CS 3800, Spring 2016
Homework 4 (70 points)
Assigned: Friday, 5 February 2016
Due: Friday, 12 February 2016

Unless stated otherwise, all languages are over the alphabet $\{0, 1\}$.

1. [4 pts] Do exercise 2.2 in the textbook (both parts).

2. [8 pts] The following grammar is ambiguous:

$$\begin{aligned} E &\to \texttt{a} \\ E &\to \texttt{(}\ E\ \texttt{)} \\ E &\to E\ \texttt{\#}\ E \\ E &\to E\ \texttt{\&}\ E \\ E &\to E\ \texttt{\%}\ E \end{aligned}$$

   Give an equivalent CFG that is unambiguous and also says the `&` operator has highest precedence and associates to the right, the `%` operator has middle precedence and associates to the left, and the `#` operator has lowest precedence and associates to the left.

3. [24 pts] Give state diagrams for pushdown automata that generate the following languages over the alphabet $\{0, 1\}$.

   (a) $\{0^i 1 0^j 1 0^k \mid i = j + k\}$
   (b) $\{0^i 0^j 0^k 1^k \mid i = j + k\}$
   (c) $\{w \mid w$ contains three times as many 1s as 0s $\}$
   (d) $\{0^i 1^j 0^k \mid i < j$ and $k = j - i\}$

4. [15 points] For each of the following languages, decide whether the language is regular. If it is, then construct a regular expression that describes the language. If it isn't, construct a CFG that generates the language.

   (a) $\{w \mid w$ contains at least as many 0s as 1s$\}$
   (b) $\{w \mid w$ is a binary numeral divisible by 3$\}$
   (c) $\{w \mid w$ contains an even number of 0s$\}$
   (d) $\{w \mid w$ contains at least two 1s$\}$
   (e) $\{w \mid w$ does not contain three consecutive 0s$\}$

5. [10 points] For each of the following languages, state whether the language is regular, context-free but not regular, or not context-free.

   (a) $\{w \mid w$ contains more 0s than 1s$\}$

   (b) $\{0^n 1^n \mid n$ is a positive integer$\}$

   (c) $\{w \mid$ the length of w is a prime number$\}$

   (d) $\{w \mid w$ contains exactly three times as many 0s as 1s$\}$

   (e) $\{w^{\mathcal{R}} w \mid w$ is non-empty$\}$

   (f) $\{w \mid w$ is a binary numeral representing a multiple of 47$\}$

   (g) $\{w \mid w$ (as a binary numeral) is congruent to 5 mod 17$\}$

   (h) $\{0^n 1^n 0^n \mid n$ is a positive integer$\}$

   (i) $\{1^n 1^n 1^n \mid n$ is a positive integer$\}$

   (j) $\{1^m 1^n \mid m$ is prime but n is not prime$\}$

6. [4 points] Which of the following grammars generates a language that is not regular?

   (a) $S \to 1 \mid 11S$

   (b) $S \to 0 \mid 1S1$

   (c) $S \to 0 \mid 1 \mid SS$

   (d) $S \to 0 \mid 1S \mid 0S$

   (e) $S \to 1 \mid 1S1$

7. [5 points] Construct a PDA that recognizes the language generated by

$$\begin{aligned} S &\to A \\ A &\to \epsilon \mid 0B \mid 1C \\ B &\to A2A \\ C &\to A3A \end{aligned}$$