Lists

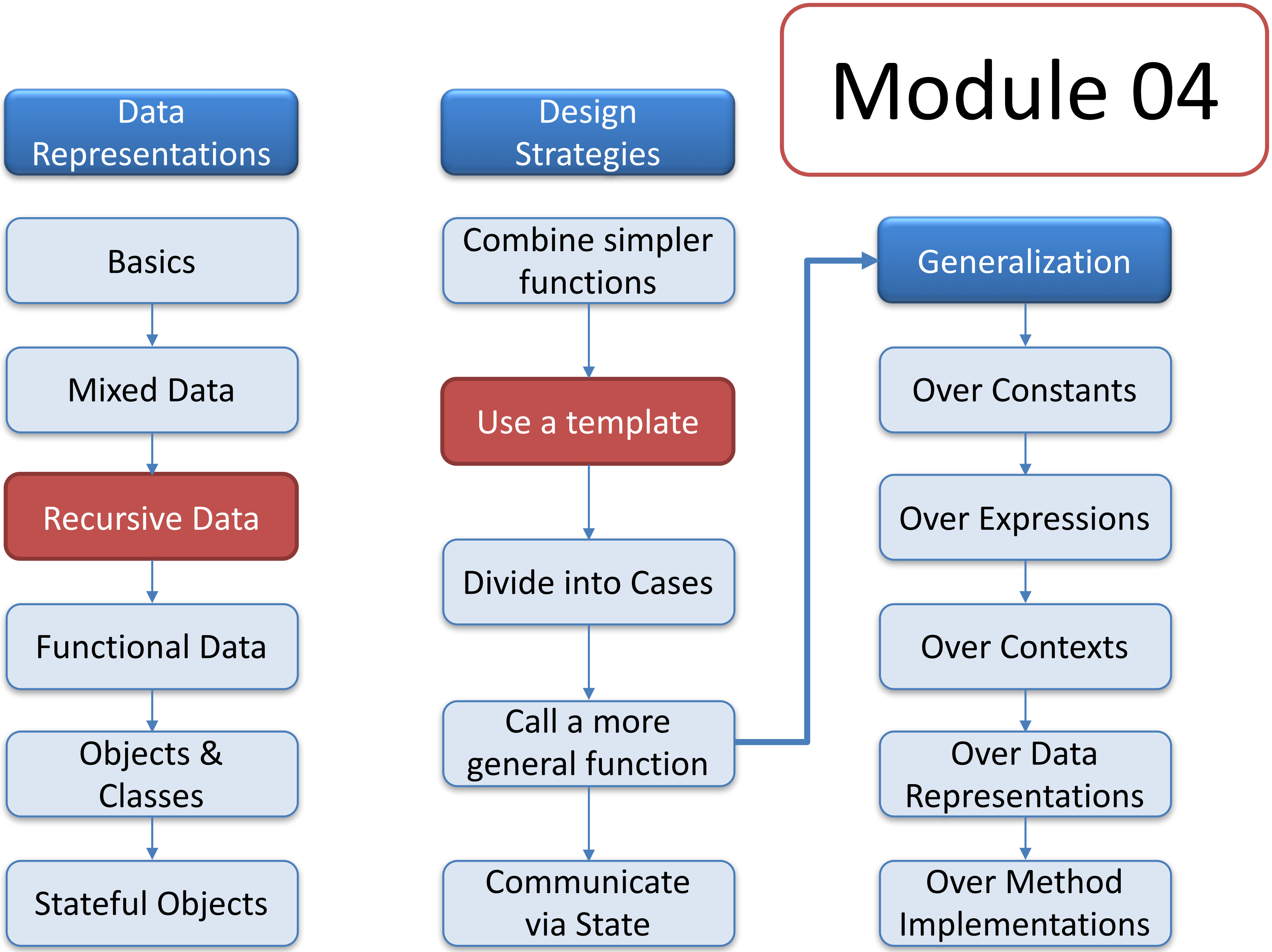
CS 5010 Program Design Paradigms

Lesson 4.1

How to represent info of arbitrary size?

- a phone book with many listings
- a space-invaders game with many invaders
- a presentation with many slides

- Each of these can be represented as a sequence of information items.
- There may be better ways for some of these, but we will start with sequences
- This is our first example of *recursive data*



Module 04
Data Representations
Basics
Mixed Data
Recursive Data
Functional Data
Objects & Classes
Stateful Objects
Design Strategies
Combine simpler functions
Use a template
Divide into Cases
Call a more general function
Communicate via State
Generalization
Over Constants
Over Expressions
Over Contexts
Over Data Representations
Over Method Implementations

Outline for the rest of this week

- The arithmetic of lists
- Using the list template
- Lists of Structures

Learning Objectives for this Lesson

At the end of this lesson, you should be able to:

- Write down a data definition for information represented as a list
- Notate lists using constructor, list, and write notations.
- Explain how lists are represented as singly-linked data structures, and how **cons**, **first**, and **rest** work on these structures
- Calculate with the basic operations on lists: **`cons`**, **`first`**, and **`rest`** .

Lists: A Handy Representation for Sequences

- Sequences of data items arise so often that most programming languages have a standard way of representing them.
- Sequence information in Racket is represented by *lists*.
- We'll see lots of examples:
  - ListOfNumbers
  - ListOfDigits
  - ListOfStrings
  - ListOfBooks

Lists of Numbers

```
A List of Numbers (LON) is one of:
-- empty
-- (cons Number LON)
```

cons is built into Racket. We don't need a define-structure for it.

List data is a kind of mixed data. Just as we did in our previous data definitions, the data definitions for lists shows the constructor for each case.
Here we have two constructors: the constant **empty** and the function **cons**. A list of numbers (or "LON") is either **empty** or the value built by applying **cons** to a number and another LON.

There's no interpretation here because these lists don't mean anything (yet). They do not refer to any real-world information.

Examples of LONs

```
                          empty
                (cons 11 empty)
       (cons 22 (cons 11 empty))
(cons 33 (cons 22 (cons 11 empty)))
                (cons 33 empty)
```

A List of Numbers (LON) is one of:
-- empty
-- (cons Number LON)

Here are some examples of LONs.

empty is a LON by the data definition.

(cons 11 empty) is a LON because **11** is a number and **empty** is a LON.

(cons 22 (cons 11 empty)) is a LON because **22** is a number and **(cons 11 empty)** is a LON.
And so on.

Lists of Digits

```
A Digit is one of
 "0" | "1" | "2" | ... | "9"

A List of Digits (LOD) is one of:
-- empty
-- (cons Digit LOD)
```

Let's do it again, this time with digits.

We define a Digit to be one of the strings **"0"**, **"1"**, etc., through **"9"**.

A List of Digits (LOD) is either empty or the cons of a Digit and a List of Digits.

Examples of LODs

```
                                empty
                      (cons "3" empty)
            (cons "2" (cons "3" empty))
  (cons "4" (cons "2" (cons "3" empty)))
```

- These are not LODs:

```
    (cons 4 (cons "2" (cons "3" empty)))
      (cons (cons "3" empty)
            (cons "2" (cons "3" empty)))
```

```
A List of Digits (LOD) is one of:
-- empty
-- (cons Digit LOD)
```

Can you explain why each of the first 4 examples are LOD's, according to the data definition?
Can you explain why the last two are not LODs?

Lists of Books

```
A Book is a (make-book ...) .

A List of Books (LOB) is one of:
-- empty
-- (cons Book LOB)
```

We can build lists of more complicated data items. Imagine we had a data definition for Book. Then we can define a List of Books in the same way as we did for lists of numbers or lists of digits: a List of Books is either empty or the cons of a Book and a List of Books.

Examples of LOBs

```
(define book1 (make-book ...))
(define book2 (make-book ...))
(define book3 (make-book ...))
```

```
                                     empty
                         (cons book1 empty)
             (cons book2 (cons book1 empty))
(cons book2 (cons book2 (cons book1 empty))
```

- Not a LOB: (Why?)

```
    (cons 4 (cons book2 (cons book1 empty))
```

```
A List of Books (LOB) is one of:
-- empty
-- (cons Book LOB)
```

This data definition is *self-referential*

```
A List of Numbers (LON) is one of:
-- empty
-- (cons Number LON)
```

The data definition for LONs contains something we haven't seen before: *self-reference*.
The second constructor uses LON, even though we haven't finished defining LONs yet. That's what we mean by self-reference.
In normal definitions, this would be a problem: you wouldn’t like a dictionary that did this.
But self-reference the way we've used it is OK. We've seen in the examples how this works: once you have something that you know is a LON, you can do a cons on it to build another LON. Since that's a LON, you can use it to build still another LON.
We also call this a *recursive* data definition.

This one is self-referential, too

```
A Digit is one of
 "0" | "1" | "2" | ... | "9"

A List of Digits (LOD) is one of:
-- empty
-- (cons Digit LOD)
```

How Lists Represent Sequences

- If X is some data definition, we define a list of X's as either empty or the cons of an X and a list of X's.
- So a list of sardines is either **empty** or the **cons** of a sardine and a list of sardines.
- The interpretation is always "a sequence of X's".
  - **empty** represents a sequence with no elements
  - **(cons x lst)** represents a sequence whose first element is **x** and whose other elements are represented by **lst**.
- If we had some information that we wanted to represent as a list of X's (say a list of people), we would have to specify the order in which the X's appear (say "in increasing order of height"), or else say “in any order.”

The General Pattern

```
A ListOfX is one of
-- empty
   interp: a sequence of X's with no elements
-- (cons X ListOfX)
   interp: (cons x lst) represents a sequence of X's
   whose first element is x and whose
   other elements are represented by lst.
```

List Notation

- There are several ways to write down lists.
- We've been using the *constructor notation*, since that is the most important one for use in data definitions.
- The second most important notation we will use is *list notation*. In Racket, you can get your output in this notation by choosing the language "Beginning Student with List Abbreviations".
- Internally, lists are represented as singly-linked lists.
- On output, lists may be notated in *write notation.*

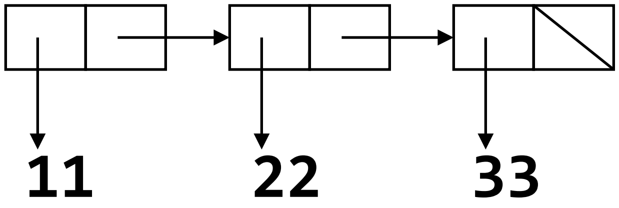
Examples of List Notation

Constructor notation:

```
(cons 11
  (cons 22
     (cons 33
           empty))))
```

List notation: `(list 11 22 33)`

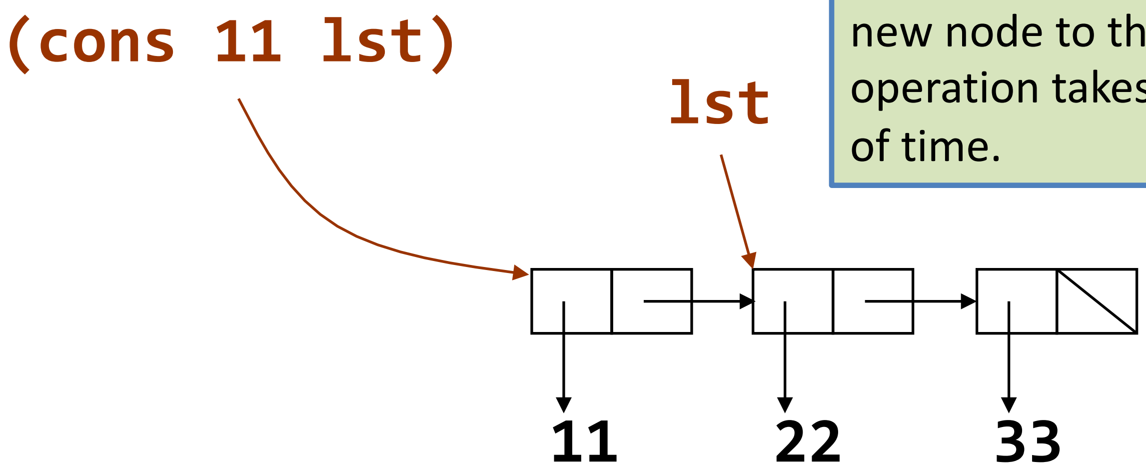
Internal representation:



`write`-style (output only): `(11 22 33)`

Implementation of **cons**

Now that we've seen the internal representation of lists, we can see how **cons** creates a new list: it simply adds a new node to the front of the list. This operation takes a short, fixed amount of time.




```
lst  =    (list 22 33)

(cons 11 lst) = (list 11 22 33)
```

Operations on Lists

```
empty? : ListOfX -> Boolean
Given a list, returns true iff the
list is empty
```

Racket provides 3 functions for inspecting lists and taking them apart. These are **empty?** , **first**, and **rest**.

The predicate **empty?** returns true if and only if the list is empty.

Operations on Lists

```
first : ListOfX -> X
GIVEN: a list
WHERE: the list is non-empty
RETURNS: its first element
```

When we write down the template for lists, we will see that when we call **first**, its argument will always be non-empty.

Operations on Lists

```
rest : ListOfX -> ListOfX
GIVEN: a list
WHERE: the list is non-empty
RETURNS: the list of all its
elements except the first
```

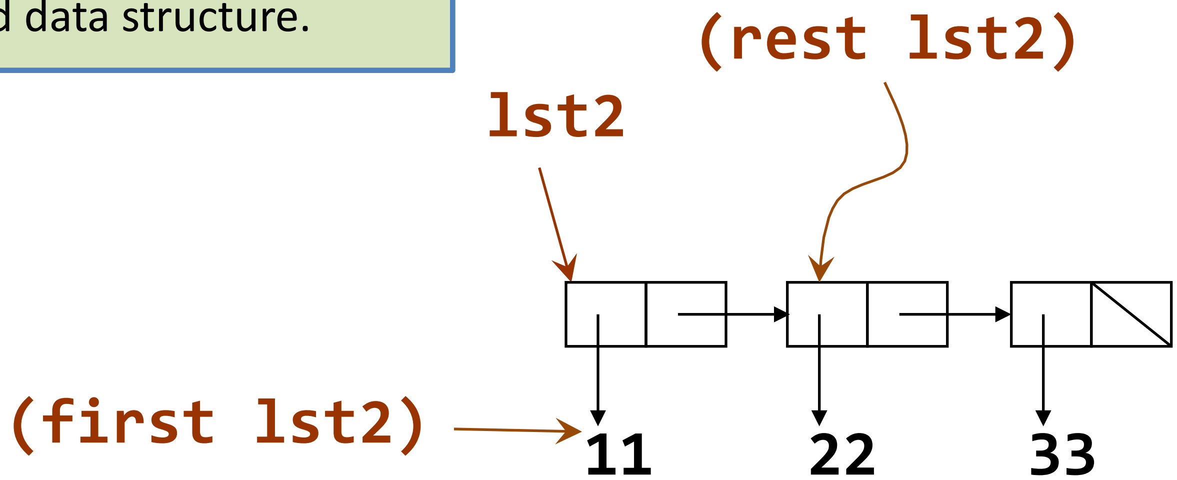
When we write down the template for lists, we will see that when we call **rest**, its argument will always be non-empty.

Examples

```
(empty?                    empty)    = true
(empty?           (cons 11 empty))   = false
(empty? (cons 22 (cons 11 empty)))   = false

(first (cons 11 empty)) = 11
(rest  (cons 11 empty)) = empty

(first (cons 22 (cons 11 empty))) = 22
(rest  (cons 22 (cons 11 empty))) = (cons 11 empty)

(first empty)  ➔ Error! (Precondition failed)
(rest  empty)  ➔ Error! (Precondition failed)
```

Implementation of **`first`** and **`rest`**

first and **rest** simply follow a pointer in the singly-linked data structure.




```
        lst2  = (list 11 22 33)
(first lst2) = 11
(rest  lst2) = (list 22 33)
```

Properties of **`cons`**, **`first`**, and **`rest`**

`(first (cons v l)) = v`

`(rest (cons v l)) = l`

If **`l`** is non-empty, then

`(cons (first l) (rest l)) = l`

Here are some useful facts about **first**, **rest**, and **cons**. Can you see why they are true?

These facts tell us that if we want to build a list whose **first** is **x** and whose **rest** is **lst**, we can do this by writing **(cons x lst)**.

Summary

At this point, you should be able to:

- Write down a data definition for information represented as a list
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Next Steps

- If you have questions about this lesson, ask them on the Discussion Board
- Do Guided Practices 4.1 and 4.2
- Go on to the next lesson