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CS 3800, Fall 2015 (Clinger's section)
Homework 1 (70 points)
Assigned:  Friday, 11 September 2015
Due:       Friday, 18 September 2015
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1. [5 pts] For each of the following set operations, specify the result by listing its elements inside curly braces.
   (a) $\{1, 2\} \cup \{2, 3, 4\} =$
   (b) $\{1, 2\} \cap \{2, 3, 4\} =$
   (c) $\{1, 2\} - \{2, 3, 4\} =$
   (d) $\{1, 2, 3\} - \{1, 4\} =$
   (e) $\{1, 2, 3\} \times \{3, 4\} =$
2. [4 pts] Write out each of the following power sets by listing their elements inside curly braces.
   (a) $\mathcal{P}(\emptyset) =$
   (b) $\mathcal{P}(\{5\}) =$
   (c) $\mathcal{P}(\{5, 6\}) =$
   (d) $\mathcal{P}(\{5, 6, 7\}) =$
3. [7 pts] If S is any set, then we use the notation $|S|$ to indicate the number of elements in S. Suppose A, B, and C are sets with $|A| = 3$, $|B| = 7$, and $|C| = 5$. Compute the number of elements in each of the following sets.
   (a) $|A \times A| =$
   (b) $|B \times C| =$
   (c) $|A \times B \times C| =$
   (d) $|\mathcal{P}(A)| =$
   (e) $|\mathcal{P}(B)| =$
   (f) $|\mathcal{P}(A \times C)| =$
   (g) $|\mathcal{P}(A \times B)| =$
4. [5 pts] For any $n \in \mathcal{N}$, we say n is even if and only if there exists $m \in \mathcal{N}$ such that $n = 2m$. We say n is odd if and only if there exists $m \in \mathcal{N}$ such that $n = 2m - 1$. From these definitions, give a rigorous proof that the sum of two odd numbers is even.
5. [5 pts] Give a rigorous proof that the product of two odd numbers is odd.
6. [5 pts] Using the definitions above, give a rigorous proof that there is no largest odd number.
7. [5 pts] Write down the formal (5-tuple) description of the DFA pictured in example 1.68(a) on page 76 of the textbook.

8. [4 pts] Draw the state transition diagram for the DFA whose formal description is

$$(\{q_1, q_2, q_3\}, \{a, b\}, \delta, q_1, \{q_1, q_2\})$$

where δ is the function listed within the following table:

	a	b
q_1	q_2	q_1
q_2	q_3	q_1
q_3	q_3	q_3

9. [4 pts] Describe the language recognized by the DFA whose formal description was given above.

10. [16 pts] For each of the following languages, draw the state transition diagram for a DFA with alphabet $\{0, 1\}$ that recognizes the language.

    (a) $\{\}$

    (b) $\{\epsilon\}$

    (c) $\{01, 10\}$

    (d) $\{w \mid w \text{ contains at least one } 0\}$

    (e) $\{w \mid w \text{ starts with } 0 \text{ and ends with } 0\}$

    (f) $\{w \mid w \text{ contains an odd number of 0s and an odd number of 1s}\}$

    (g) $\{w \mid w \text{ is a binary numeral that is divisible by } 3\}$

    (h) $\{w \mid w \text{ there exist strings } x \text{ and } y \text{ such that } w = x101y\}$

11. [5 pts] Do problem 1.37 in the textbook.

12. [5 pts] Prove the following theorem. If B is a language over an alphabet Σ, and $B = B^*$, then $BB \subseteq B$.