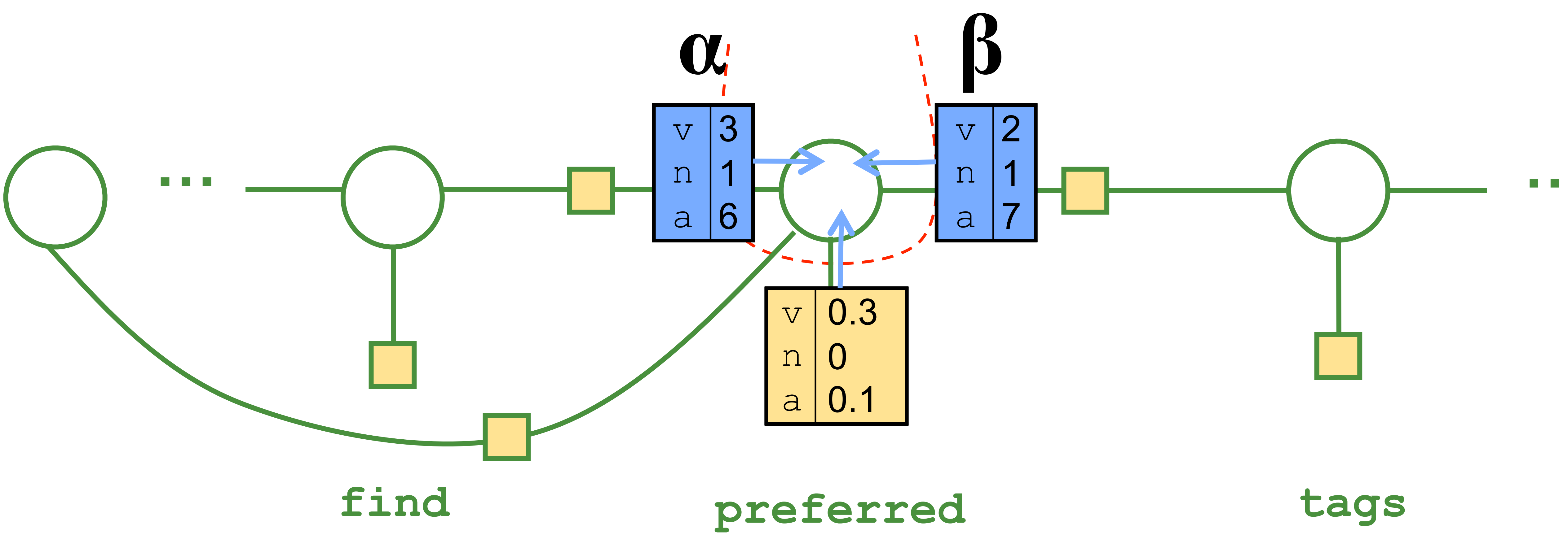
- Extend CRF to “skip chain” to capture non-local factor



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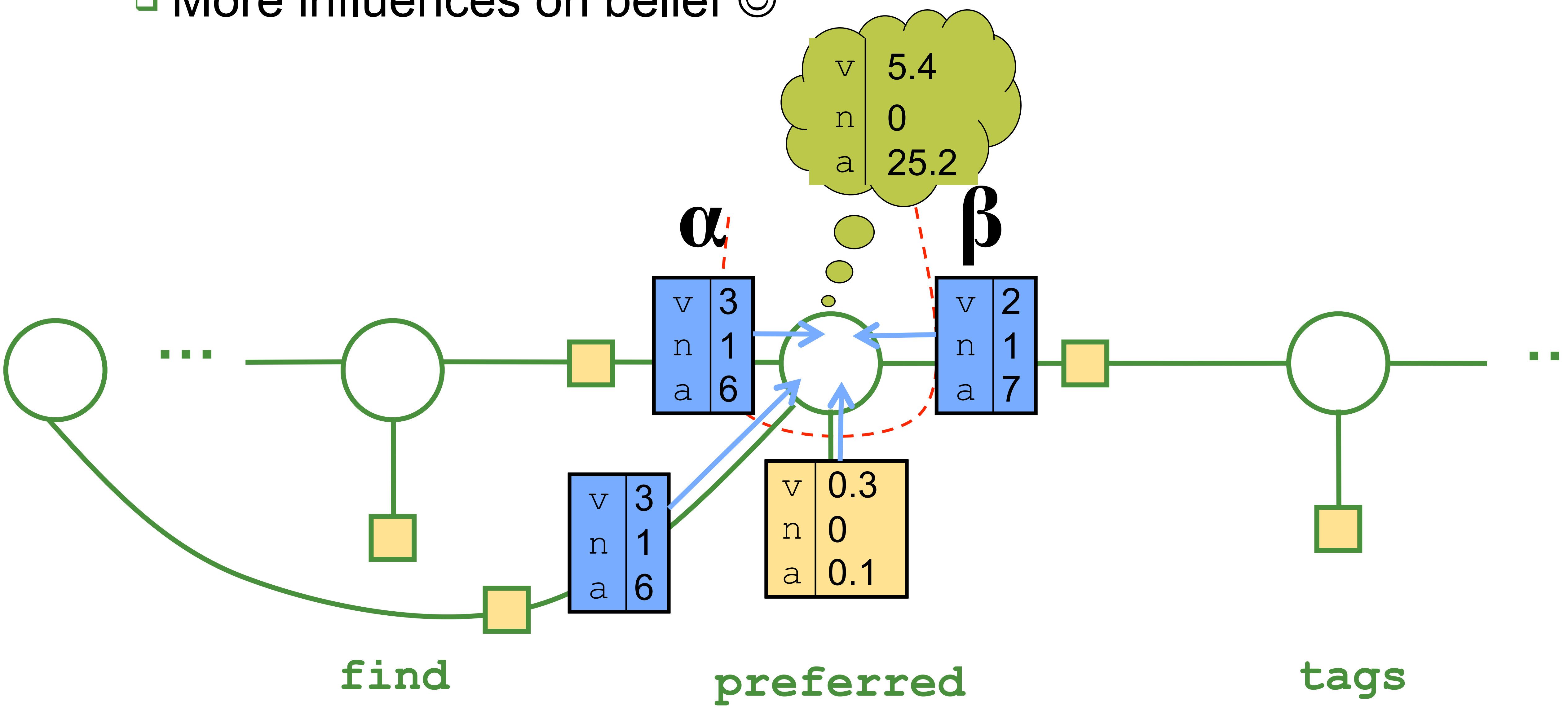
  - More influences on belief ☺



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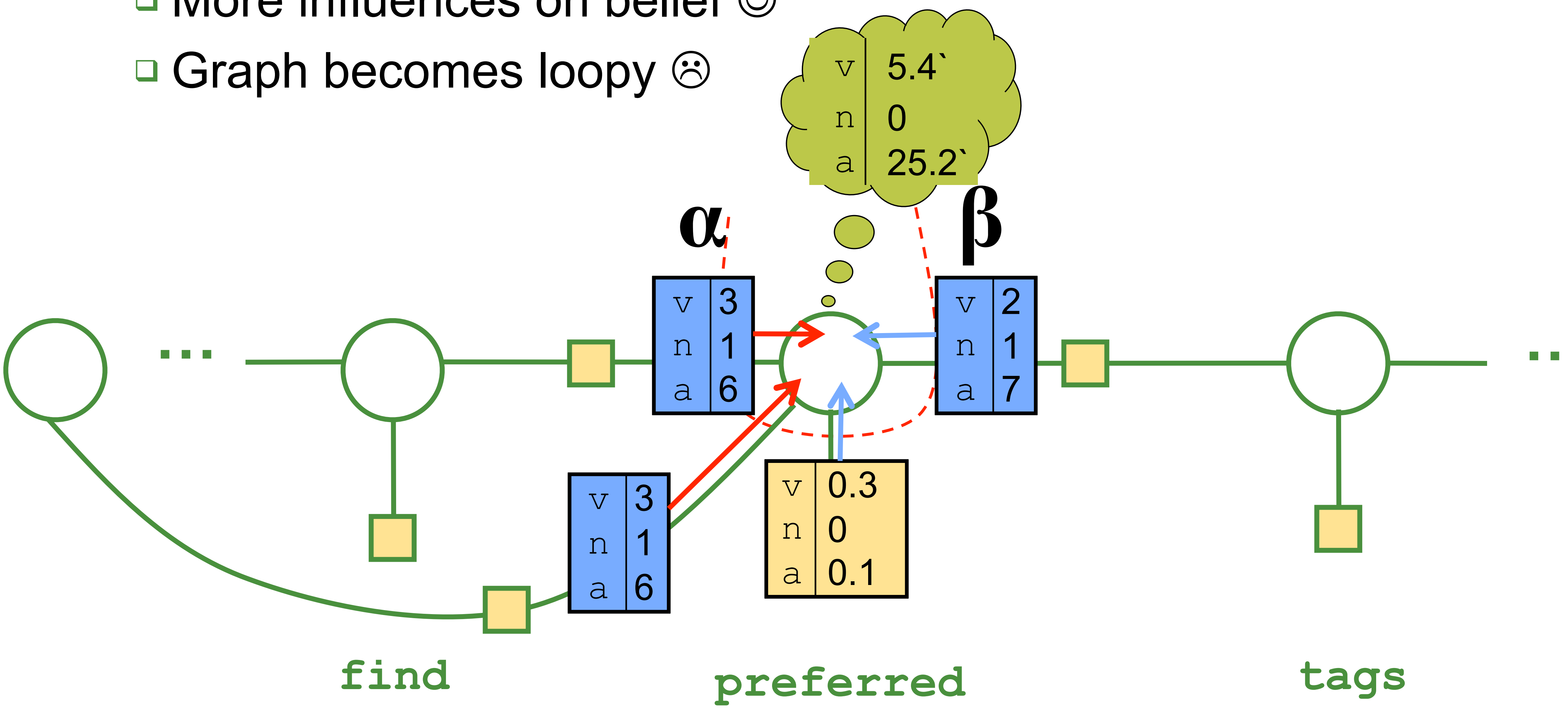
  - Graph becomes loopy ☹



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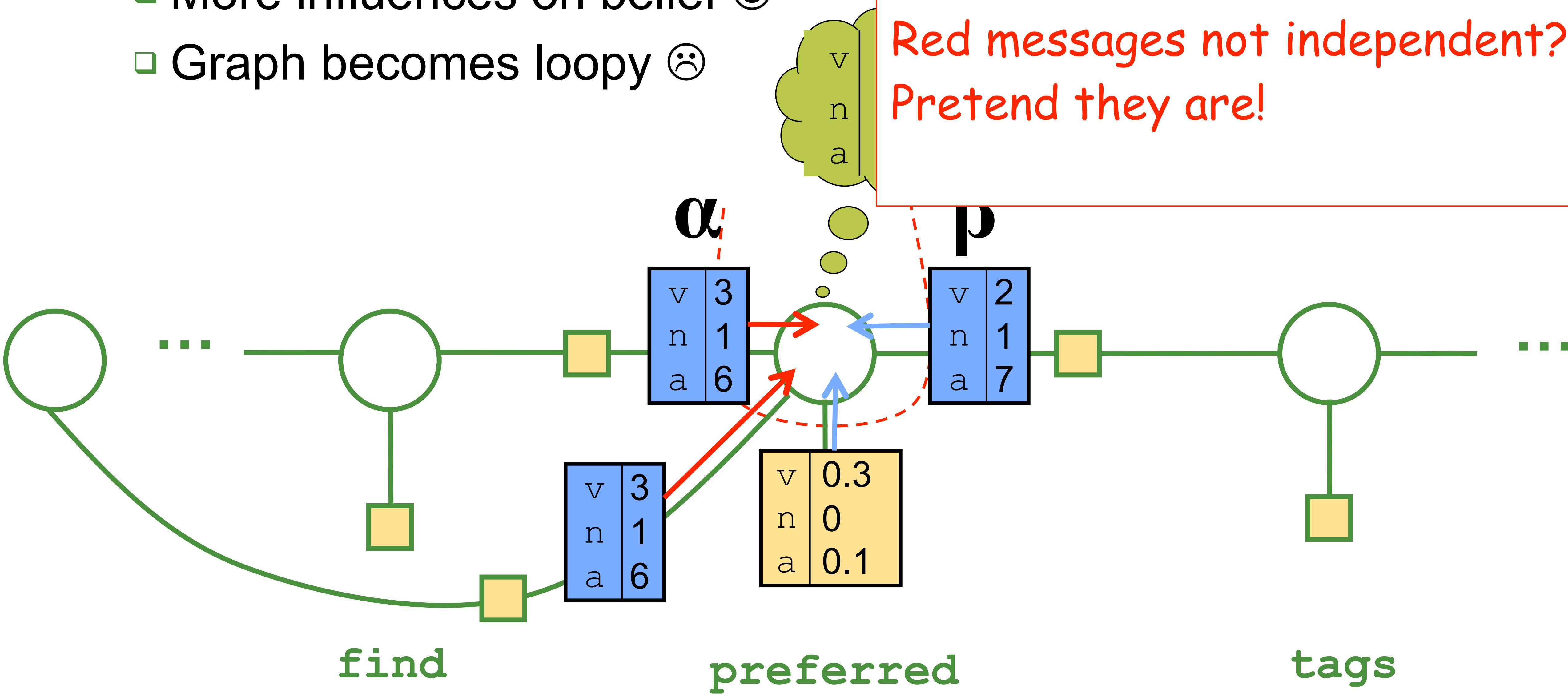
Red messages not independent?
Pretend they are!



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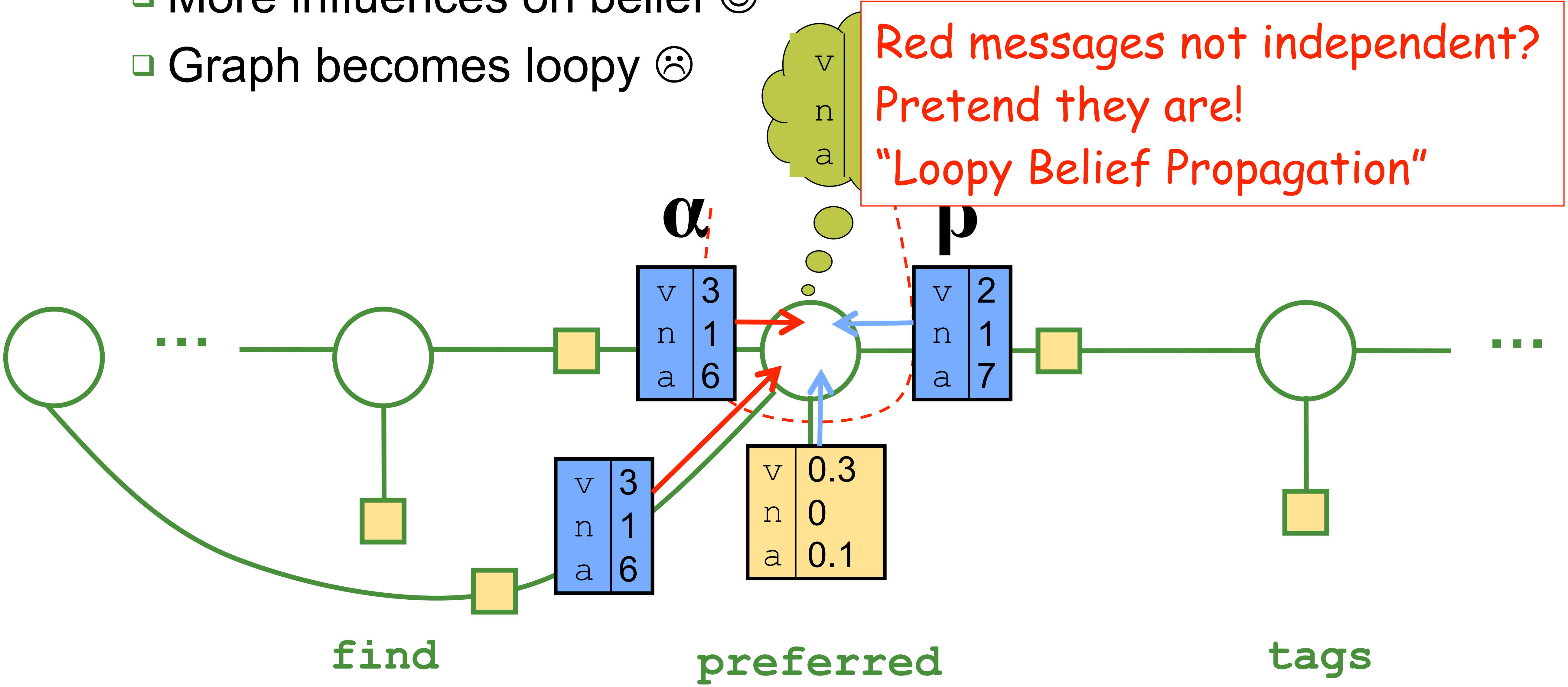

Terminological Clarification

propagation

Terminological Clarification

`belief propagation`

Terminological Clarification

`loopy` `belief` `propagation`

Terminological Clarification

$\left[\texttt{loopy} \quad \texttt{belief}\right]$ $\texttt{propagation}$

Terminological Clarification

[loopy belief] propagation

Terminological Clarification

[loopy belief] propagation

loopy [belief propagation]

Terminological Clarification

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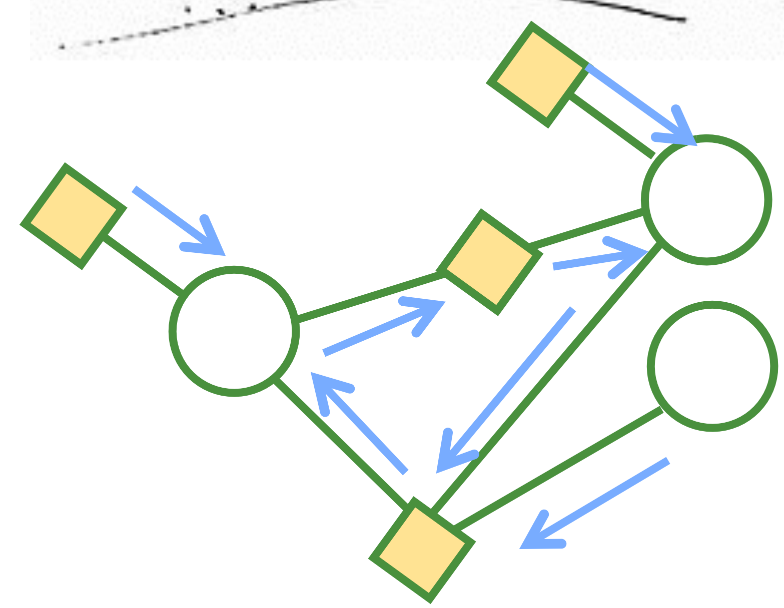
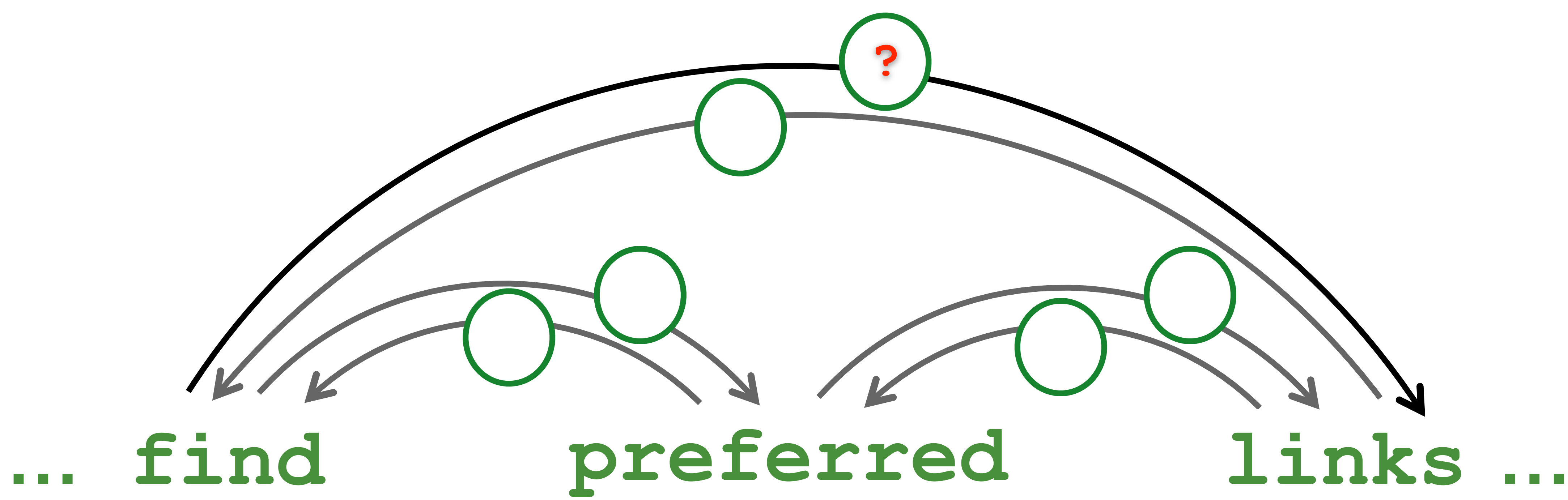


Propagating Global Factors

- Loopy belief propagation is easy for local factors
- How do combinatorial factors (like TREE) compute the message to the link in question?
  - "Does the TREE factor think the link is probably **t** given the messages it receives from *all* the other links?"



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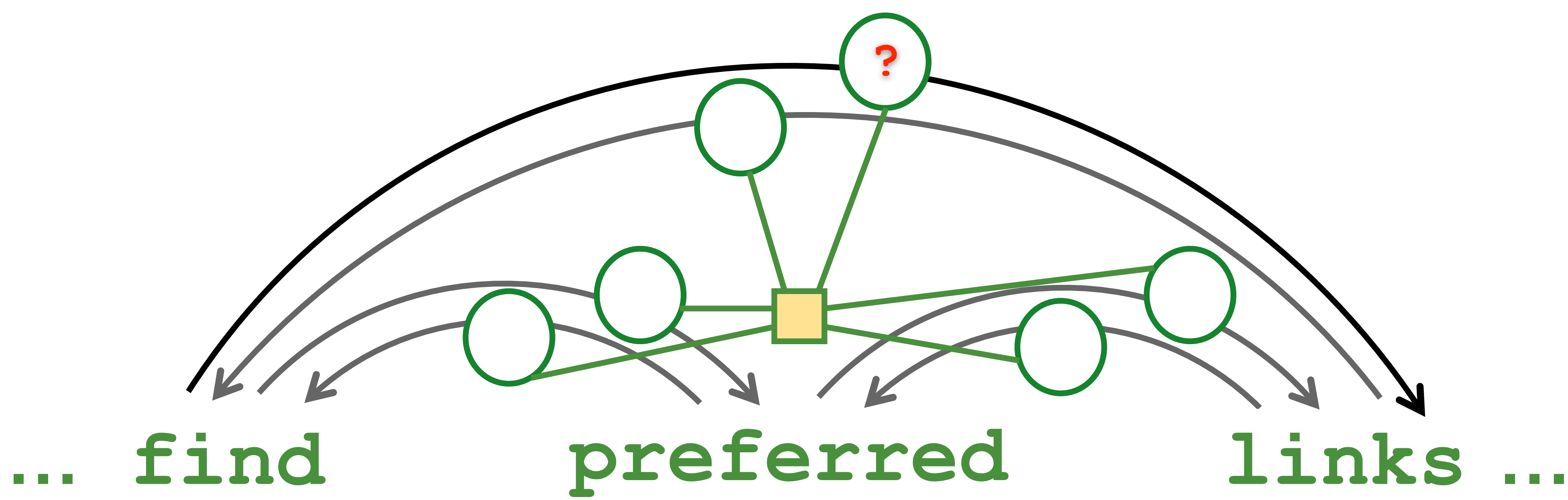

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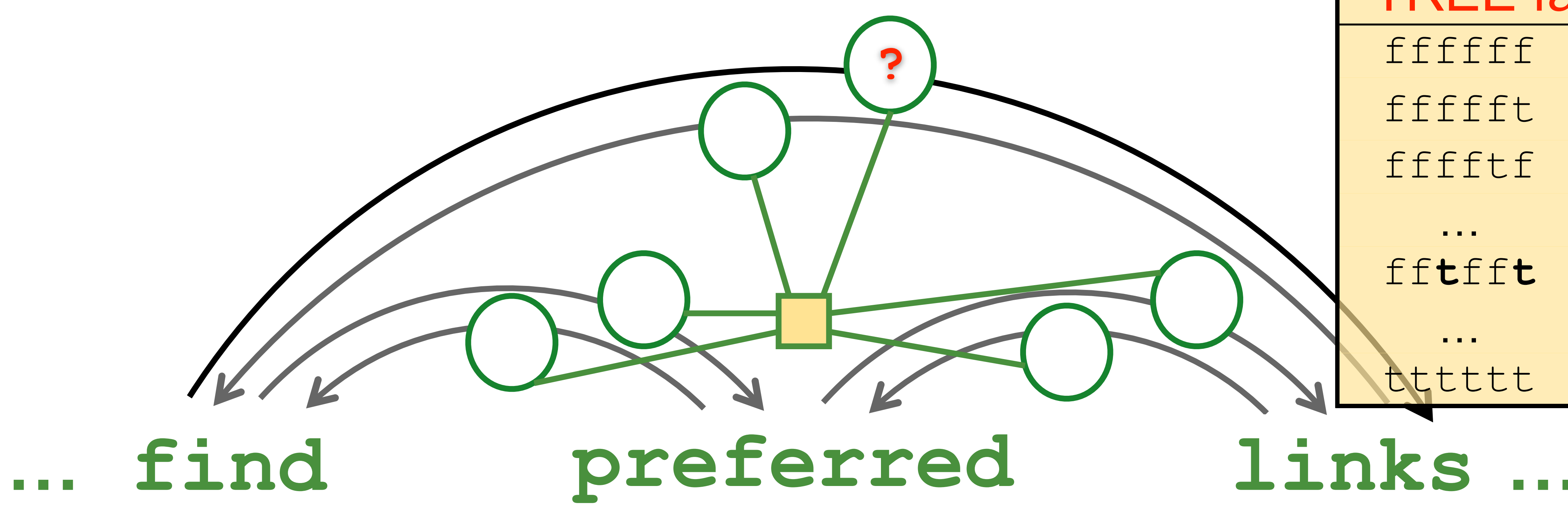
TREE factor	
ffffff	0
ffffft	0
fffftf	0
…	…
fftfft	1
…	…
tttttt	0



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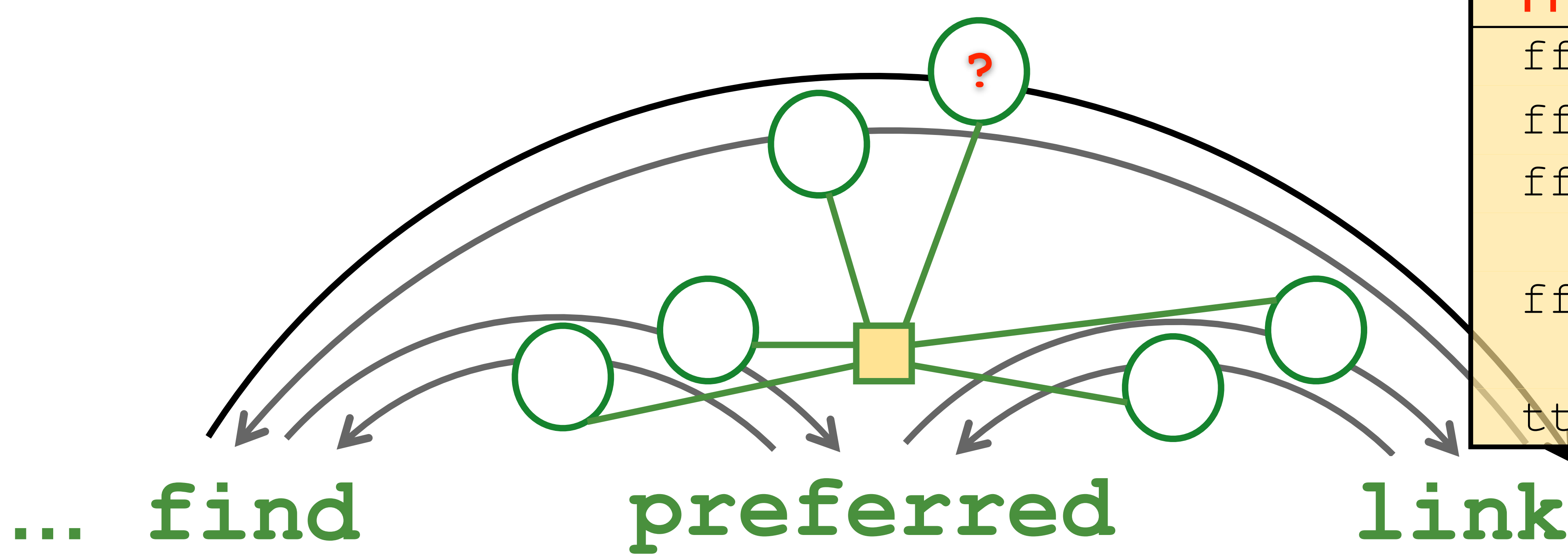

TREE factor	
ffffff	0
ffffft	0
fffftf	0
…	…
ff**t**ff**t**	1
…	…
tttttt	0

Propagating Global Factors

- How does the TREE factor compute the message to the link in question?
  - "Does the TREE factor think the link is probably **t** given the messages it receives from *all* the other links?"

Old-school parsing to the rescue!

This is the outside probability of the link in an *edge-factored* parser!

∴ TREE factor computes all outgoing messages at once (given all incoming messages)

Projective case: total $O(n^3)$ time by inside-outside

Non-projective: total $O(n^3)$ time by inverting Kirchhoff matrix

Graph Theory to the Rescue!

Tutte's **Matrix-Tree Theorem** (1948)

The **determinant** of the Kirchoff (aka Laplacian) adjacency matrix of directed graph *G* without row and column *r* is equal to the **sum of scores of all directed spanning trees** of *G* rooted at node *r*.

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Exactly the *Z* we need!

Graph Theory to the Rescue!

$O(n^3)$ time!

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Exactly the Z we need!

Kirchoff (Laplacian) Matrix

$$\begin{bmatrix} 0 & -s(1,0) & -s(2,0) & \cdots & -s(n,0) \\ 0 & 0 & -s(2,1) & \cdots & -s(n,1) \\ 0 & -s(1,2) & 0 & \cdots & -s(n,2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -s(1,n) & -s(2,n) & \cdots & 0 \end{bmatrix}$$

- Negate edge scores
- Sum columns (children)
- Strike root row/col.
- Take determinant

Kirchoff (Laplacian) Matrix

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Kirchoff (Laplacian) Matrix

$$\begin{bmatrix} 0 & -s(1,0) & -s(2,0) & \cdots & -s(n,0) \\ 0 & \sum_{j\neq 1} s(1,j) & -s(2,1) & \cdots & -s(n,1) \\ 0 & -s(1,2) & \sum_{j\neq 2} s(2,j) & \cdots & -s(n,2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -s(1,n) & -s(2,n) & \cdots & \sum_{j\neq n} s(n,j) \end{bmatrix}$$

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Kirchoff (Laplacian) Matrix

$$
\begin{vmatrix}
\sum_{j \neq 1} s(1,j) & -s(2,1) & \cdots & -s(n,1) \\
-s(1,2) & \sum_{j \neq 2} s(2,j) & \cdots & -s(n,2) \\
\vdots & \vdots & \ddots & \vdots \\
-s(1,n) & -s(2,n) & \cdots & \sum_{j \neq n} s(n,j)
\end{vmatrix}
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- Negate edge scores
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- Take determinant

N.B.: This allows multiple children of root, but see Koo et al. 2007.

Transition-Based Parsing

- Linear time
- Online
- Train a classifier to predict next action
- Deterministic or beam-search strategies
- But... generally less accurate

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Start state: $([\ ], [1, \ldots, n], \{\ \})$

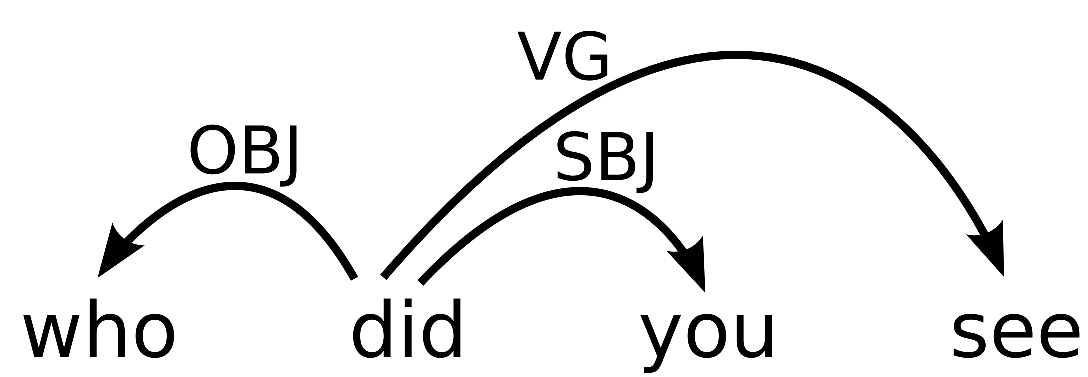
Final state: $(S, [\ ], A)$

Shift:	$(S, i\|B, A)$	$\Rightarrow$	$(S\|i, B, A)$
Reduce:	$(S\|i, B, A)$	$\Rightarrow$	$(S, \mathrm{B}, \mathrm{A})$
Right-Arc:	$(S\|i, j\|B, A)$	$\Rightarrow$	$(S\|i\|j, B, A \cup \{i \rightarrow j\})$
Left-Arc:	$(S\|i, j\|B, A)$	$\Rightarrow$	$(S, j\|B, A \cup \{i \leftarrow j\})$

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\ ]_S$	$[\text{who, did, you, see}]_B$	{ }

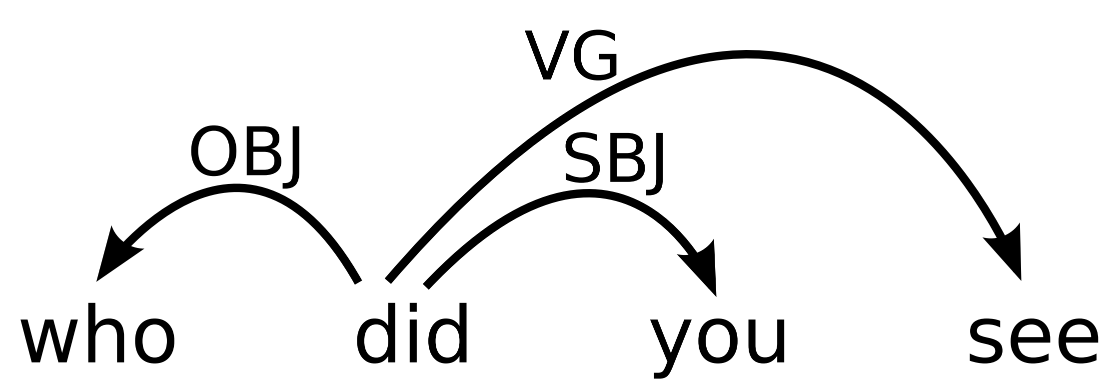


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{who}]_S$	$[\text{did, you, see}]_B$	{ }

Shift

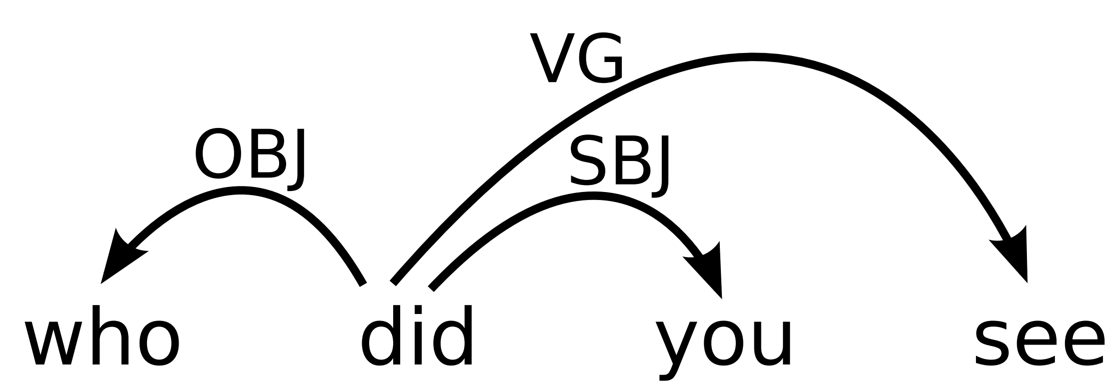


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	Buffer	Arcs
$[\ ]_S$	$[\text{did, you, see}]_B$	$\{ \text{who} \xleftarrow{\text{OBJ}} \text{did} \}$

Left-arc OBJ

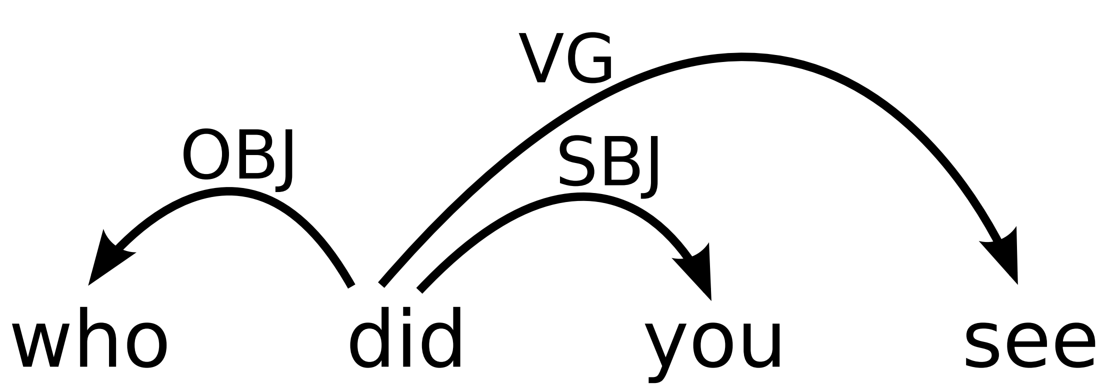


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did}]_S$	$[\text{you, see}]_B$	$\{\ \text{who} \xleftarrow{\text{OBJ}} \text{did}\ \}$

Shift

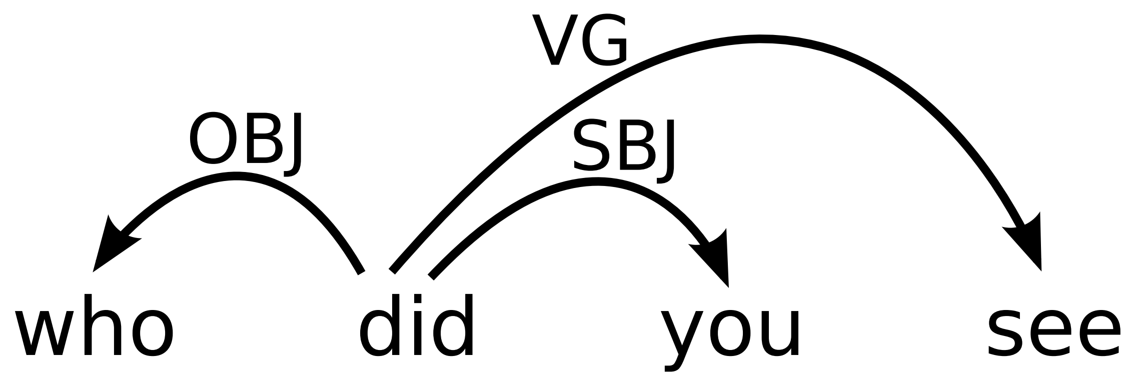


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did, you}]_S$	$[\text{see}]_B$	$\{\ \text{who} \xleftarrow{\text{OBJ}} \text{did},$ $\text{did} \xrightarrow{\text{SBJ}} \text{you}\ \}$

Right-arc SBJ

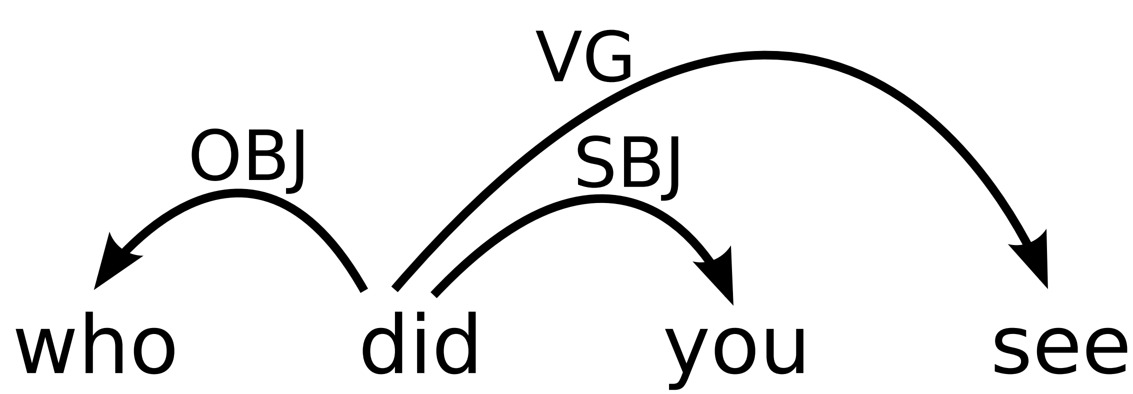


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did}]_S$	$[\text{see}]_B$	$\{\ \text{who} \xleftarrow{\text{OBJ}} \text{did},\ \text{did} \xrightarrow{\text{SBJ}} \text{you}\ \}$

Reduce

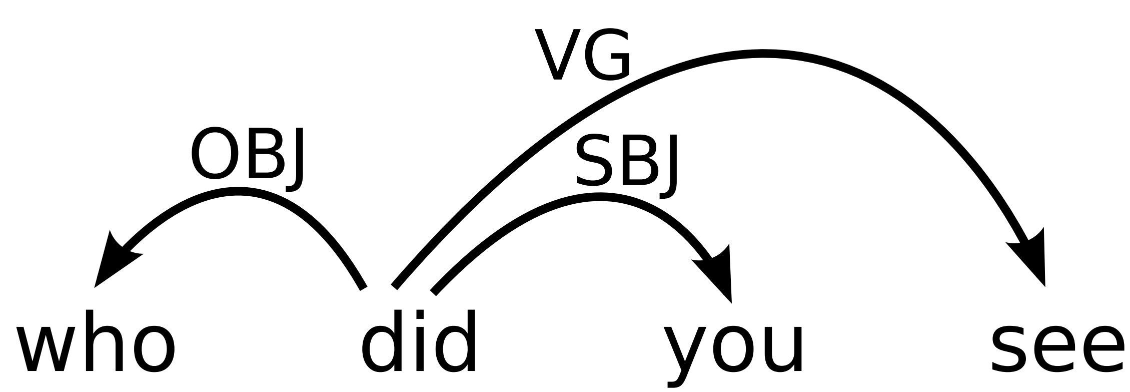


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did, see}]_S$	$[\ ]_B$	$\{ \text{who} \xleftarrow{\text{OBJ}} \text{did},$ $\text{did} \xrightarrow{\text{SBJ}} \text{you},$ $\text{did} \xrightarrow{\text{VG}} \text{see} \}$

Right-arc VG

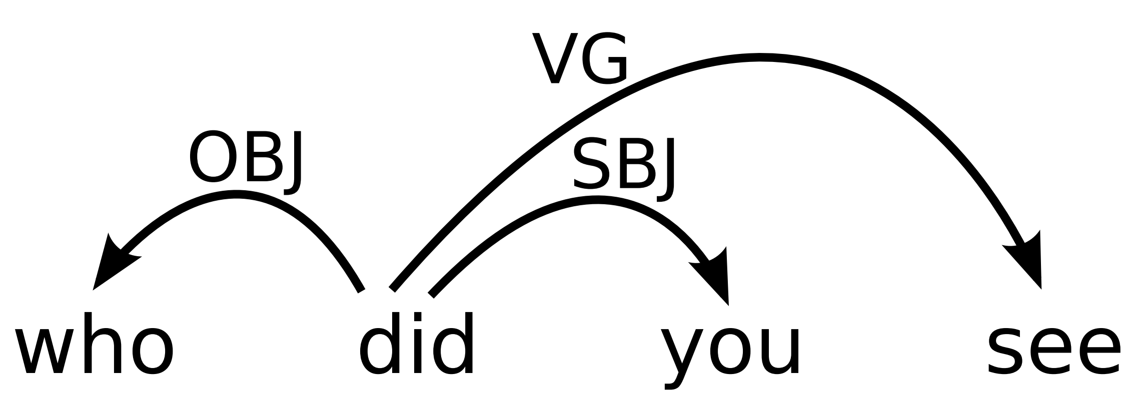


Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did, you}]_S$	$[\text{see}]_B$	$\{\ \text{who} \xleftarrow{\text{OBJ}} \text{did},$ $\text{did} \xrightarrow{\text{SBJ}} \text{you}\ \}$

Right-arc SBJ



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack	**Buffer**	**Arcs**
$[\text{did, you}]_S$	$[\text{see}]_B$	$\{ \text{who} \xleftarrow{\text{OBJ}} \text{did}, \; \text{did} \xrightarrow{\text{SBJ}} \text{you} \}$

Right-arc SBJ

VG

Choose action w/best classifier score
100k - 1M features

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack **B**

[did, you]$_S$ [s

Very fast linear-time performance
WSJ 23 (2k sentences) in 3 s

did $\longrightarrow$ you }

Right-arc
SBJ

VG

Choose action w/best classifier score
100k - 1M features