CS 5100: Foundations of Artificial Intelligence

Search Problems and Solutions

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Outline

- Review inference in FOL
  - Most general unifiers
  - Conversion to CNF
  - Unification algorithm
  - Backward chaining
- Search Problems
  - Uninformed search
  - Informed search (maybe…)

Review: Find a MGU for the following pairs (if one exists)

1. $Isa(Fido, Dog)$
   $Isa(x, Dog)$

2. $Isa(x, Dog)$
   $Isa(y, z)$

3. $Likes(x, Owner(x))$
   $Likes(John, Owner(Fido))$

4. $Likes(x, Owner(y))$
   $Likes(John, z)$

5. $Likes(x, Owner(y))$
   $Likes(Fido, Father(John))$

Review: Find a MGU for the following pairs (if one exists)

1. $Isa(Fido,Dog)$
   $Isa(x, Dog)$

   $\theta = \{x/Fido\}$

2. $Isa(x,Dog)$
   $Isa(y,z)$

   $\theta = \{x/y, z/Dog\}$

3. $Likes(x,Owner(x))$
   $Likes(John, Owner(Fido))$

   None

4. $Likes(x,Owner(y))$
   $Likes(John, z)$

   $\theta = \{x/John, z/Owner(y)\}$

5. $Likes(x,Owner(y))$
   $Likes(Fido, Father(John))$

   None

Review: Steps in converting FOL to CNF

- Remove implications
- Move negation inward
- Standardize variable names
- Skolemize (drop existential quantifiers)
  - $\forall x\, \forall y\, \exists z\, [\, Knows(x, z) \wedge Knows(y, z)\,]$ **becomes** $\forall x\, \forall y\, [Knows(x, F(x,y)) \wedge Knows(y, F(x,y))\,]$
  - $\forall x\, \exists z\, \forall y\, [\, Knows(x, z) \wedge Knows(y, z)\,]$ **becomes** $\forall x\, \forall y\, [Knows(x, F(x)) \wedge Knows(y, F(x))\,]$
- Drop universal quantifiers
- Distribute ∨ over ∧

Conversion to CNF Examples

1. $\exists y\, \exists z\, [\, City(z) \wedge Team\text{-}from(y, z) \wedge Winner(y, SuperBowl)\,]$

2. $\forall x\, \forall y\, [\, Student(x) \wedge TakingClass(x, y) \Rightarrow \exists z\, [\, Grade(z) \wedge GradeFor(x, y, z)]\,]$

3. $\forall w\ [\, \exists z\, [\, House(z) \wedge (\, Owns(w, z) \vee Rents(w, z)\,)\ ] \Rightarrow CreditWorthy(w)\,]$

Conversion to CNF Examples

1. $\exists y\, \exists z\, [\, City(z) \wedge Team\text{-}from(y, z) \wedge Winner(y, SuperBowl)\,]$
   $\boldsymbol{City(g2) \wedge Team\text{-}from(g1, g2) \wedge Winner(g1, SuperBowl)}$
2. $\forall x\, \forall y\, [\, Student(x) \wedge TakingClass(x, y) \Rightarrow \exists z\, [\, Grade(z) \wedge GradeFor(x, y, z)]\,]$
   $\boldsymbol{(\neg Student(x) \vee \neg TakingClass(x, y) \vee Grade(z)) \wedge (\neg Student(x) \vee \neg TakingClass(x, y) \vee GradeFor(x, y, z))}$
3. $\forall w\ [\, \exists z\, [\, House(z) \wedge (\, Owns(w, z) \vee Rents(w, z)\,)\ ] \Rightarrow CreditWorthy(w)\,]$
   $\boldsymbol{(\neg House(z) \vee \neg Owns(w, z) \vee CreditWorthy(w)) \wedge (\neg House(z) \vee \neg Rents(w, z) \vee CreditWorthy(w))}$

Unification Algorithm

function UNIFY(x, y, θ) **returns** a substitution to make x and y identical

inputs: x, a variable, constant, list, or compound
y, a variable, constant, list, or compound
θ, the substitution built up so far

if θ = failure **then return** failure
else if $x = y$ **then return** θ
else if VARIABLE?(x) **then return** UNIFY-VAR(x, y, θ)
else if VARIABLE?(y) **then return** UNIFY-VAR(y, x, θ)
else if COMPOUND?(x) **and** COMPOUND?(y) **and Op(x) = Op(y) then**
return UNIFY(ARGS[x], ARGS[y], [redacted], θ))
else if LIST?(x) **and** LIST?(y) **then**
return UNIFY(REST[x], REST[y], UNIFY(FIRST[x], FIRST[y], θ))
else return failure

Unification Algorithm (cont.)

function UNIFY-VAR(var, x, θ) **returns** a substitution

inputs: var, a variable
x, any expression
θ, the substitution built up so far

if $\{var/val\} \in \theta$ **then return** UNIFY(val, x, θ)
else if $\{x/val\} \in \theta$ **then return** UNIFY(var, val, θ)
else if OCCUR-CHECK?(var, x) **then return** failure
else return add $\{var/x\}$ to θ **Using compose**

Unification algorithm in action…

- Unify *Likes*(*y*,*x*) with *Likes*(*x*,*John*)

Unification algorithm in action…

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else if COMPOUND?(x) **and** COMPOUND?(y) **and** *Op*(x) = *Op*(y) **then**
return UNIFY(ARGS[x], ARGS[y], , θ))
else if LIST?(x) **and** LIST?(y) **then**
return UNIFY(REST[x], REST[y], UNIFY(FIRST[x], FIRST[y], θ))
else return failure

call UNIFY(*Likes*(*y*,*x*), *Likes*(*x*,*John*), [ ])

return UNIFY([*y*,*x*], [*x*,*John*], [ ])

Unification algorithm in action…

function UNIFY(x, y, θ) **returns** a substitution to make x and y identical

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else return failure

call UNIFY([*y*,*x*], [*x*,*John*], [ ])

return UNIFY([*x*], [*John*], UNIFY([*y*],[*x*],[ ]))

Unification algorithm in action…

function UNIFY(x, y, θ) **returns** a substitution to make x and y identical
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call UNIFY(*y*,*x*,[ ])

return UNIFY-VAR(*y*,*x*,[ ])

Unification algorithm in action…

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else if OCCUR-CHECK?(var, x) **then return** failure
else return add $\{var/x\}$ to θ # Using compose

call UNIFY-VAR(*y*,*x*,[ ])

return θ = {y/x}

Unification algorithm in action…

- Unify *Likes*(*y*,*x*) with *Likes*(*x*,*John*)
  - First step: $\theta = \{y/x\}$

Unification algorithm in action…

function UNIFY(x, y, θ) **returns** a substitution to make x and y identical

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call UNIFY([*y,x*], [*x,John*], [ ])

return UNIFY(*x*, *John*, θ = {y/x})

Unification algorithm in action…

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call UNIFY(*x*, *John*, θ = {y/x})

return UNIFY-VAR(*x*,*John*, θ = {y/x})

Unification algorithm in action…

function UNIFY-VAR(var, x, θ) **returns** a substitution

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else if OCCUR-CHECK?(var, x) **then return** failure

else return add $\{var/x\}$ to θ # Using compose

call UNIFY-VAR(*x*,*John*, θ = {y/x})

Unification algorithm in action…

- Unify *Likes*(*y,x*) with *Likes*(*x,John*)

  - First step: $\theta = \{y/x\}$
  - Second step: *unify-var*(*x, John,* θ)

  - **add**{x/John} to θ yields what?
    Push(*newpair*, θ) → {x/John, y/x}
    Compose(*newpair*, θ): s.t. when the lhs (L) of the new pair appears in the rhs (R) of an existing element of theta, replace R with *subst*(*newpair*, R)→{x/John, y/John}

Unification algorithm in action…

function UNIFY-VAR(var, x, θ) **returns** a substitution

inputs: var, a variable

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if $\{var/val\} \in \theta$ **then return** UNIFY(val, x, θ)

else if $\{x/val\} \in \theta$ **then return** UNIFY(var, val, θ)

else if OCCUR-CHECK?(var, x) **then return** failure

else return add $\{var/x\}$ to θ # Using compose

call UNIFY-VAR(*x*,*John*, θ = {y/x})

return θ = {x/John, y/John}

Backward chaining algorithm

function FOL-BC-ASK(KB, goals, θ) **returns** a set of substitutions

inputs: KB, a knowledge base

$goals$, a list of conjuncts forming a query

θ, the current substitution, initially the empty substitution $\{\}$

local variables: ans, a set of substitutions, initially empty

if $goals$ is empty **then return** $\{\theta\}$

$q' \leftarrow \text{SUBST}(\theta, \text{FIRST}(goals))$

for each r **in** KB where $\text{STANDARDIZE-APART}(r) = (p_1 \wedge \ldots \wedge p_n \Rightarrow q)$

and $\theta' \leftarrow \text{UNIFY}(q, q')$ succeeds

$ans \leftarrow \text{FOL-BC-ASK}(KB, [p_1, \ldots, p_n | \text{REST}(goals)], \text{COMPOSE}(\theta, \theta')) \cup ans$

return ans

Some slight changes/corrections

- NOTE:

for each r **in** KB **where** STANDARDIZE-APART$(r) = (p_1 \wedge \ldots \wedge p_n \Rightarrow q)$
and $\theta' \leftarrow$ UNIFY(q, q') **succeeds**

Should be: for each *r*, θ combination in *KB* where …

- An empty θ = [ ] means an exact match, so *subst*(θ,*p*) does nothing
- Why can't we standardize apart once at the beginning?

- Remember, backward chaining is a depth-first search procedure
  - Search methods can be used to solve many problems in AI

Syntax of AND/OR trees

How do we search for backward chaining solutions?

- Facts are queried first (in the order they appear)
- Then rules in the order they appear
- Premises are also "solved" in the order they appear

Alternating node types (except for leaf nodes)

- A goal/subgoal node—**OR node** representing all the ways the goal can be proved
  - Children are rules or facts
- A rule node—**AND node** since the premises are conjoined
  - Children are subgoals
- Fact node—**leaf node** that satisfies a subgoal

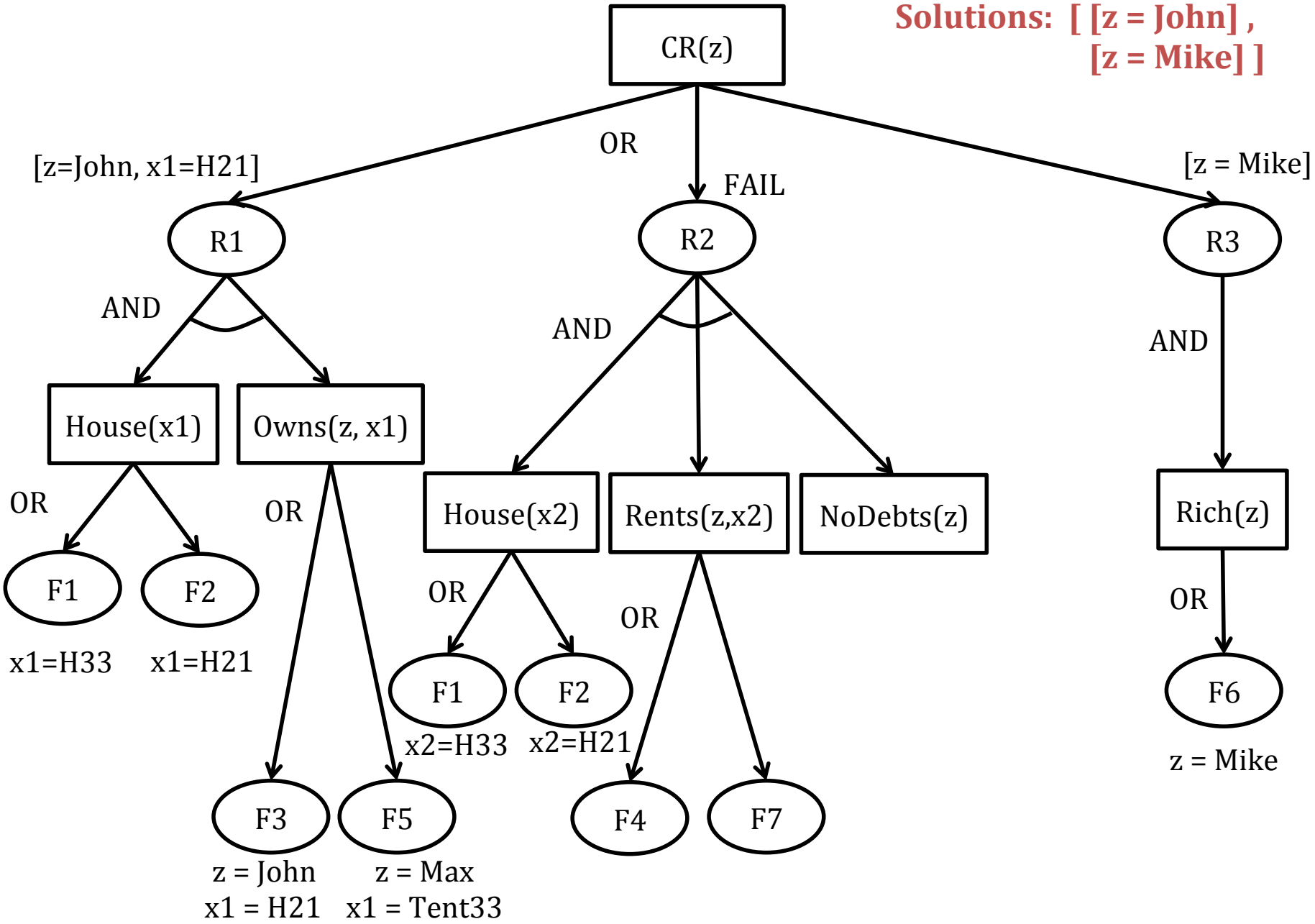
Example: AND/OR tree for backward chaining process

- NOTE: Order of the KB is important

R1. $House(x) \wedge \text{Owns}(y, x) \Rightarrow Cr(y)$
R2. $House(x) \wedge Rents(y, x) \wedge NoDebts(y) \Rightarrow Cr(y)$
R3. $Rich(x) \Rightarrow \text{Cr}(x)$
F1. $House(H33)$
F2. $House(H21)$
F3. $Owns(John, H21)$
F4. $Rents(Sally, Apt\ 12)$
F5. $Owns(Max, Tent33)$
F6. $Rich(Mike)$
F7. $Rents(Jim, H33)$

Query: $?Cr(z)$ who is credit worthy??

Solutions: [[z = John] , [z = Mike]]



Search problems

- Classic search problems
  - Missionaries and cannibals
  - 8-puzzle
  - 4 color map problem
  - Path finding
  - Resource allocation
  - Adversarial games extend the basic model
- Relationship to operations research
  - Factor scheduling (e.g., painting cars)
  - Logistics (e.g., delivering cars to dealers)
- Finding optimal or good schedules

Search framework for problem solving

- **Problem formulation:**
  - A set of discrete states (usually finite, but often very large) that define the problem space
  - A designated start state
  - A subset of states designated as goal states
  - A set of operators or actions the agent can perform at each state
  - A transition function *Next: State* × *OP* → *State*
- **Objective:**
  - Find a sequence of operators that, if applied in the start state, will lead to a goal state
    - **Optimal** solution—find the **shortest** or **least-cost** such sequence
    - Cost determined by agent's performance measure

Explore the problem space using a search tree

- State space forms a **directed graph** or tree structure
  - Root = start state
  - Each node represents a state
  - Actions are branches—children are all the possible next-states

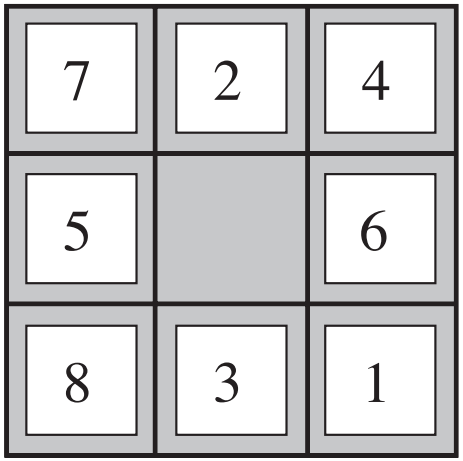
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

Graph search (dynamic programming)

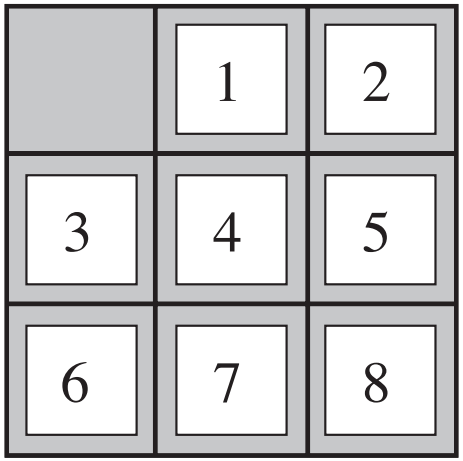
- Graph contains at most **one copy** of each state
  - Growing tree directly on state-space graph

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Example: The 8-puzzle



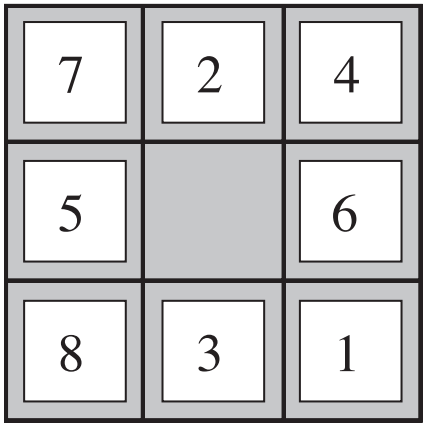
Start State



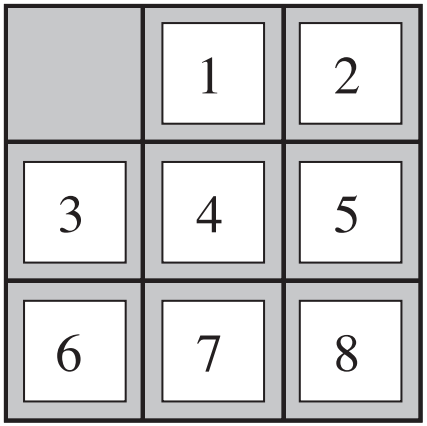
Goal State

- States?
- Actions?
- Goal test?
- Path cost?

Example: The 8-puzzle



Start State



Goal State

- States? **Locations of tiles**
- Actions? **Move blank left, right, up, down**
- Goal test? **==Goal State (given)**
- Path cost? **1 per move**

[NOTE: optimal solution of *n*-puzzle family is NP-hard]

Example: Path finding

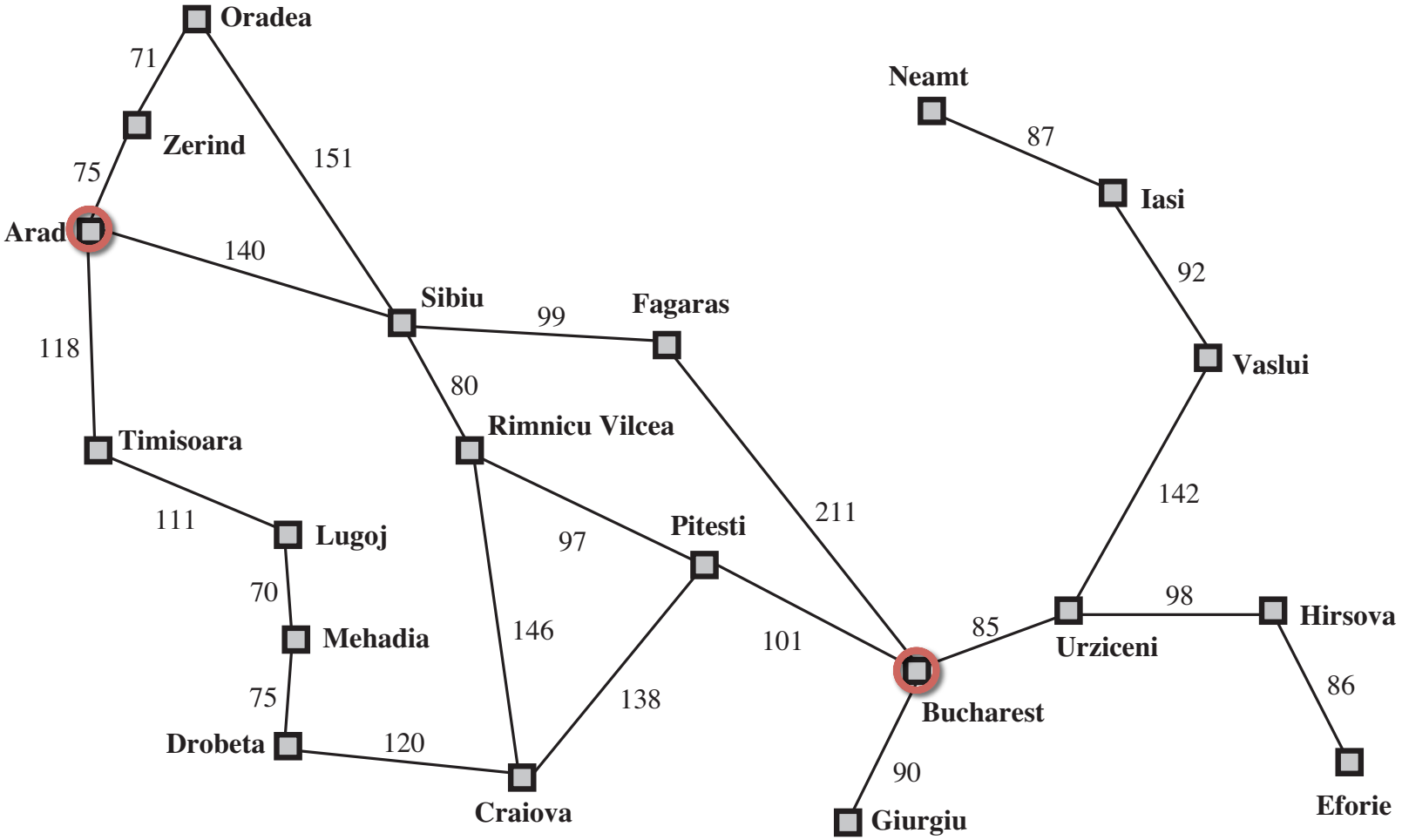
- Sample problem
  - On holiday in Romania; currently in Arad
  - Flight leaves tomorrow from Bucharest

- Formulate goal:
  - Be in Bucharest

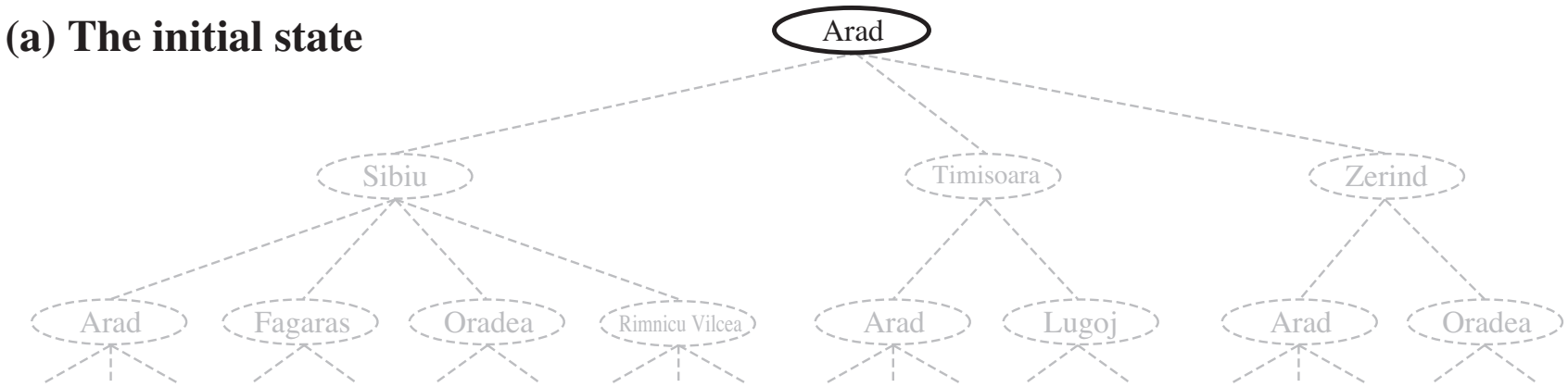
- Formulate problem:
  - **States**—cities in Romania
  - **Actions**—drive between cities

- Find solution:
  - Sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Path finding (cont.)



Solve path finding with tree search



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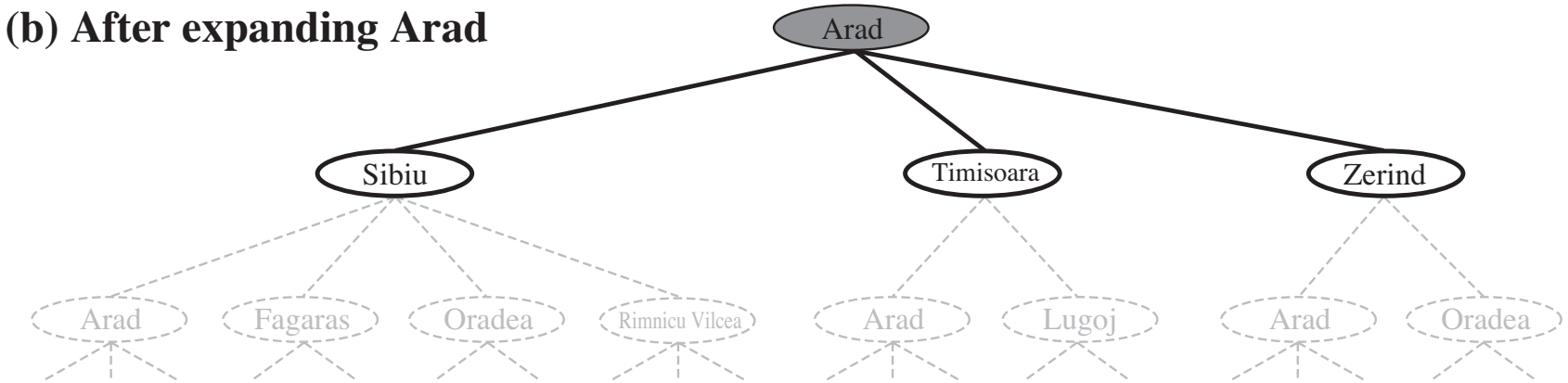

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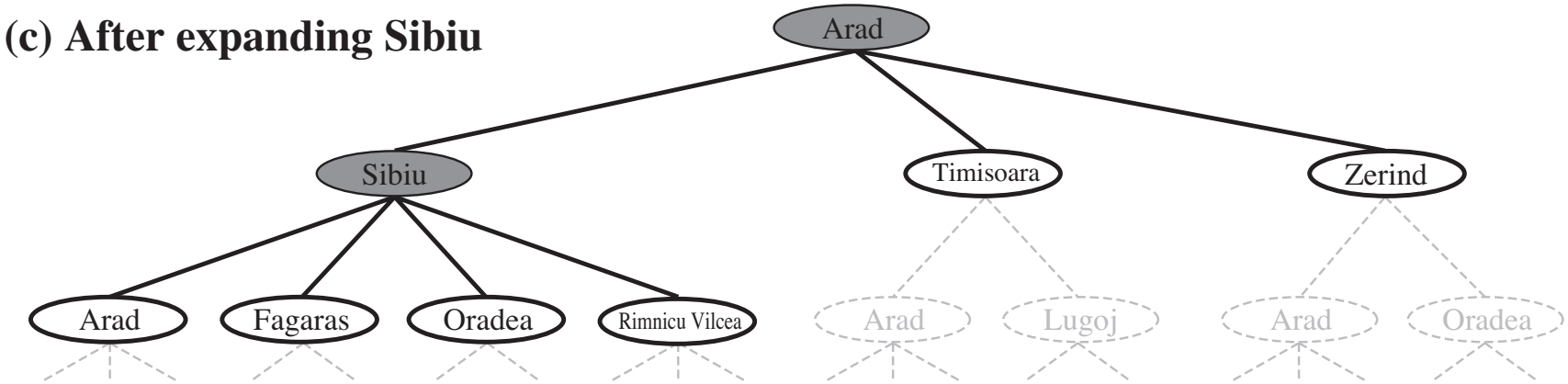
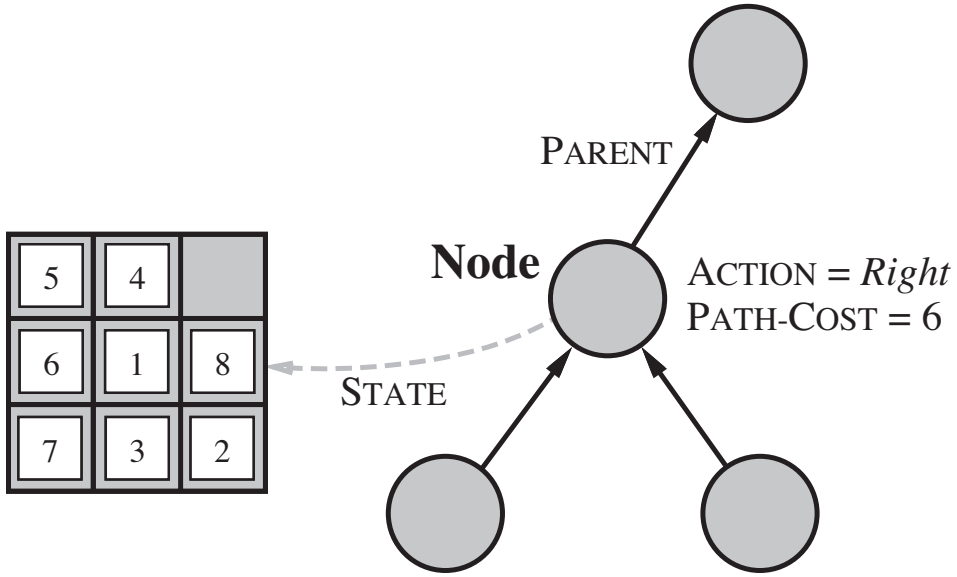


Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree
  - Representative data structure must include state, parent node, action, path cost
  - More than one node can correspond to a single world state



Search strategies

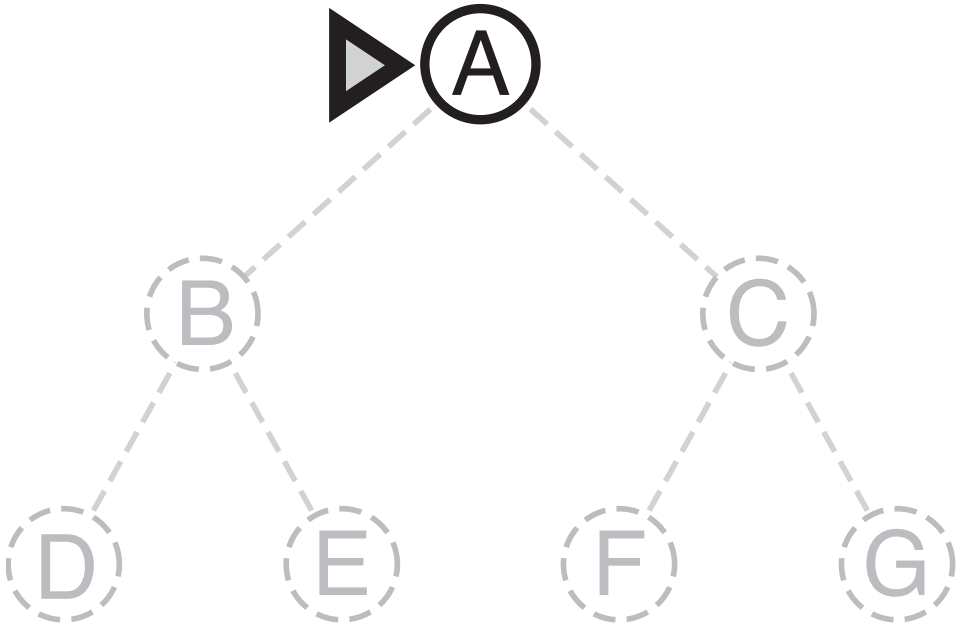
- Search strategy defined by picking the **order of node expansion**
- Strategies are evaluated along the following dimensions
  1. **Completeness**—does it always find a solution if one exists?
  2. **Time complexity**—number of nodes generated
  3. **Space complexity**—maximum number of nodes in memory
  4. **Optimality**—does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b*—Maximum branching factor for the search tree
  - *d*—Depth of the least-cost solution
  - *m*—Maximum depth of the state space (may be ∞)

Uninformed search strategies

- **Uninformed** (blind) search strategies use only the information available in the problem definition
- Search strategies
  1. Breadth-first search
  2. Uniform-cost search
  3. Depth-first search
  4. Depth-limited search
  5. Iterative deepening search

Breadth-first search

- Expand **shallowest** unexpanded node
  - All nodes at a given depth expanded before any at the next level
- Implementation
  - *Frontier* is a FIFO queue, i.e., new successors go at the end



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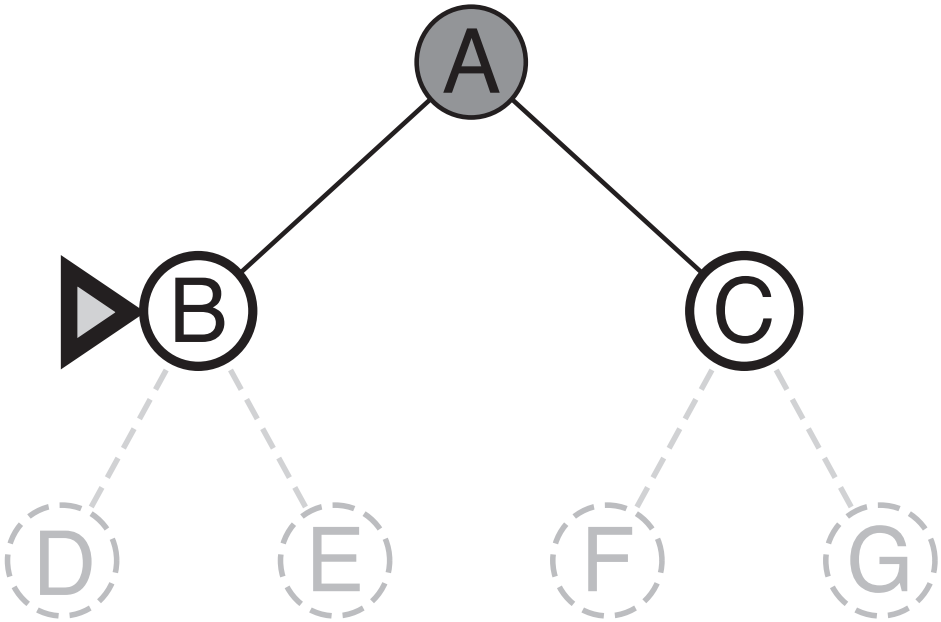

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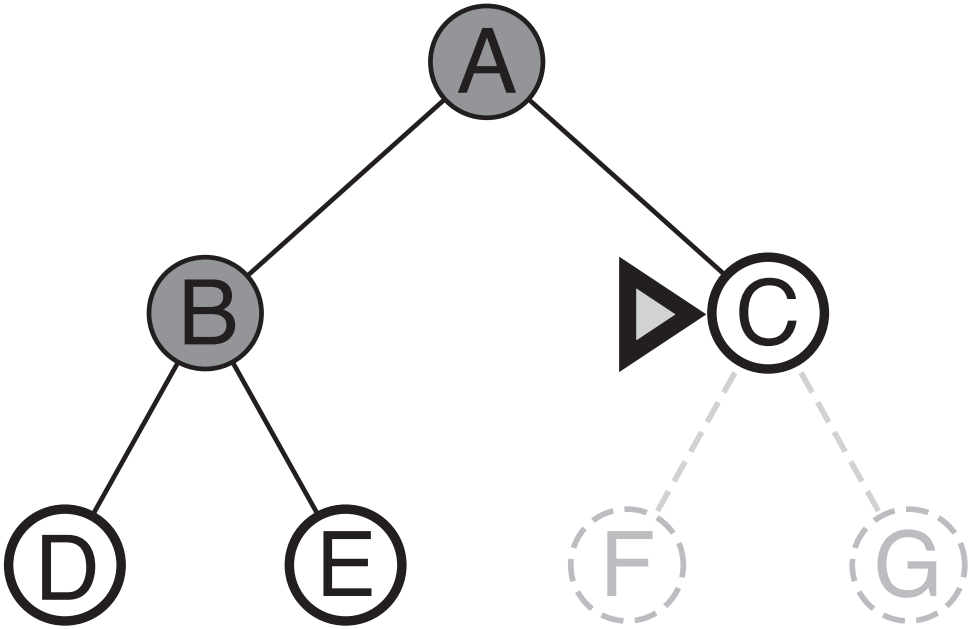

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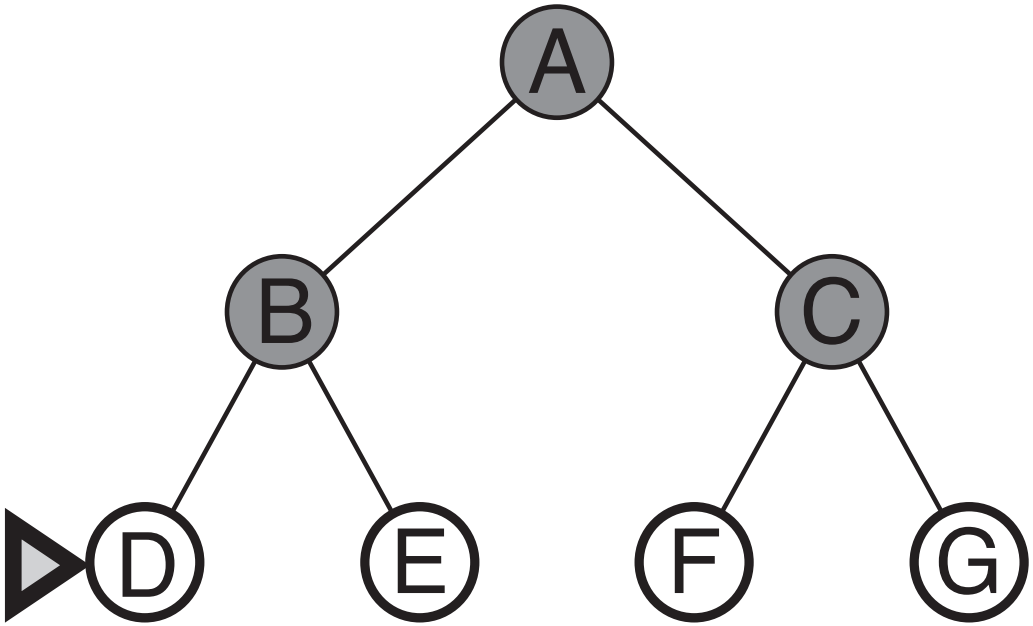


Properties of breadth-first search

- Complete? **Yes** (if b is finite)
- Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$
- Space? $O(b^{d+1})$—keeps every node in memory
- Optimal? **Yes** (if cost is non-decreasing function of depth)

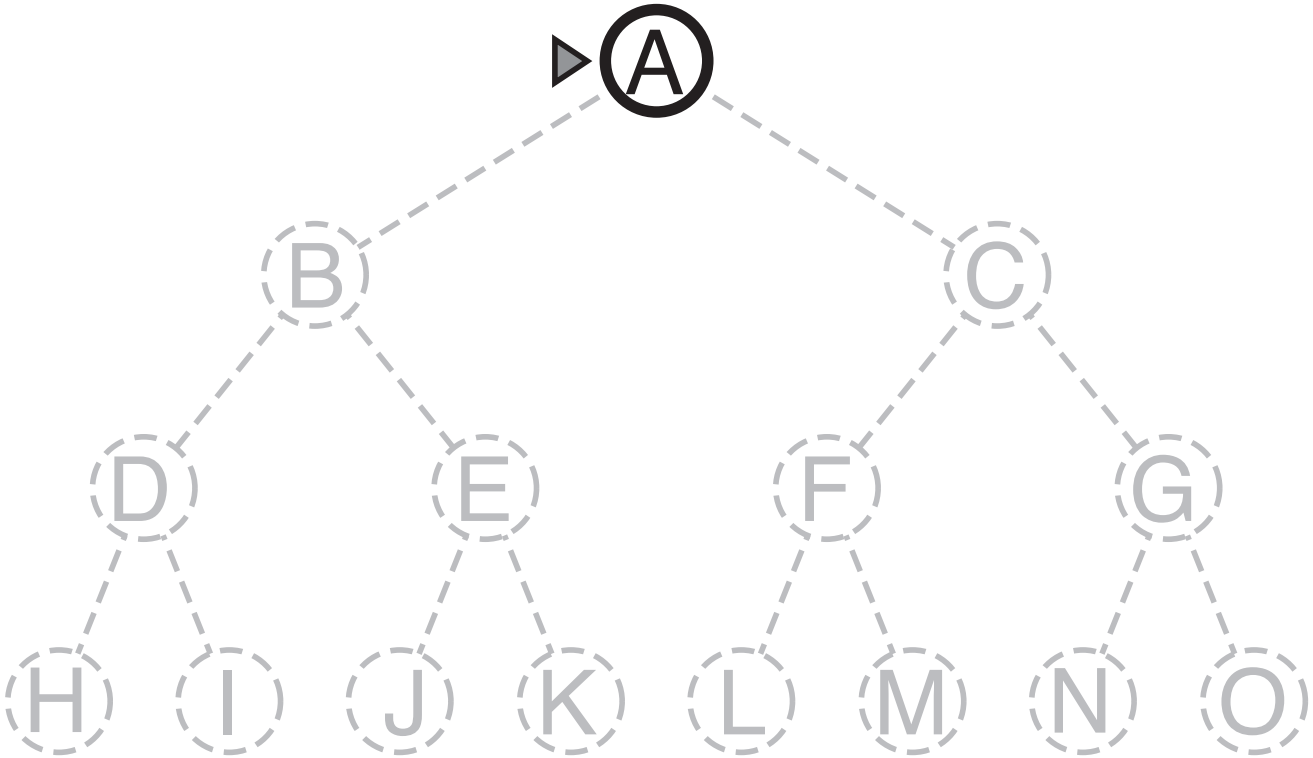
- **Space** is the bigger problem (more than time)
  - Suppose b = 10, a node uses 1000 bytes, and 1 million nodes per second can be generated
    - Solution at depth 10 takes 3 hours and uses 10 terabytes
    - At level 12 it is 13 days and 1 petabyte; at 14 it is 3.5 years, 99 petabytes
  - **Exponential complexity** bound search problems cannot be solved by uninformed strategies—not scalable

Uniform-cost search

- Expand **least-cost** unexpanded node
- Implementation
  - *Frontier* is a queue ordered by path cost
- Equivalent to breadth-first search if step costs are all equal
  - Complete? **Yes**, if step cost ≥ ε
  - Time? # of nodes with $g \leq$ cost of optimal solution, $\mathbf{O}(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
  - Space? # of nodes with $g \leq$ cost of optimal solution, $\mathbf{O}(b^{ceiling(C^*/\varepsilon)})$
  - Optimal? **Yes**—nodes are expanded in increasing order of $g(n)$

Depth-first search

- Expand deepest unexpanded node
- Implementation
  - *Frontier* is a LIFO queue (stack), i.e., new successors go at the front



Depth-first search

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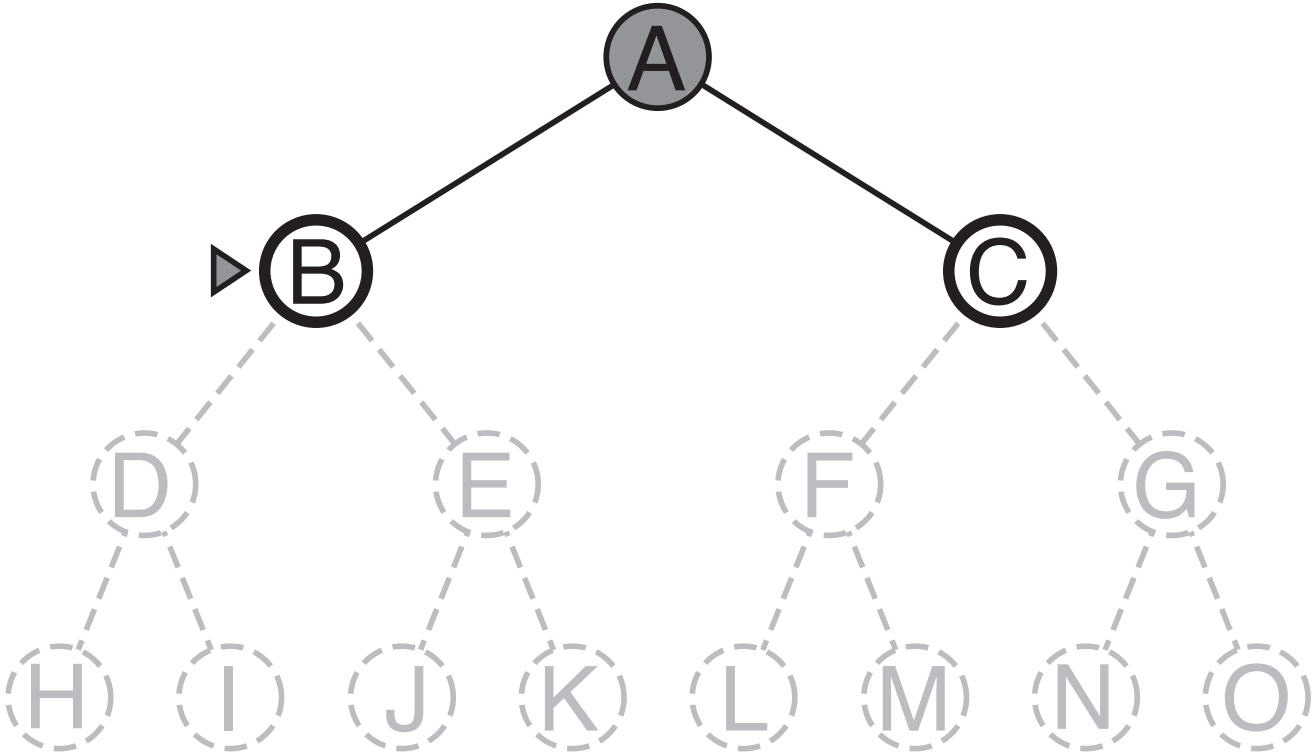

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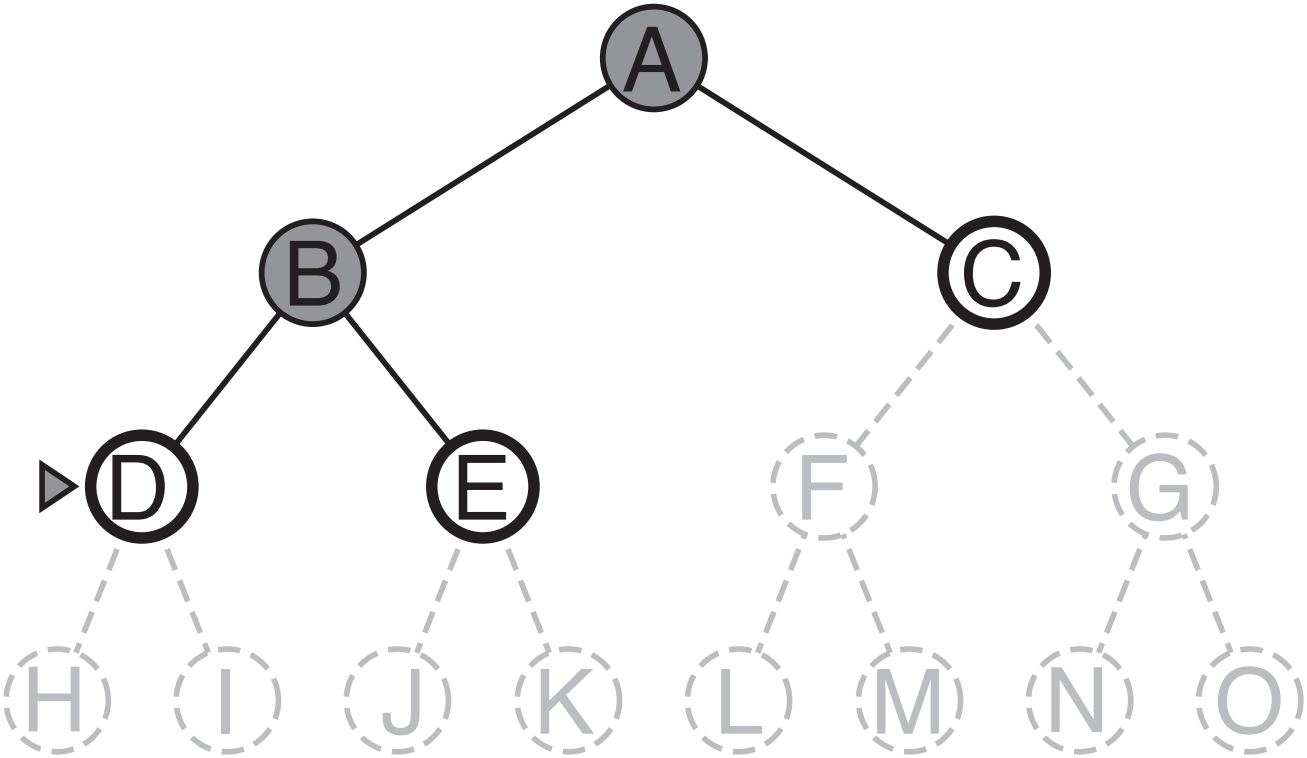

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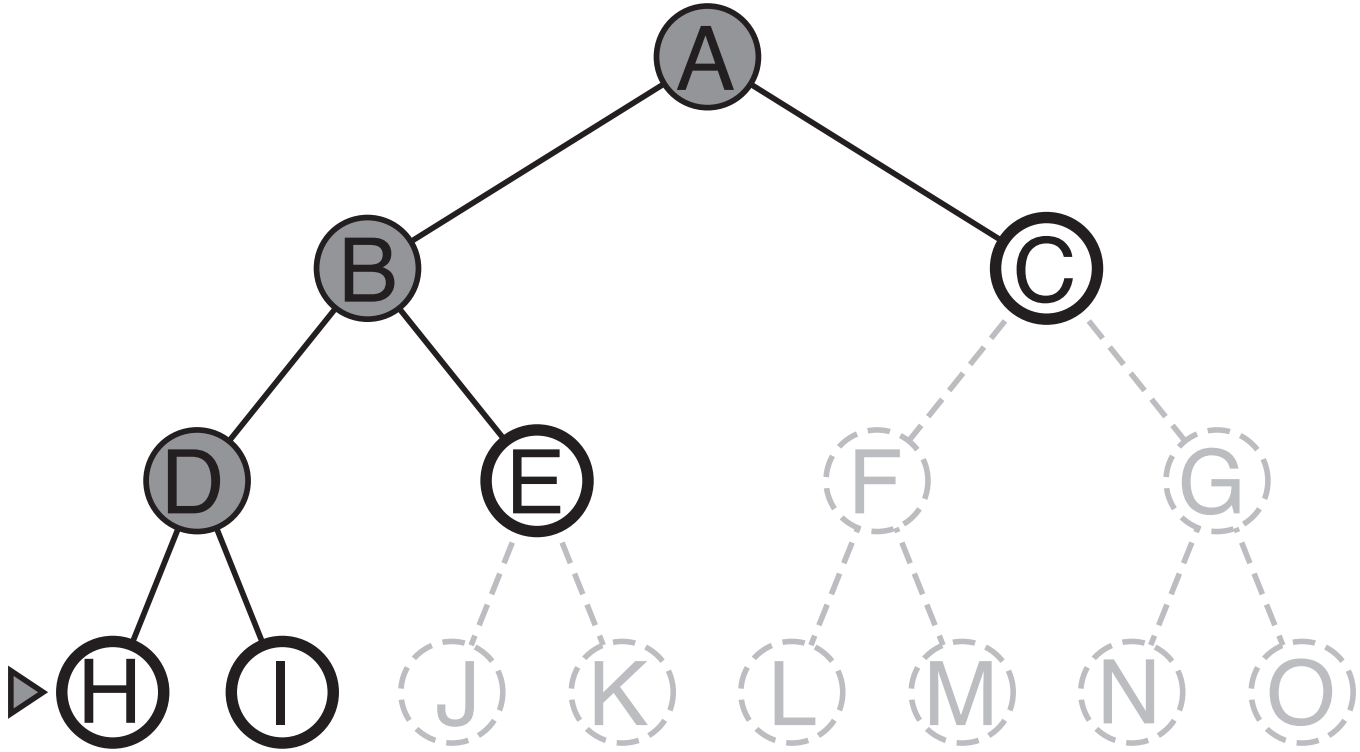

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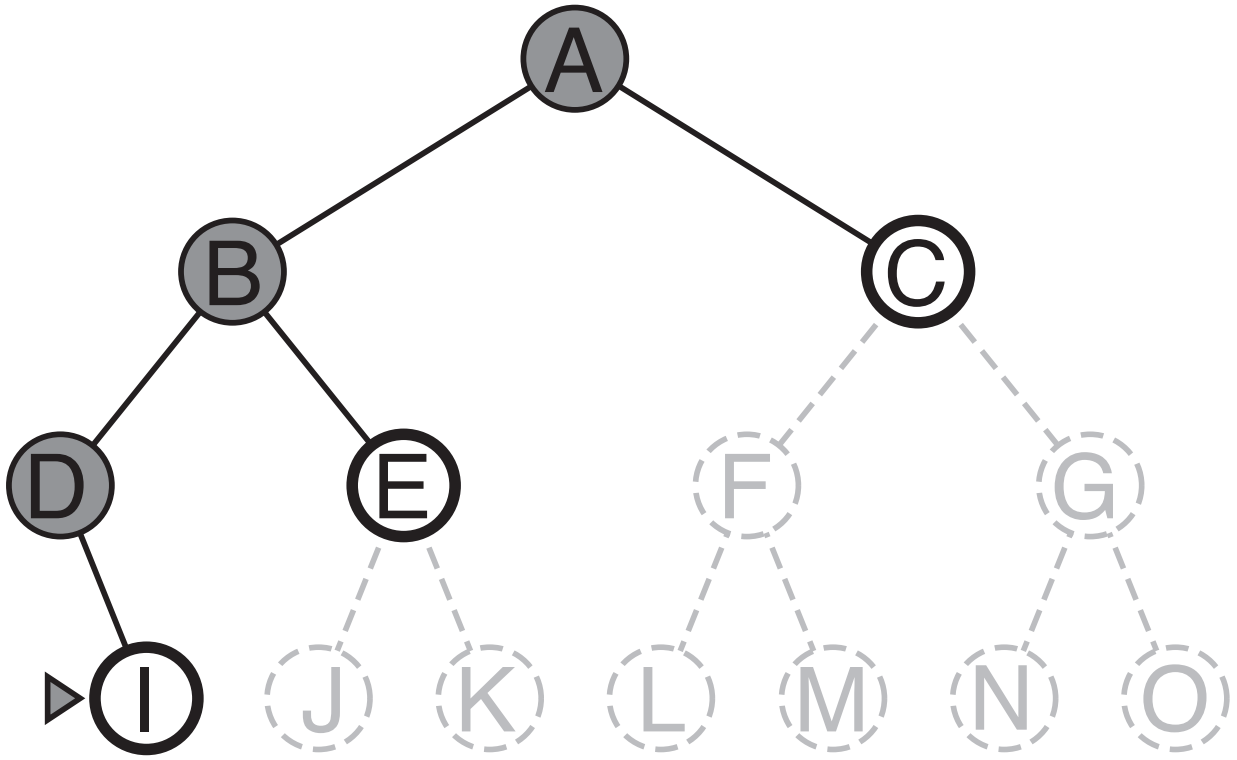

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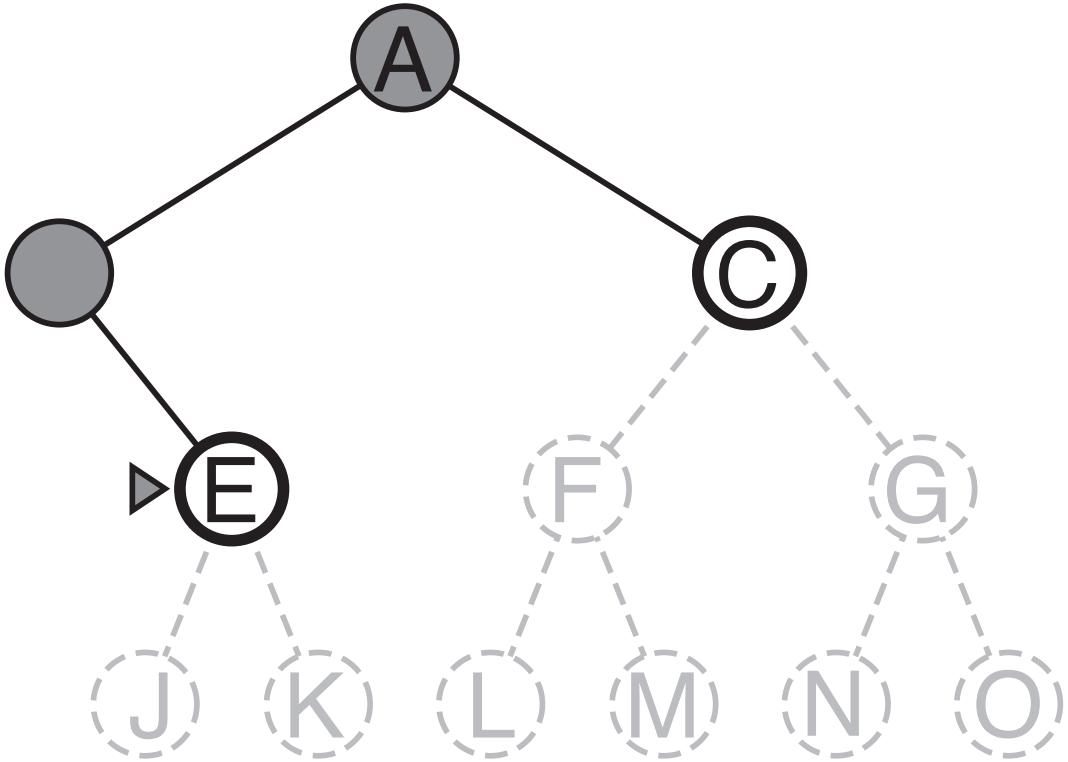

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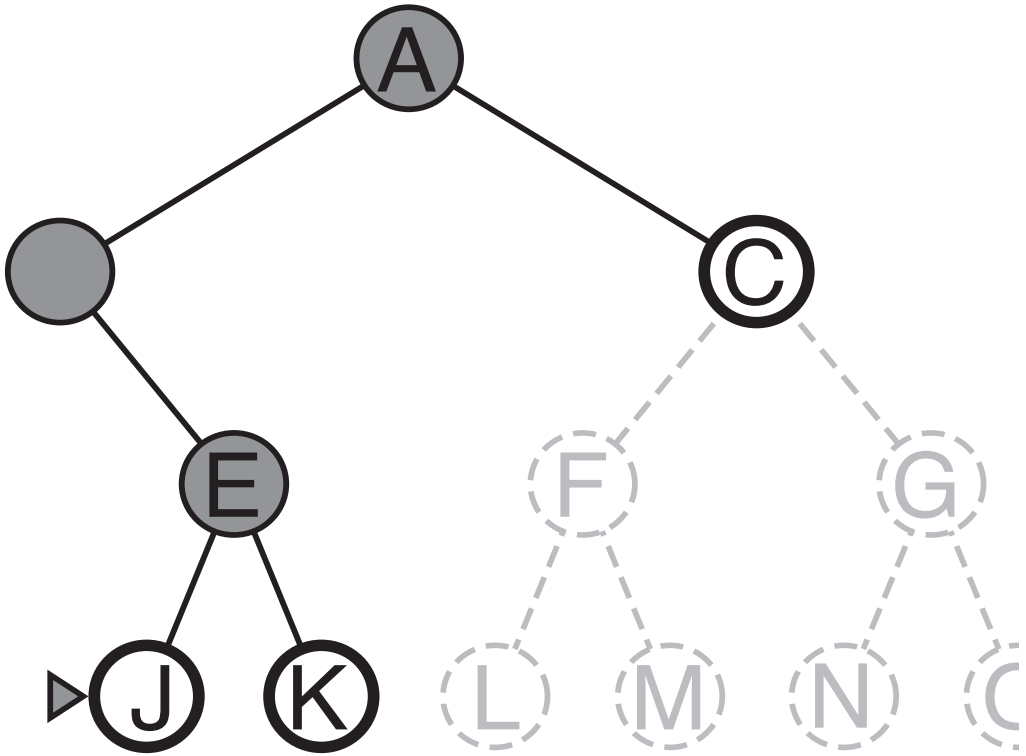

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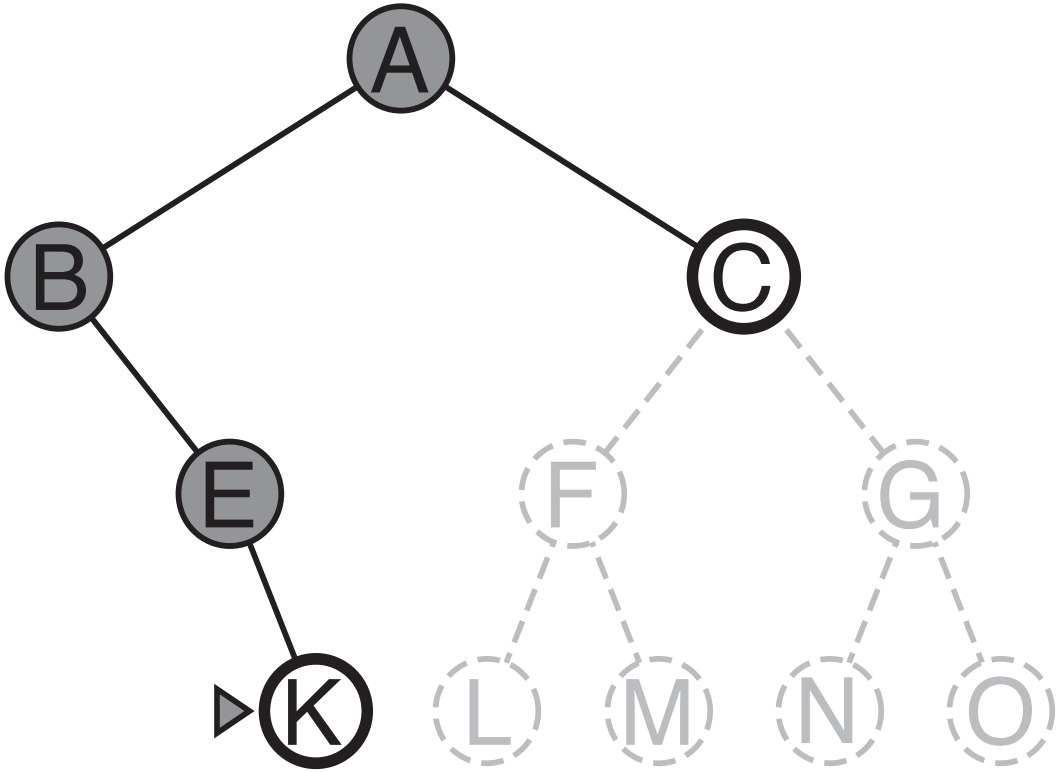

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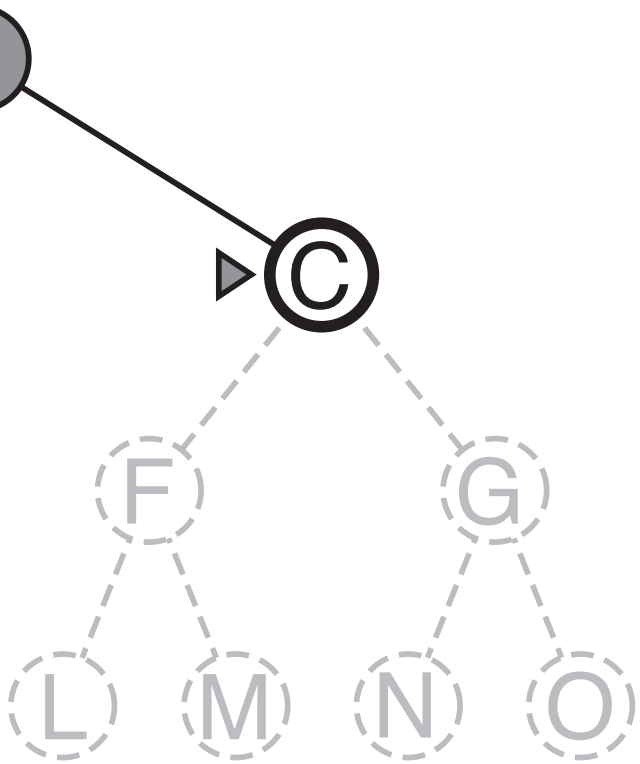

Depth-first search

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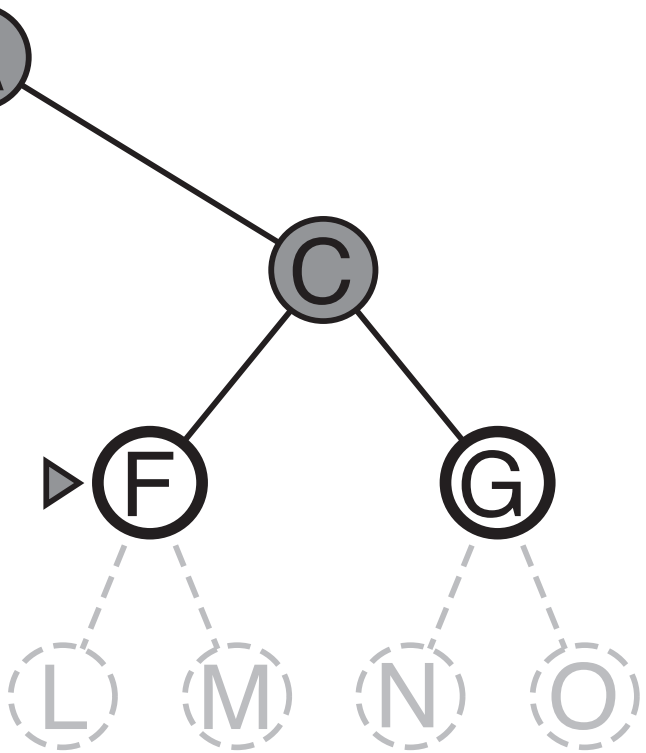

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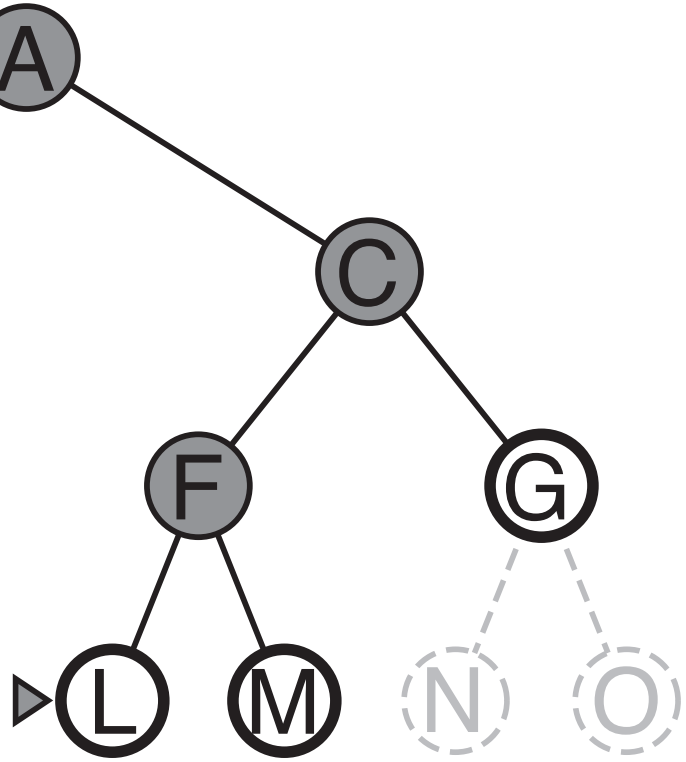

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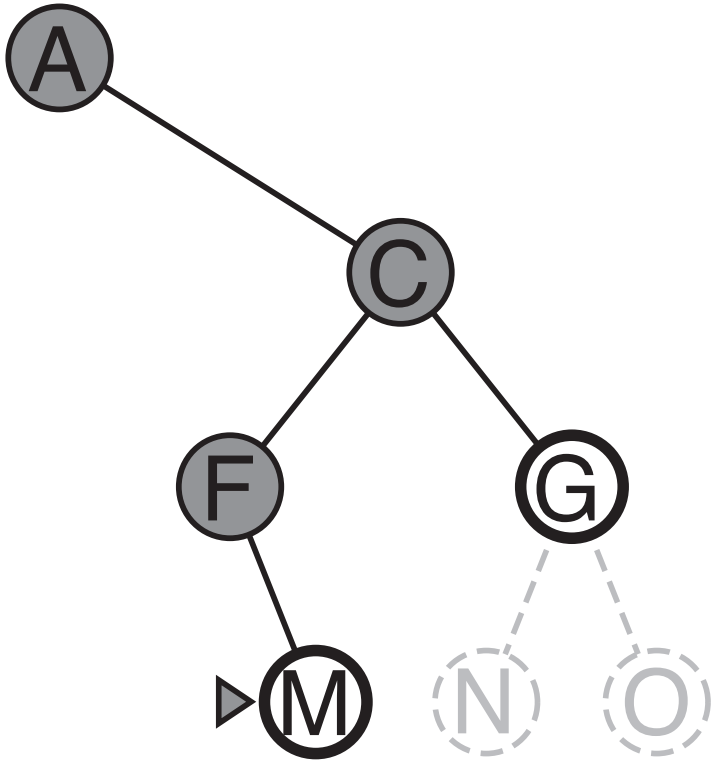


Properties of depth-first search

- Complete? **No**—fails in infinite depth spaces or spaces with loops (NOTE: our algorithm prevents loops)
  - Modify to avoid repeated states along path—complete in finite spaces
- Time? **$O(b^m)$**—terrible if m is much larger than d
  - Solutions are dense, so may be much faster than breadth-first
- Space? **$O(bm)$**—linear space!
- Optimal? **No**

Depth-limited search

- Depth-first search with **depth limit** *l*
  - Nodes at depth *l* have no successors

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

- Gradually **increase** the depth limit in depth-first search
  - Combines benefits of breadth-first and depth-first search

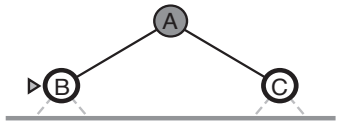
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

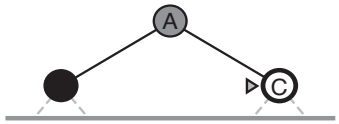
Iterative deepening search

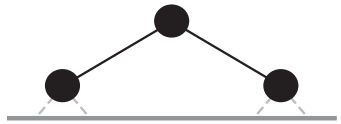
Limit = 0

Iterative deepening search

Limit = 1

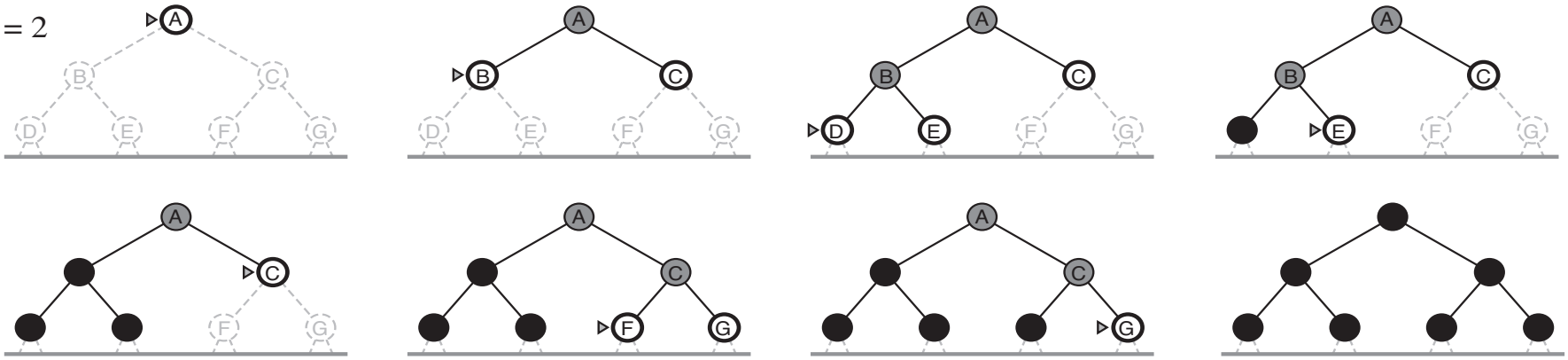




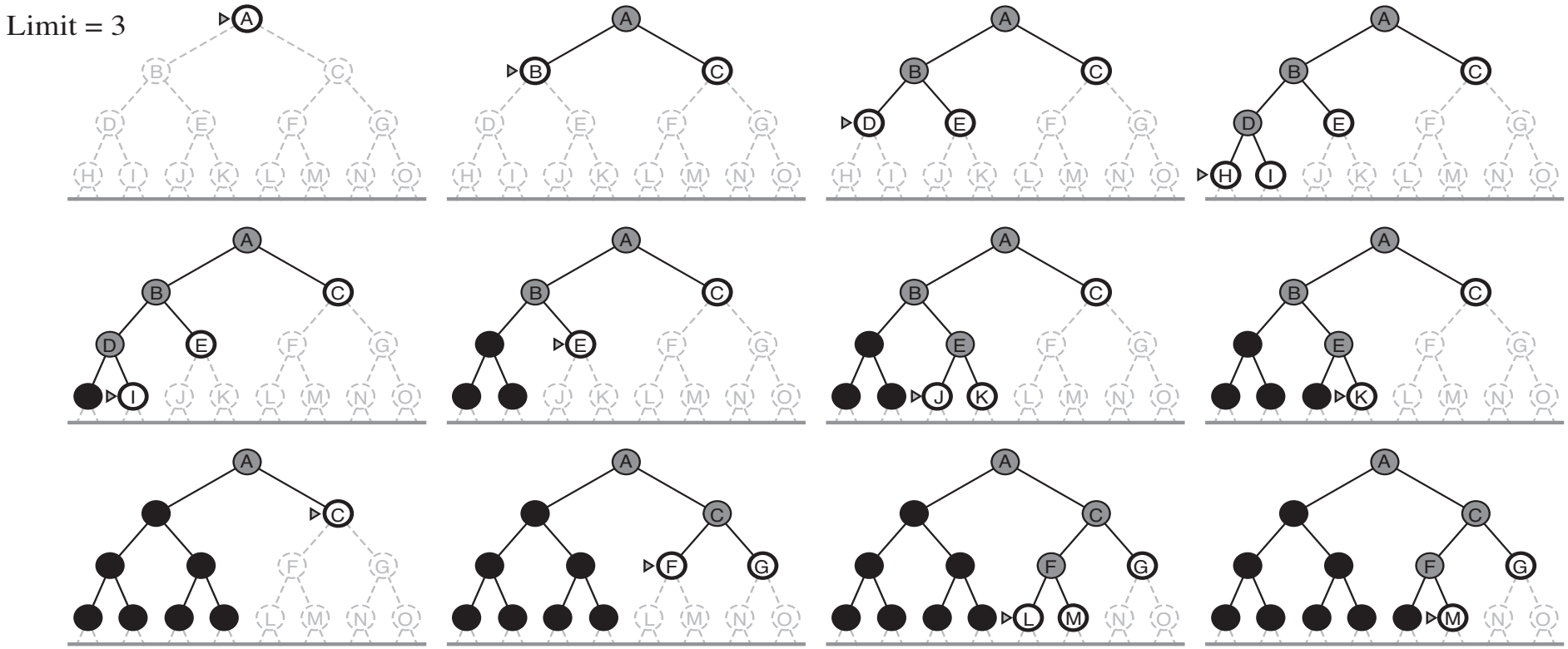


Iterative deepening search

Limit = 2



Iterative deepening search



Iterative deepening search

- **Number of nodes** generated in iterative deepening search to depth d with branching factor b
  $N(\text{IDS}) = (d+1)b^0 + (d)b^1 + (d-1)b^2 + \ldots + (3)b^{d-2} + (2)b^{d-1} + (1)b^d$
  - **Time complexity** = $O(b^d)$
  - Same as breadth-first search
- Time complexity **comparison** to breadth-first search
  - Suppose $b = 10$ and $d = 5$
    $N(\text{IDS}) = 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}450$
    $N(\text{BFS}) = 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}110$
  - Overhead difference = (123,450 - 111,110) / 111,110 = 11%

Properties of iterative deepening

- Complete? **Yes**
- Time? $(d+1)b^0 + (d)b^1 + (d-1)b^2 + \ldots + (3)b^{d-2} + (2)b^{d-1} + (1)b^d = \mathbf{O(}\boldsymbol{b^d}\mathbf{)}$
- Space? $\mathbf{O(}\boldsymbol{bd}\mathbf{)}$—same as depth-first search
- Optimal? **Yes**, if step cost = 1

Summary of uninformed search algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Informed search

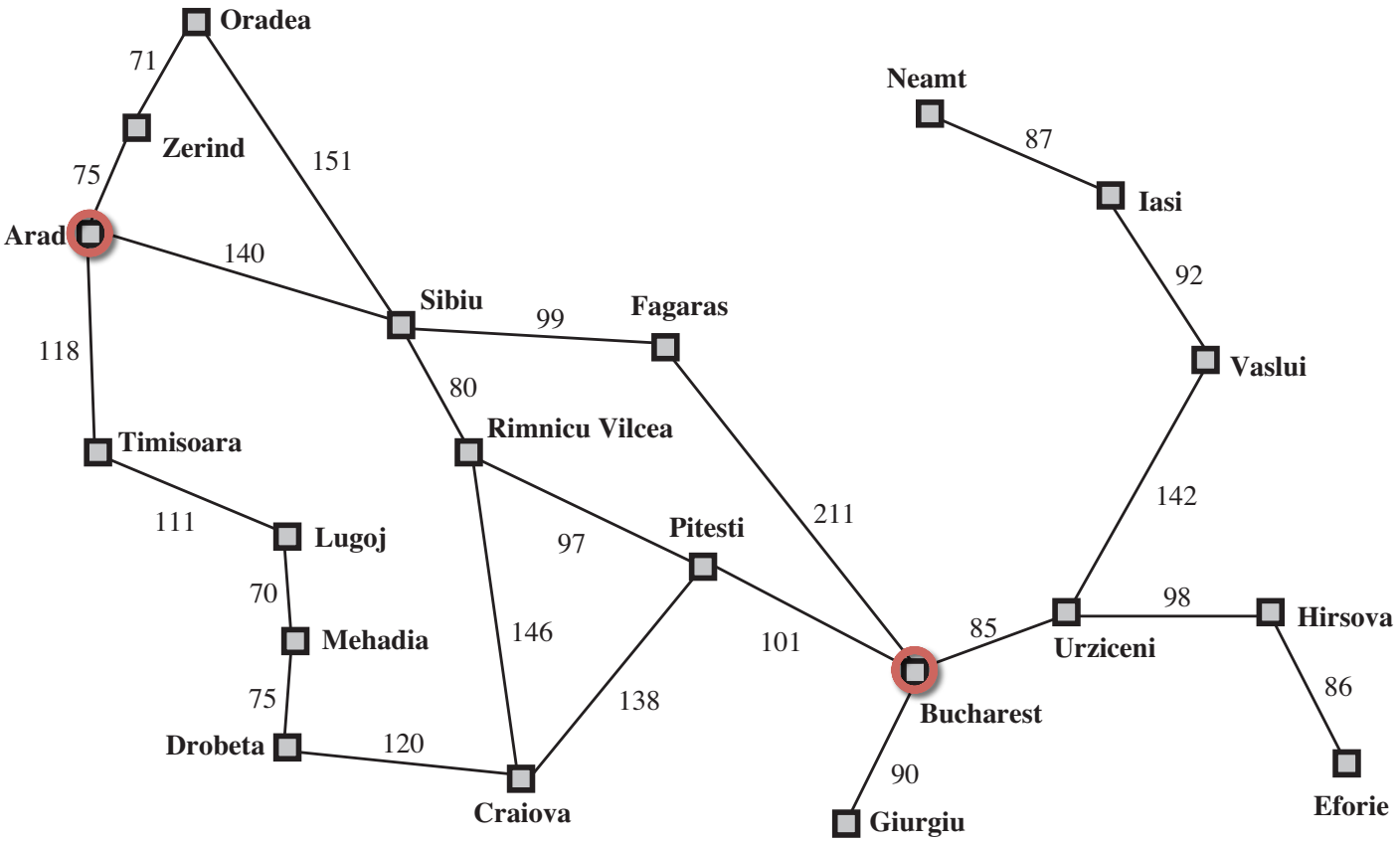
- Uses **problem-specific** knowledge beyond the problem definition to find solutions more efficiently
- Outline of topics
  - Heuristics
  - Best-first search
  - Greedy best-first search
  - A* search
- Remember, a search strategy is defined by picking the **order of node expansion**

Best-first search

- Use an **evaluation function** $f(n)$ for each node
  - Estimate of "desirability"
  - Includes a **heuristic function** to add more knowledge to the problem
    $h(n)$ = estimated cost of cheapest path from state at n to the goal
    - Heuristics are problem-specific
- Expand **most desirable** unexpanded node
- Implementation
  - Order nodes in the frontier in decreasing order of desirability
- Special cases
  - Greedy best-first search
  - A* search

Heuristic for path-finding

- Romania example: estimate the straight-line distance from each node to Bucharest (the goal)



Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search

- Evaluation function
  $f(n) = h(n)$ = estimate cost from n to *goal*
  - e.g., $h_{SLD}(n)$ = straight line distance from n to Bucharest

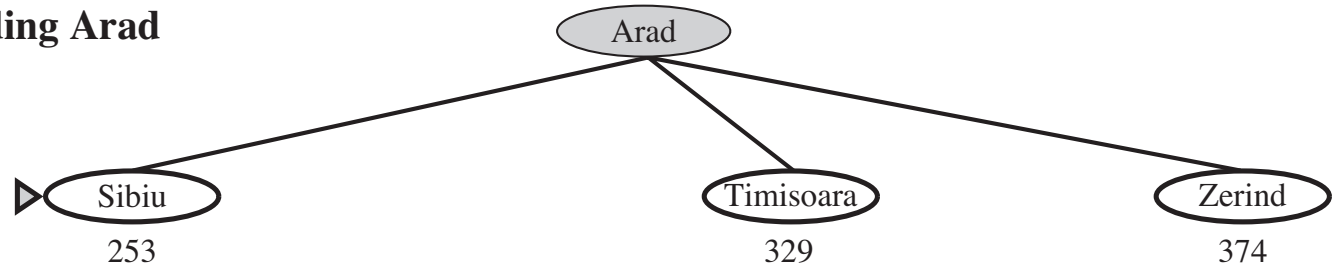
- Search strategy
  - Expand the node that **appears** to be closest to the goal (i.e., has the lowest $h(n)$)

Greedy best-first search

(a) The initial state

Greedy best-first search

(b) After expanding Arad



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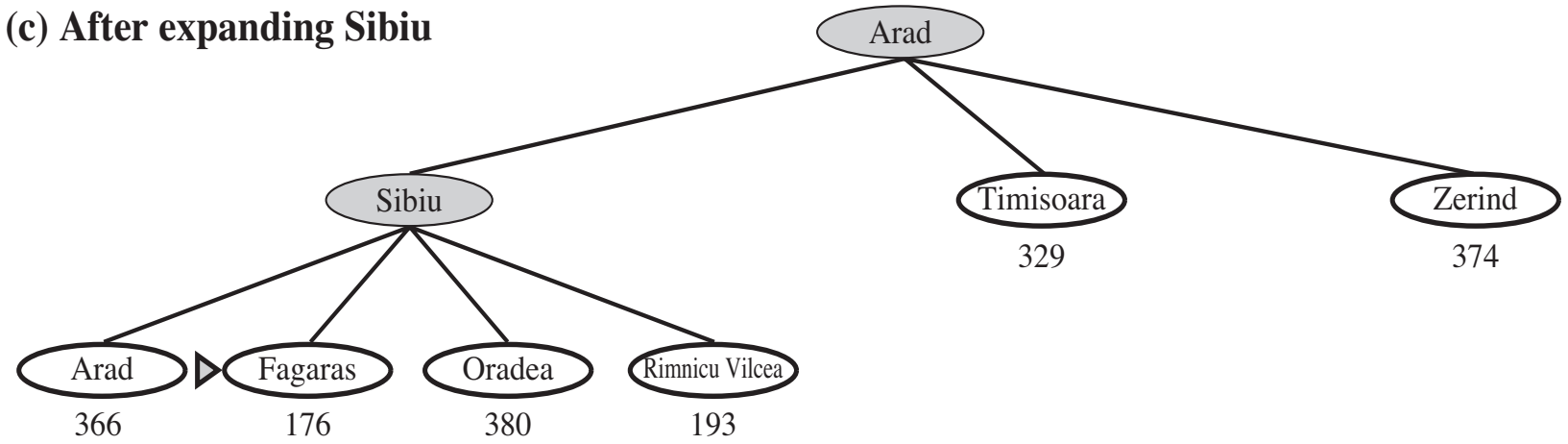

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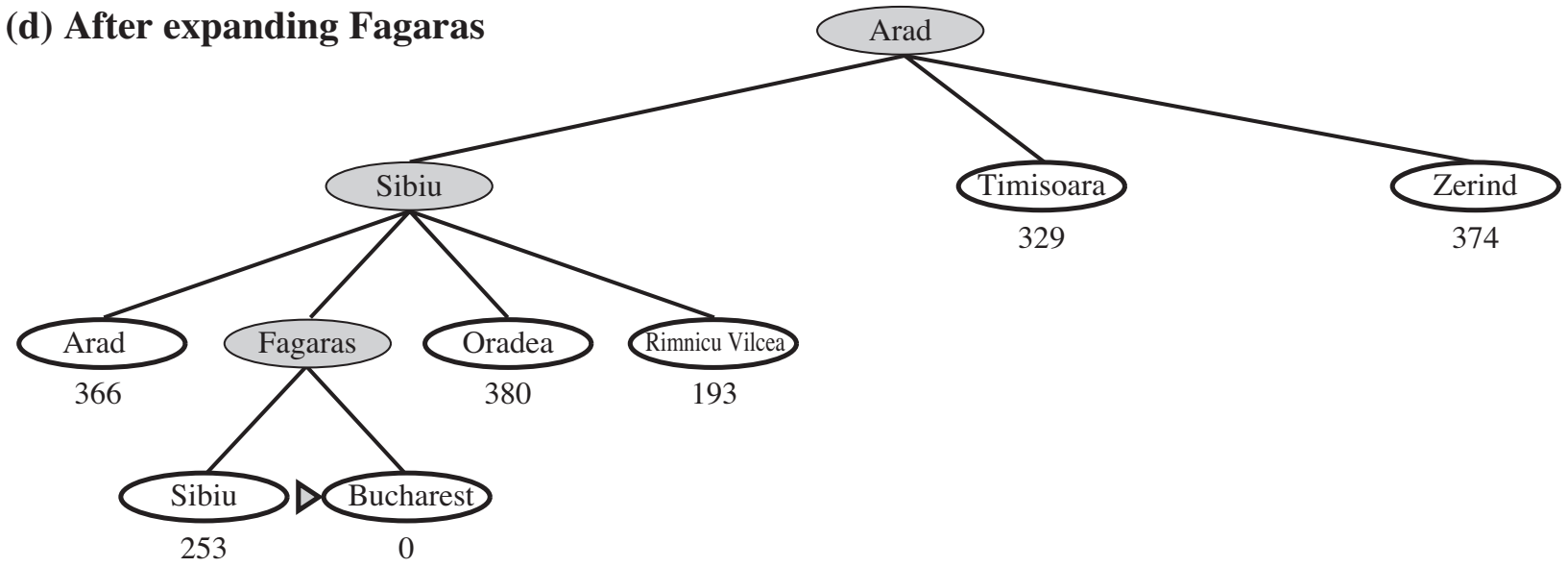


Properties of greedy best-first search

- Complete? **No**—can get stuck in loops
  e.g., Iasi → Neamt → Iasi → Neamt → …
- Time? **$O(b^m)$**, but a good heuristic can give dramatic improvement
- Space? **$O(b^m)$**—keeps all nodes in memory
- Optimal? **No**

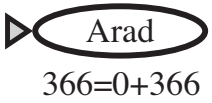
A* Search

- Avoid expanding paths that are already expensive

- **Evaluation function** $f(n) = g(n) + h(n)$
  - $g(n)$ = cost of path so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal

A* Search

(a) The initial state



A* Search

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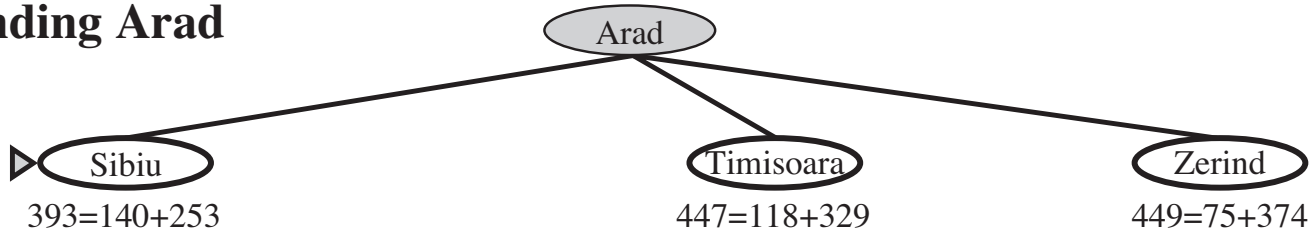
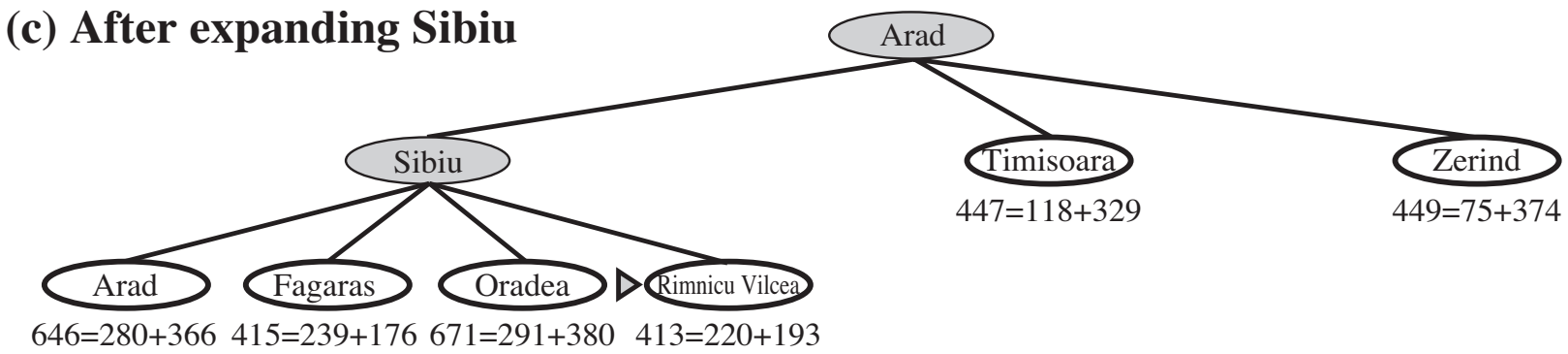
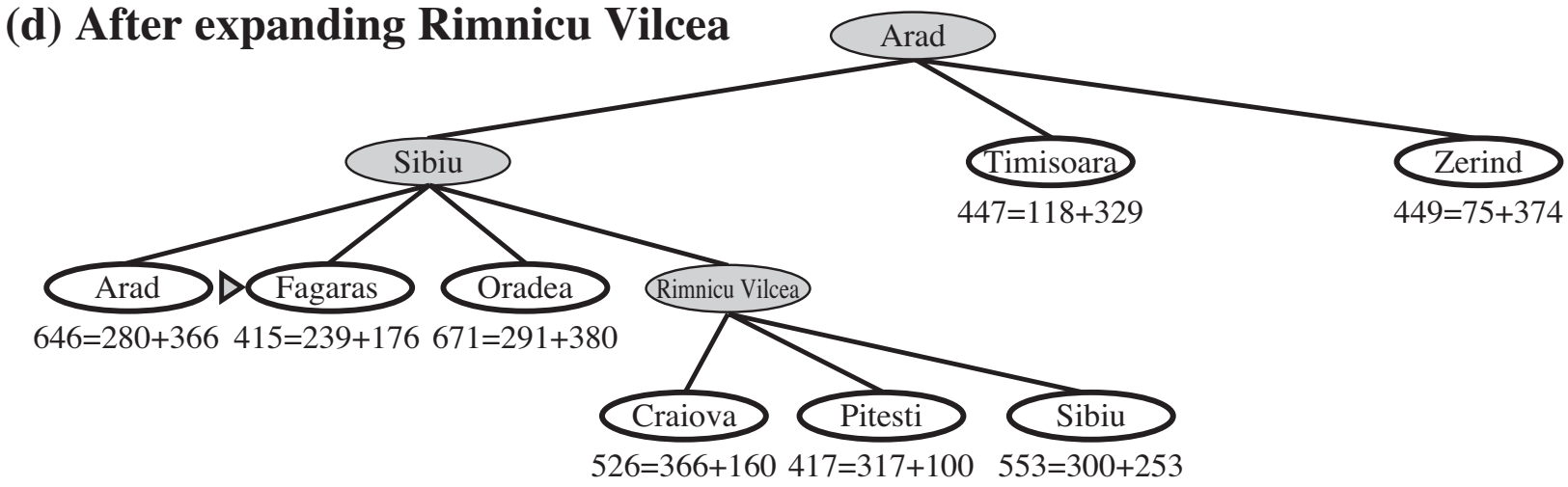


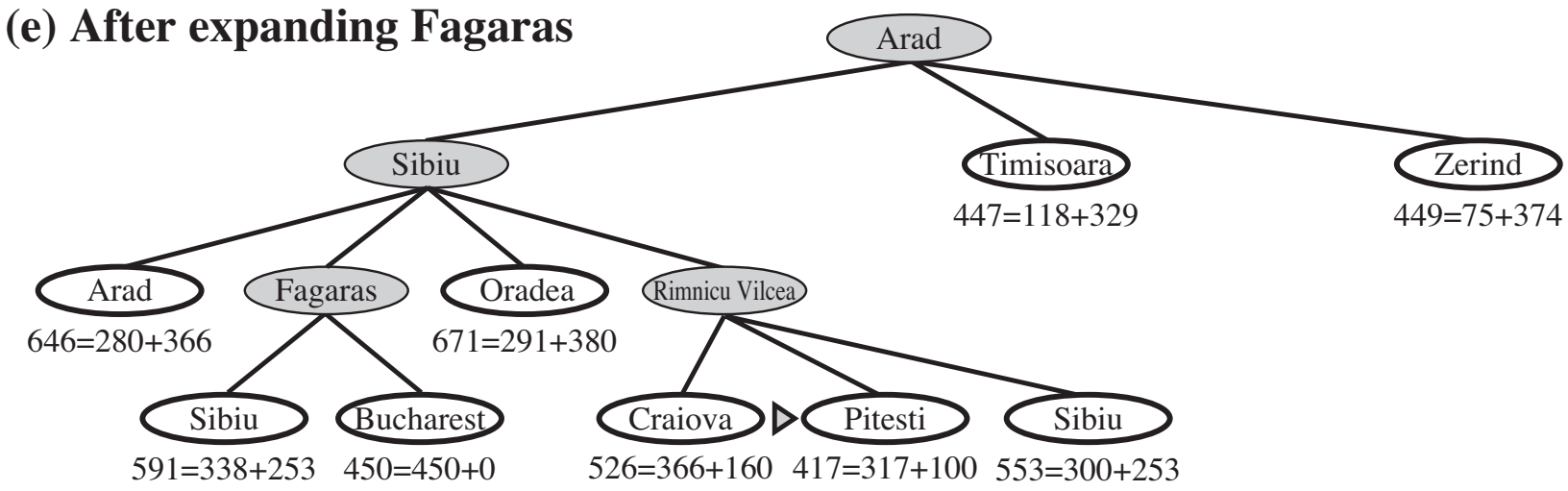
A* Search


(c) After expanding Sibiu
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193

A* Search


(d) After expanding Rimnicu Vilcea
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

A* Search


(e) After expanding Fagaras
Arad
Sibiu
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447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
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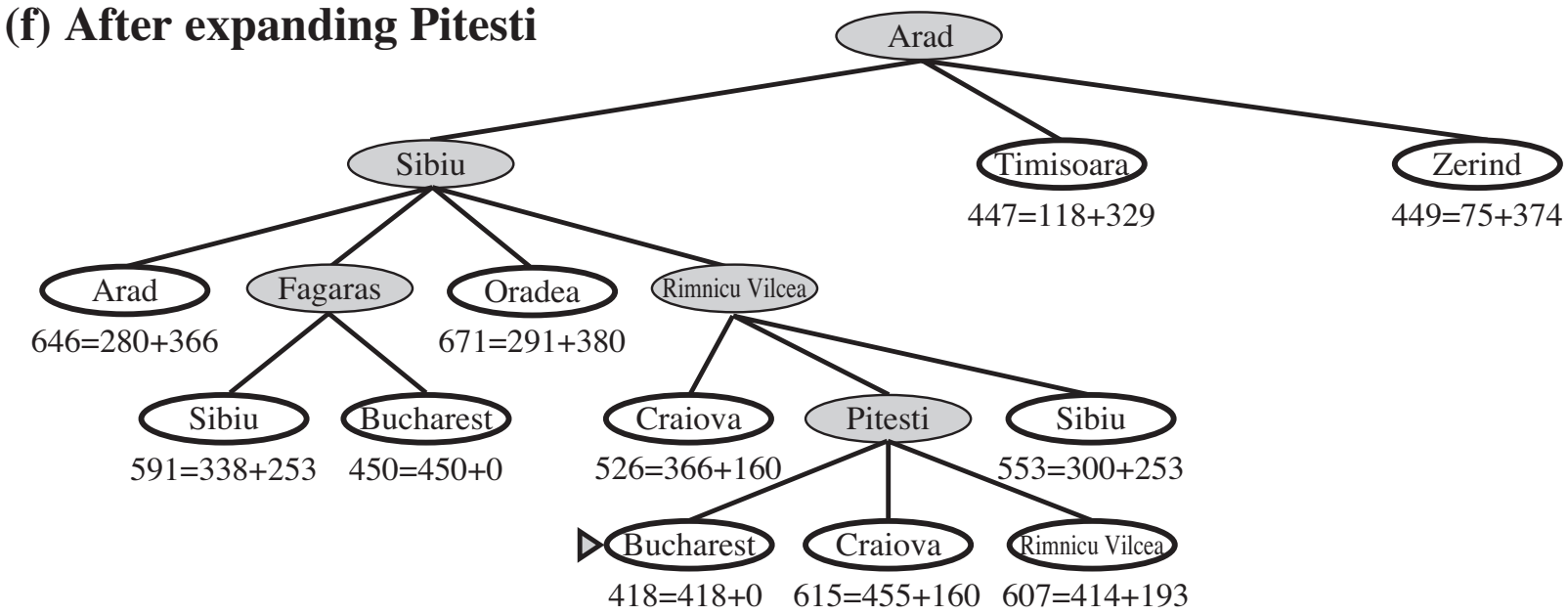
A* Search


(f) After expanding Pitesti
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Pitesti
Sibiu
553=300+253
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

Properties of A*

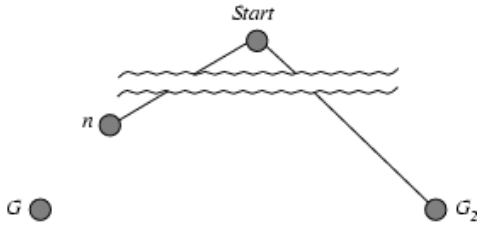
- Complete? **Yes** (unless there are infinitely many nodes with $f(n) < C^*$
- Time? **Exponential**—depends on assumptions made about state space
- Space? Keeps **all nodes** in memory
- Optimal? **Yes**

Admissible heuristics

- Heuristic $h(n)$ is **admissible** if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n
  - **Never overestimates** the cost to reach the goal, i.e., it is **optimistic**
- Example
  - $h_{SLD}(n)$ never overestimates the actual road distance by using straight-line distance
- **Theorem**
  - If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 is in the frontier. Let n be an unexpanded node in the frontier s.t. n is on a shortest path to an optimal goal G



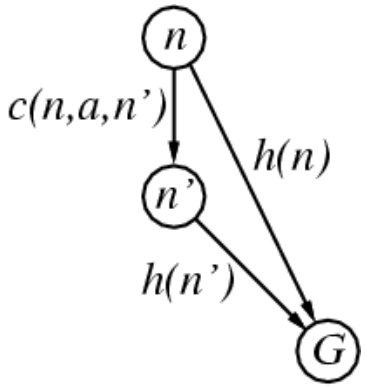
- Proof:
  - $f(G_2) > f(G)$ from above
    $h(n) \leq h^*(n)$ since h is admissible
    $g(n) + h(n) \leq g(n) + h^*(n)$
    $f(n) \leq f(G)$
  - Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

Consistent heuristics

- Heuristic is **consistent** if for every node n, every successor n' of n generated by action a,
  $h(n) \leq c(n,a,n') + h(n')$
- If h is consistent we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

  - i.e., $f(n)$ is non-decreasing along any path



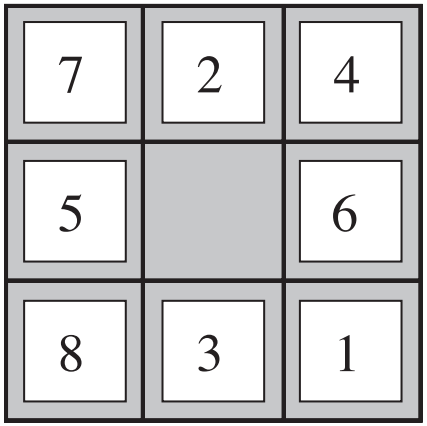
- **Theorem**
  - If h(n) is consistent, A* using `GRAPH-SEARCH` is optimal

Generating admissible heuristics

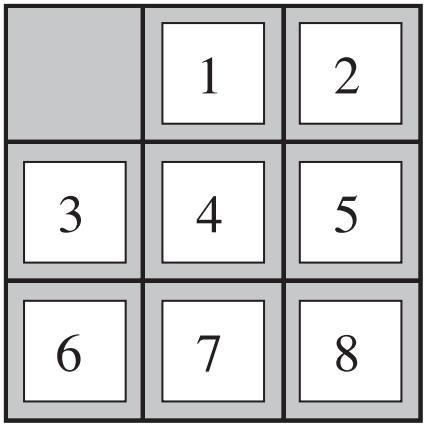
- Possible heuristics for the 8-puzzle
  - Generate from a **relaxed** problem—fewer restrictions than original

    $h_1(n)$ = number of misplaced tiles

    $h_2(n)$ = total Manhattan distance to the goal position



Start State



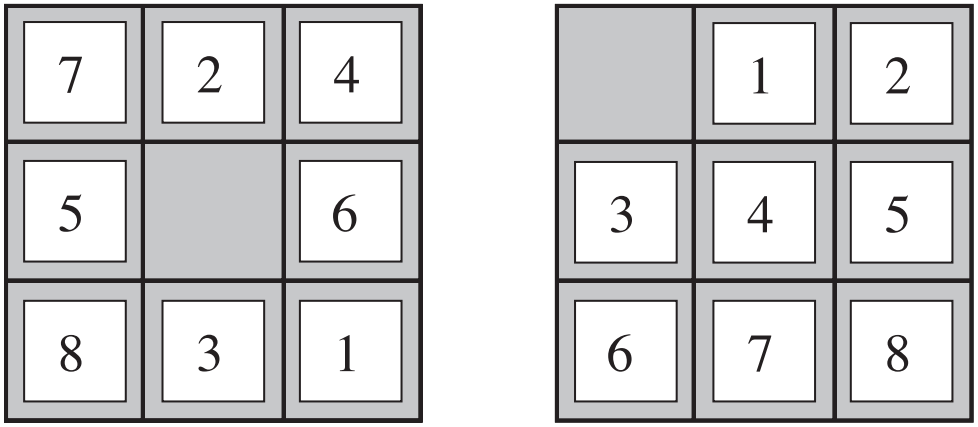
Goal State

$h_1(S)$ =?

$h_2(S)$ =?

Generating admissible heuristics

- Possible heuristics for the 8-puzzle
  - Generate from a **relaxed** problem—fewer restrictions than original
    $h_1(n)$ = number of misplaced tiles
    $h_2(n)$ = total Manhattan distance to the goal position



Start State Goal State

$h_1(S)$ =? **8**

$h_2(S)$ =? **3 + 1 + 2 + 2 +2 + 3 +3 + 2 = 18**

$h^*(S)$ = 26 so both are admissible heuristics (lower bounds)