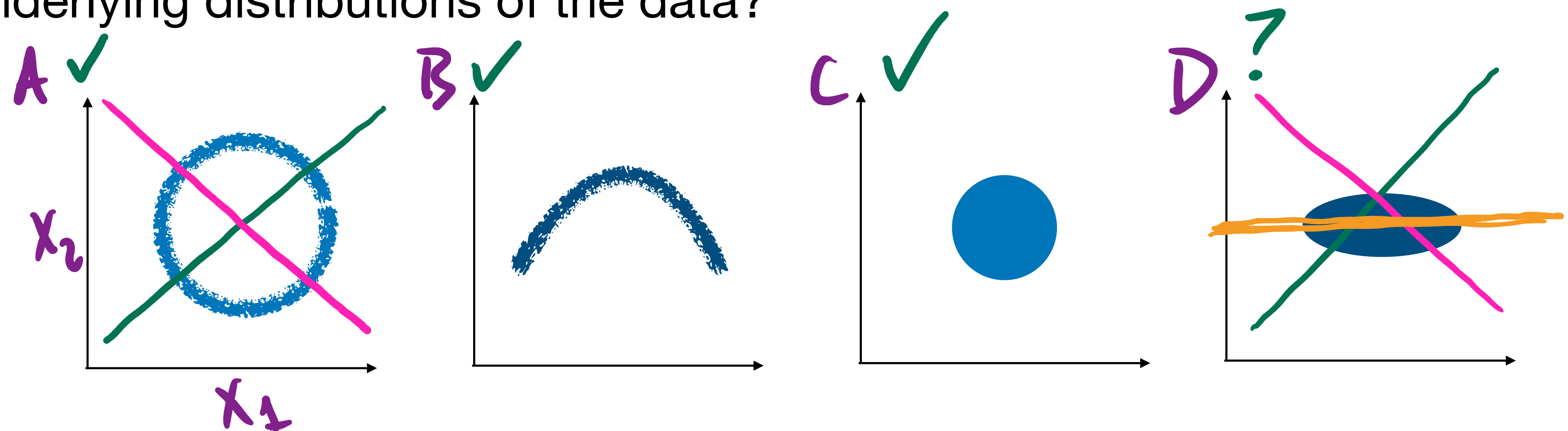
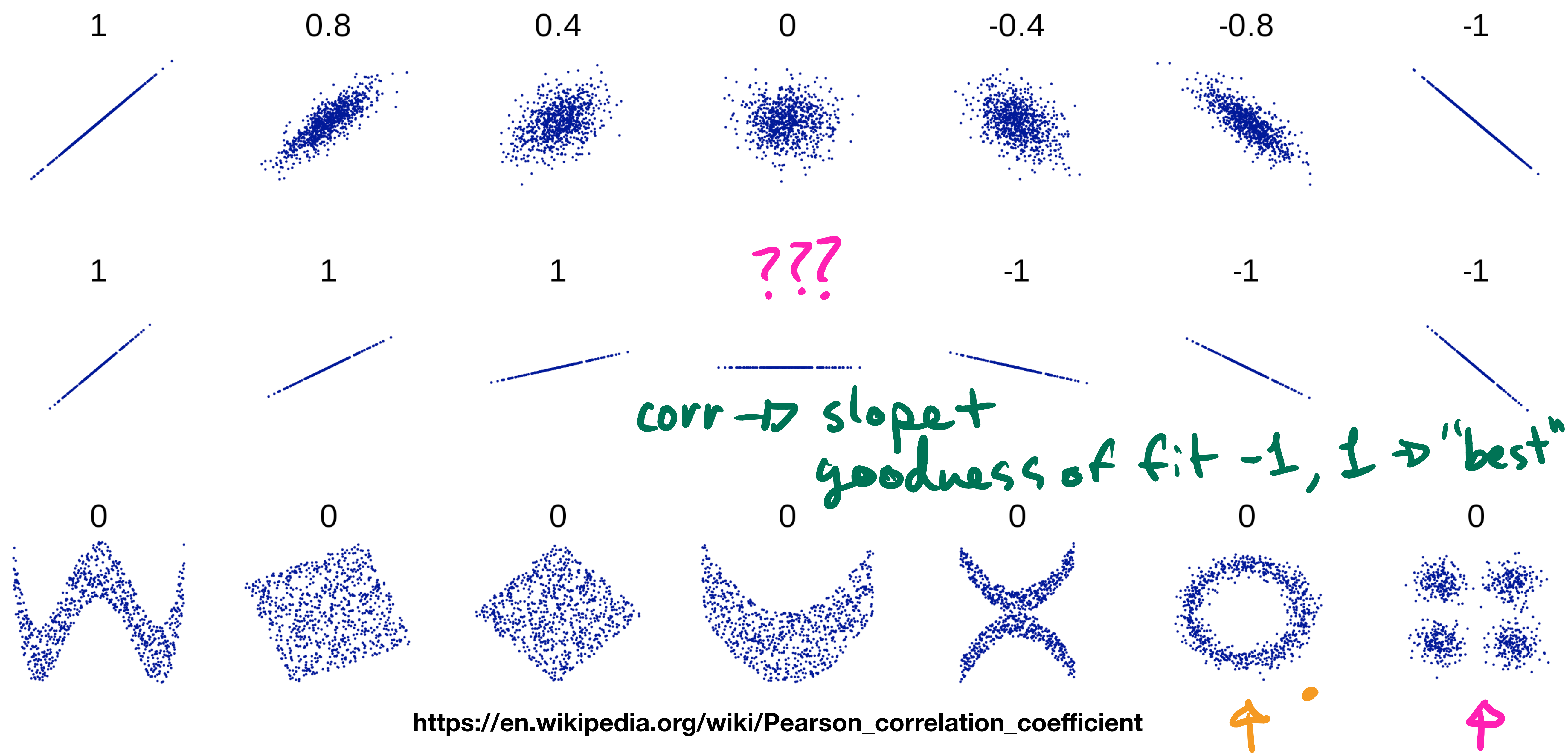
Bayes' rule, conditional ind., Bayes Nets (pt. 1)

You are told that $\underline{correlation}(x_1, x_2) = 0$. Which of the following might be underlying distributions of the data?



Pearson's Correlation Coefficient examples



https://en.wikipedia.org/wiki/Pearson_correlation_coefficient

ICA Question 1: Conditional Probabilities

You are given a gift. What is the probability that you were given a book?

$$P(G=\text{book}) = \frac{\#\text{books}}{\#\text{gifts}} = \frac{5}{8} = 0.625$$

You are given a gift by Felix. What is the probability that you were given a book?

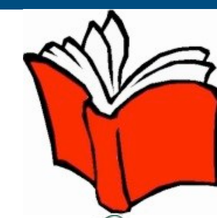
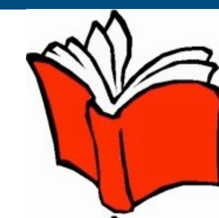
$$P(\overset{A}{G=\text{book}} \mid \overset{B}{\text{From}=\text{Felix}}) = \frac{2}{2} = 1$$

#books+Felix

#of Felix gifts

$$P(G=\text{puppy} \mid \text{From}=\text{felix}) = \frac{0}{2} = 0$$

$$\rightarrow \frac{P(A,B)}{P(B)}$$

Giver	Gift 1	Gift 2
Felix		
Swati		
Camilla		
Parth		

ICA Question 2: Bayes Rule

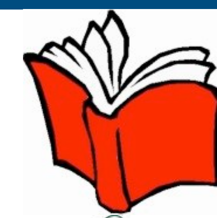
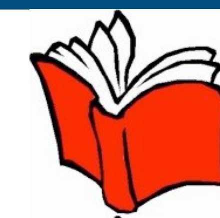
You are given a gift at a mystery gift exchange. What is the probability that Felix was the gift bringer?

$$P(\text{From} = \text{Felix}) = \frac{1}{4} = \frac{2 \text{ Felix gifts}}{8 \text{ gifts}}$$

You are given a **book** at a mystery gift exchange. What is the probability that Felix was the gift bringer?

$$P(\text{From} = \text{Felix} \mid G = \text{book}) = \frac{\#\text{ Felix books}}{\#\text{ total books}}$$

$$\frac{P(\text{book} \mid \text{Felix}) P(\text{Felix})}{P(\text{book})} = \frac{1 * 1/4}{5/8}$$

Giver	Gift 1	Gift 2
Felix		
Swati		
Camilla		
Parth		

Bayes' rule

- Bayes' rule denotes the relationship between $P(A \mid B)$ and $P(B \mid A)$
- $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$
- When calculating $P(B)$ for the denominator, it's often useful to calculate this as the sum of $\sum_i P(B \mid A_i)P(A_i)$
- (for the previous example that's $\sum_{people} P(book \mid person)P(person)$)

$$\underset{1}{P(book \mid Felix)} \; \underset{*\,1/4}{P(Felix)} + \ldots + \underset{1/2}{P(book \mid Swati)} \; \underset{*\,1/4}{P(Swati)} + \ldots$$

Bayes' rule in pictures

- Say we'd like to know P(have covid|negative test) given that the false positive rate for our Super Official covid tests is **5%** and our false negative rate for our Super Official covid tests is **20%**. Further, we know that the current distribution of people with covid is 10% of the population.

→ $P(covid) = .1$

→ 90% of pop

$$P(covid|-) = \frac{P(-|covid)P(covid)}{P(-|covid)P(covid) + P(-|no\ covid)P(no\ covid)}$$

true negatives

20% → not % of pop

5%

$$\frac{.2 * .1}{(.2 * .1) + (.95 * .9)} = 2.3\%$$

Bayes' rule in pictures

Sol'n on next page for both

- Say we'd like to know P(have covid|negative test) given that the false positive rate for our Super Official covid tests is **20%** and our false negative rate for our Super Official covid tests is **5%**. Further, we know that the current distribution of people with covid is 10% of the population.

- What about P(no covid | positive test)?

Updated P(covid|−):

$$P(\text{covid}\mid -) = \frac{P(-\mid \text{covid})P(\text{covid})}{P(-\mid \text{covid})P(\text{covid}) + P(-\mid \text{no covid})P(\text{no covid})}$$

$$= \frac{.05 * .1}{(.05 * .1) + (.8 * .9)} = .0069$$

$$P(\text{no covid}\mid +) = \frac{.2 * .9}{(.2 * .9) + (.95 * .1)} = .65$$

Wow, that's not good for our test, but maybe we don't care?

Bayes nets

- So, in the real world, we often want to incorporate more information than one random variable.

  - **BUT** this often leads to *very* complex joint probability distributions

    ↳ prob. of mult. variables

  - A **Bayes Net** (also known as a graphical model) is a way to encode conditional interdependencies and simplify the logic behind what's happening

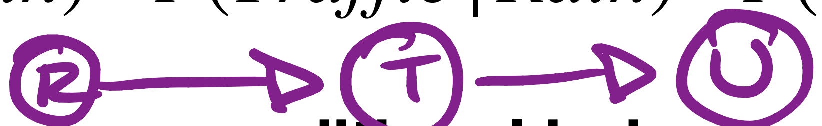
- e.g. I want to compute P(illness | symptoms) or P(illness1,illness2,illness3| symptoms)

Conditional independence & the chain rule

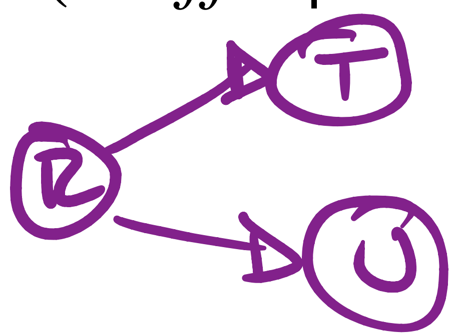
- If we have multiple variables, the **chain rule** defines their joint probability
$P(X_1, X_2, \ldots, X_n) = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1, X_2)\ldots P(X_N \mid X_1, \ldots, X_{N-1})$

P(Rain=True, Traffic=lots, Umbrella=True)

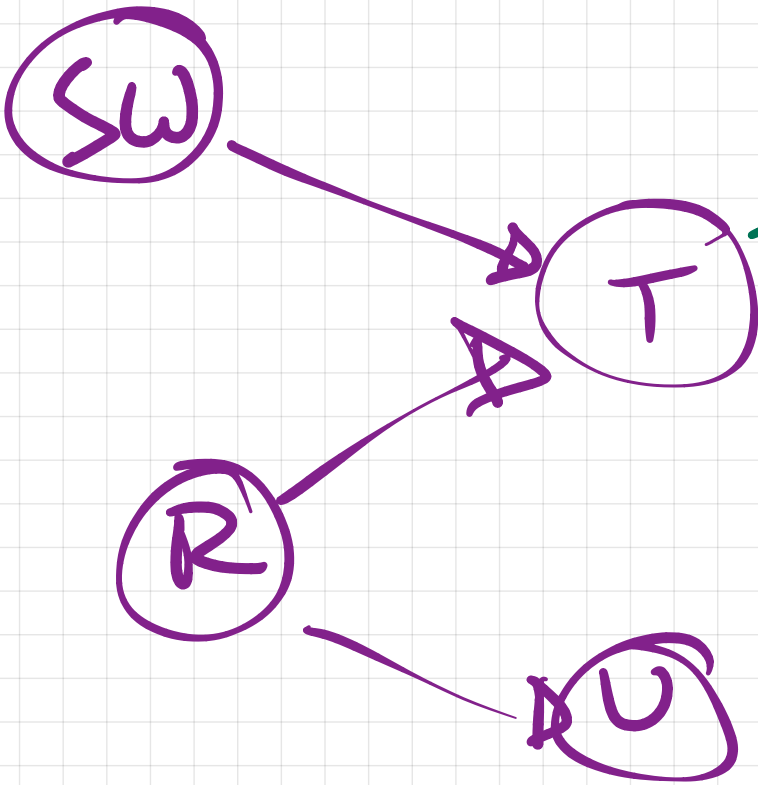
- So, if we want to know $P(Rain, Traffic, Umbrella)$, then we can calculate
    - $P(Rain) * P(Traffic \mid Rain) * P(Umbrella \mid Traffic, Rain)$



- If we assume **conditional independence** between the traffic and my umbrella, then this becomes
    - $P(Rain) * P(Traffic \mid Rain) * P(Umbrella \mid Rain)$



Rain, Traffic, Umbrella, Street Width



computation should include a term $P(T|R,SW)$

we'll look at examples that look more like this next Thursday

Bayes nets

- e.g. I want to compute P(illness | symptoms) or P(illness1,illness2,illness3| symptoms)

- Instead of looking to calculate $P(A \mid B)$ here, we really want to calculate $P(A \mid B, C)$ or $P(A, B \mid C, D)$ or even $P(A, B, C \mid D, E, F)$ (Or more)

- Remember, $P(A \mid B) = \dfrac{P(A, B)}{P(B)}$

- $P(A, B, C \mid D, E, F) = \dfrac{P(A, B, C, D, E, F)}{P(D, E, F)}$

conditional ind.

$P(\underline{A},\underline{B}|C) = P(A|C)\,P(B|C)$

↳ conditionally independent

ind.

$P(A|B) = P(A)$

if A + B are independent

Bayes nets

- Notation:
- nodes: random variables
- arrows: dependency relationships

$$A \longrightarrow B$$

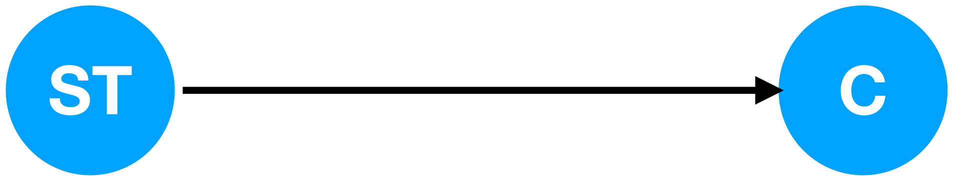
$P(B|A)$

B is dep. on A

Bayes nets

- I want to calculate the probability that I have covid based on whether or not I have a Sore Throat.

ST_yes	ST_no
0.2	0.8



	C_yes	C_no
ST_yes	0.05	0.95
ST_no	0.01	0.99

base probs of St.

$P(\text{covid} \mid St)$

joint prob of $P((C, St)$

- Algebraically, what is the probability that I have covid?

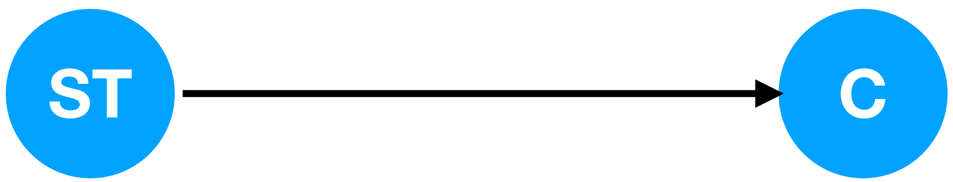
$\rightarrow P(\text{covid}_y, St_y) + P((C_y, St_n)$

$P(\text{covid}_y \mid St_y) P(St_y)$

Bayes nets

- I want to calculate the probability that I have covid based on whether or not I have a Sore Throat.

ST_yes	ST_no
0.2	0.8



	C_yes	C_no
ST_yes	0.05	0.95
ST_no	0.01	0.99

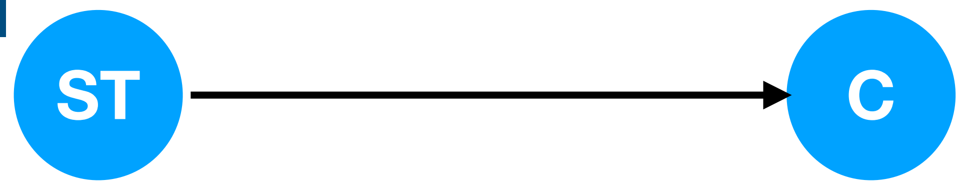
- Algebraically, what is the probability that I have covid?

- $$P(C_y) = \sum_{ST} P(C_y, ST) = P(C_y, ST_y) + P(C_y, ST_n) = P(C_y \mid ST_y)P(ST_y) + P(C_y \mid ST_n)P(ST_n)$$

Bayes nets

- I want to calculate the probability that I have covid based on whether or not I have a Sore Throat.

ST_yes	ST_no
0.2	0.8



	C_yes	C_no
ST_yes	0.05	0.95
ST_no	0.01	0.99

- Okay, are ST and C independent?
  - No, $P(C \mid ST) \neq P(C)$!

Bayes nets

- (Spreadsheet screenshot)

A	B	C	D	E	F	G
sore throat	p(st)	covid	sore throat	p(st, c)		
yes	0.2	yes	yes	0.01	<- B2 * .05	<- P(st) * P(c|st)
no	0.8	yes	no	0.008	<- B3 * .01	
Sanity Check	1	no	yes	0.19		
		no	no	0.792		
			Sanity Check	1		

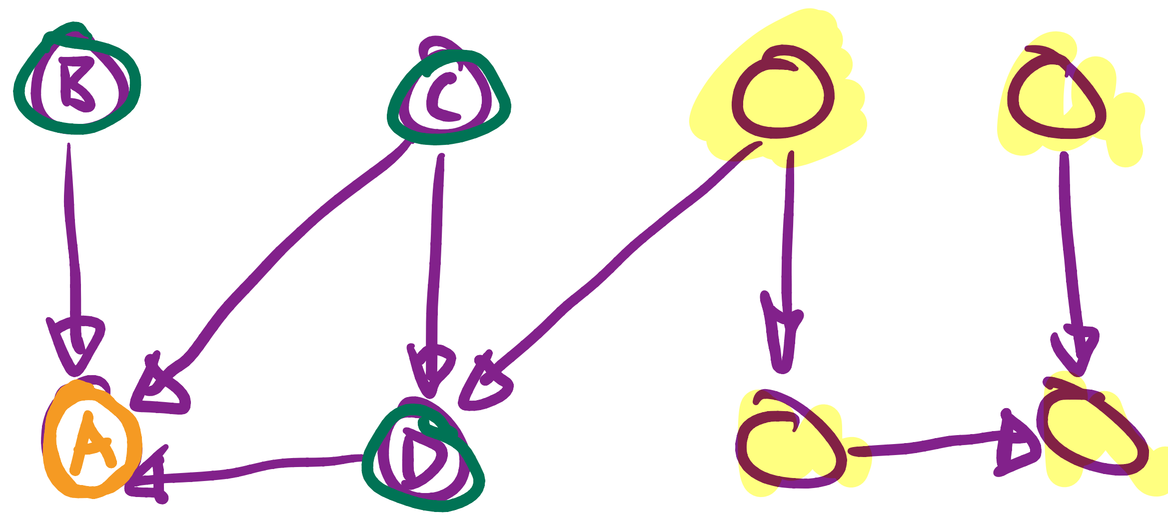
Bayes nets

- Wait, why Bayes Nets???
  - We get power from Bayes Nets when we are modeling things with more complex relationships (more than two variables)

Bayes nets

- Assumptions:
  - each node is conditionally independent of everything except its parents:
    - $P(x_i \mid x_1, \ldots, x_{i-1}) = P(x_i \mid parents(X_i))$

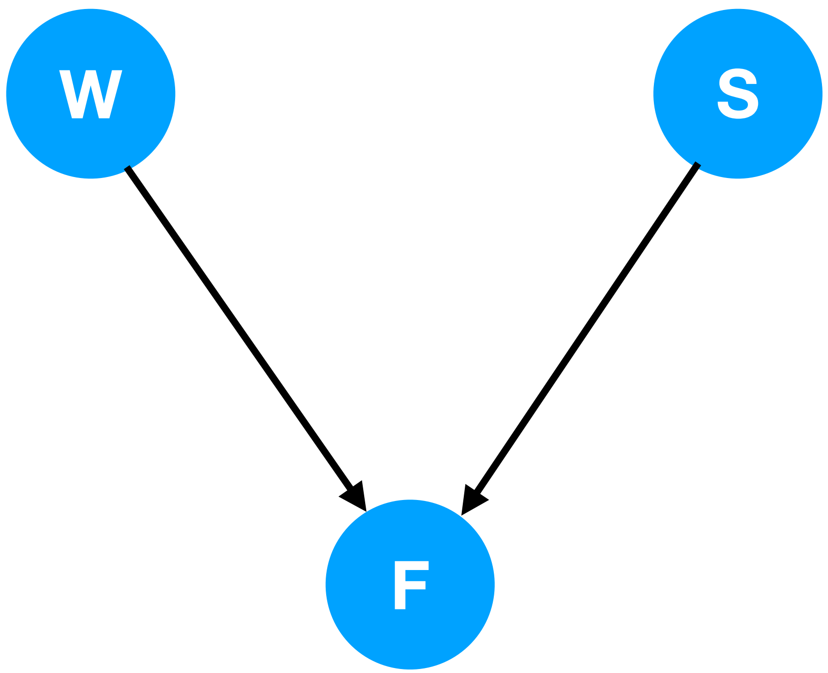
$P(A \mid B, C, D)$



Bayes nets

- I want to calculate the probability that I'll finish the Boston marathon.

W_hot	W_warm	W_cold
0.5	0.3	0.2



S_7	S_8
0.6	0.4

	F_yes	F_no
W_h, S_7	0.1	0.9
W_h, S_8	0.6	0.4
W_w, S_7	0.7	0.3
W_w, S_8	0.8	0.2
W_c, S_7	0.5	0.5
W_c, S_8	0.6	0.4

Bayes nets

updated 4/21/22

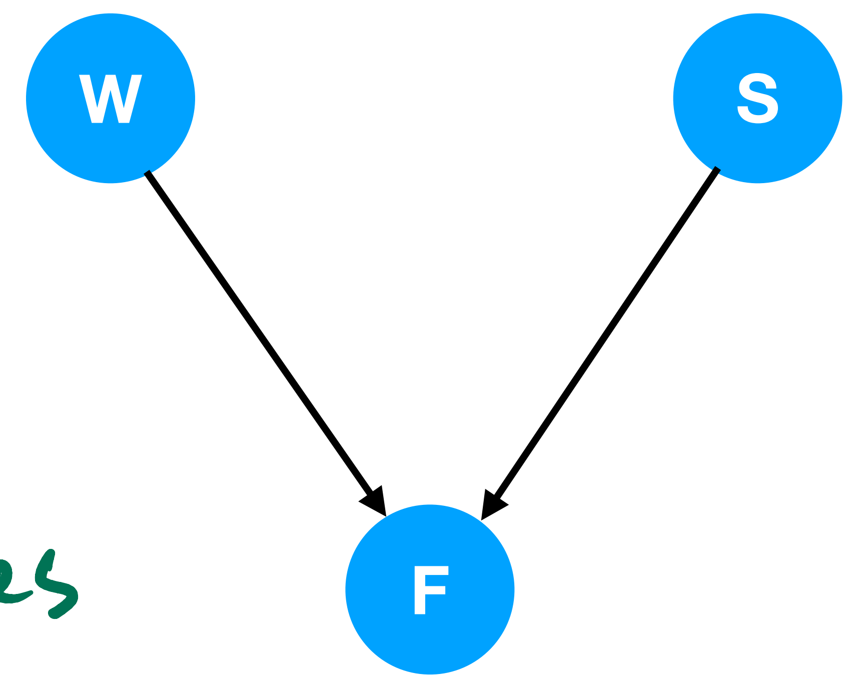
- In this graph, W and S are ~~conditionally~~ independent, and
- $P(W, S \mid F) = P(W \mid F)P(S \mid F)$

But!

- $P(W \mid S, F) \neq P(W \mid F)$
- $P(S \mid W, F) \neq P(S \mid F)$

→ example in lecture 23

→ when either of these variables is conditioned on F, they become linked



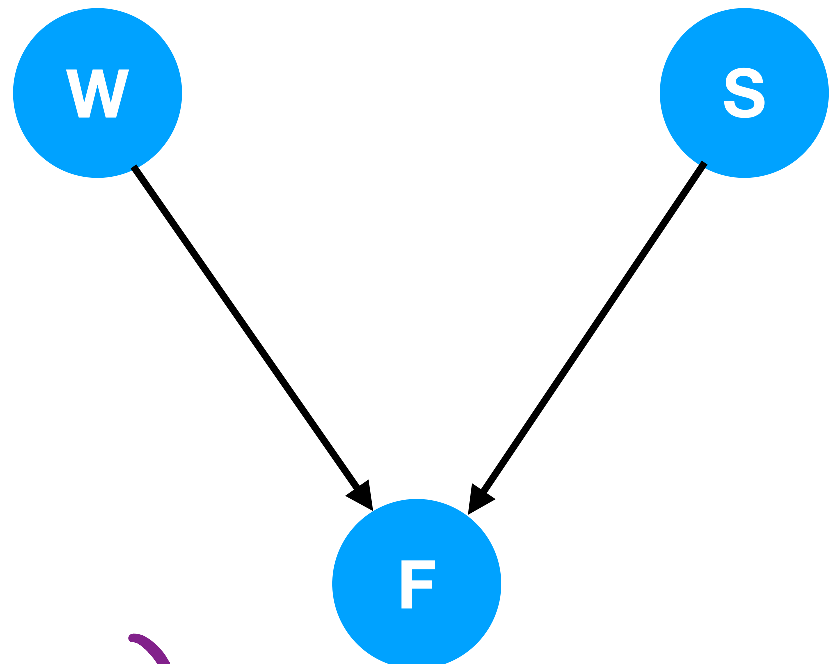
Bayes nets

- we can calculate the probability of any one variable taking on a specific value by using the joint probability distribution and combining with our conditional independence assumptions

- $$P(F) = \sum_{W,S} P(F, W, S) = 1$$

- $$P(F_{yes}) = \sum_{W,S} P(F_{yes}, W, S)$$

$$P(F_y | W, S) P(W, S)$$



Bayes nets

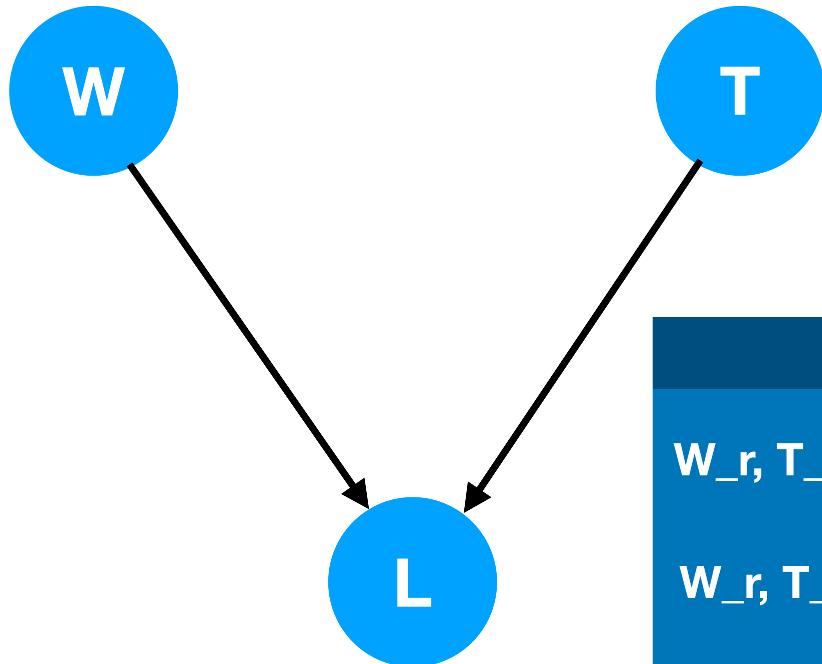
- (Spreadsheet screenshot)

	A	B	C	D	E	F	G	H	I	J	K
	Weather	P(w)	Speed	P(s)	Weather	Speed	P(w, s)	Finished	Weather	Speed	P(F, W, S)
	hot	0.5	seven	0.6	hot	seven	0.3	yes	hot	seven	0.03
	warm	0.3	eight	0.4	hot	eight	0.2	yes	hot	eight	0.12
	cold	0.2	Sanity check	1	warm	seven	0.18	yes	warm	seven	0.126
	Sanity check	1			warm	eight	0.12	yes	warm	eight	0.096
					cold	seven	0.12	yes	cold	seven	0.06
					cold	eight	0.08	yes	cold	eight	0.048
						Sanity check	1	no	hot	seven	0.27
								no	hot	eight	0.08
								no	warm	seven	0.054
								no	warm	eight	0.024
								no	cold	seven	0.06
								no	cold	eight	0.032
										Sanity check	1

ICA Question 3: Bayes Nets

Given the following Bayes Net, use a spreadsheet to calculate the probability Felix is not late (L_no).

W_rain	W_clear
0.6	0.4



T_bike	T_t
0.7	0.3

	L_yes	L_no
W_r, T_b	0.3	0.7
W_r, T_t	0.6	0.4
W_c,T_b	0.1	0.9
W_c, T_t	0.7	0.3

Link to spreadsheet computations for these Bayes Nets!

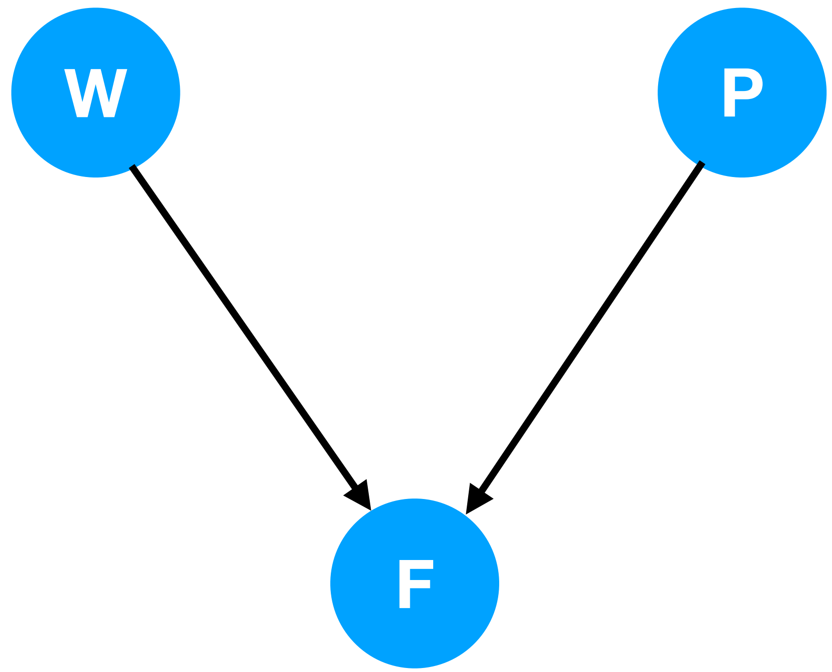
- https://bit.ly/sec1bayes

Bayes nets

- So, to algebraically compute $P(F_{yes}) = \sum_{W,S} P(F_{yes}, W, S)$, we need:

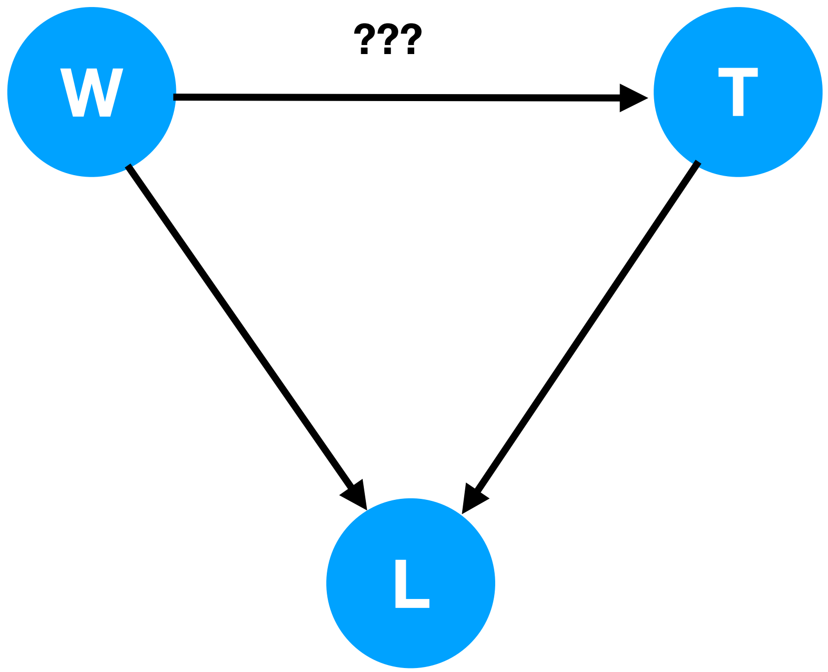
- $P(F_{yes}) = P(F_{yes}, W_h, S_7) + P(F_{yes}, W_h, S_8) + P(F_{yes}, W_w, S_7) + P(F_{yes}, W_w, S_8) + P(F_{yes}, W_c, S_7) + P(F_{yes}, W_c, S_8)$

- And for each individual term, we need:
  - $P(F_{yes}, W_h, S_7) = P(F_{yes} \mid W_h, S_7) P(W_h, S_7)$



Bayes nets

- More examples next Thursday!



Schedule

Turn in **ICA 22** on Canvas (submit by 2pm!) - passcode is "thursday"

4/21: Updated to Bayes Nets part 2 (about 1 hr) & a mini project work day (about 40 min)

HW 9: (released 4/19) you have everything you need, we'll do more examples on 4/21

Mini-project: must email Felix to request an extension (by default no late passes)

Test 4: May 4th, 1 - 3pm, Snell Engineering 108

Mon	Tue	Wed	Thu	Fri	Sat	Sun
April 11th Lecture 21 - conditional probabilities, bayes	**Felix OH Calendly**	**Felix OH Calendly**	**Felix OH Calendly** Lecture 22 - conditional independence, bayes nets			**HW 8 due @ 11:59pm**
April 18th **No lecture - Patriot's Day**	**Felix OH Calendly**	**Felix OH Calendly**	**Felix OH Calendly** Lecture 23 - Bayes Nets, part 2, mini-project work			
April 25th Lecture 24 - presentations, review **Mini-project due @ 11:45am**		**HW 9 due @ 11:59pm**				

More recommended resources on these topics

- YouTube: 3Blue1Brown, Bayes theorem, the geometry of changing beliefs
- YouTube: 3Blue1Brown, The medical test paradox, and redesigning Bayes' Rule
- YouTube: Berkeley AI, Section 5: Probability, Bayes Nets
- UW CSE 473, Bayes' Nets: https://courses.cs.washington.edu/courses/cse473/19sp/slides/cse473sp19-BayesNets.pdf