Set-up!

- we'll start @ ~11:47
- if you haven't set your zoom profile to a picture, please do so!

1) get out your notes

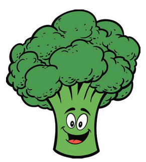
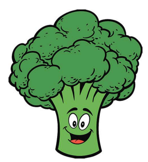
2) get out someplace to do today's ICA, then, at the top, write:
   - your name
   - my name (Felix Muzny / Prof. Muzny)
   - date (1/20/2022)
   - ICA 2

3) If you forgot to turn in ICA 1, do that now. If you forgot to do it, don't panic!

⚠ zoom: rename → Felix ⚠

Part 1: Linear Systems Review & RREF

Is the following equation linear? Prove it.

f() = |  |

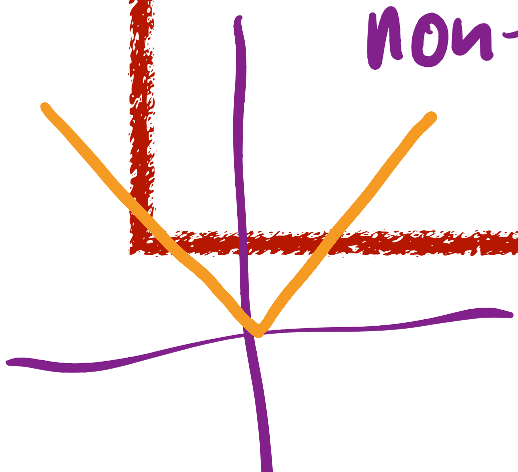
→ scaling
→ adding

$f(\alpha x + \beta y) = f(\alpha x) + f(\beta y)$

$\hookrightarrow \alpha f(x) + \beta f(y)$

non-linear → choose $\alpha, \beta = -1$, $x, y = 1$

$f(\alpha x + \beta y) = |(-1 * 1 + -1 * 1)| = |-2| = 2$



$-1 f(1) + -1 f(1) = -1|1| + -1|1| = -2$

Hello! Who am I?

- Felix Muzny ("Muse-knee", IPA: /mjuzni/) - call me "Felix", "Professor Felix", or "Professor Muzny"
    - pronouns: they/them & he/him
- I'm from Colorado (a rectangular state with lots of mountains)
- I did my graduate research working on mostly on "digital humanities" (using computational techniques to investigate humanities-driven inquiries)
    - I read old books with computers and modeled how dialogue changed, looking at how that was associated with different literary movements

↳ Natural Lang. Processing → modelling human lang. w/ math

Remote lectures: expectations

- Remote learning can be weird! We'll be doing our best to reduce weirdness.
- Here are my expectations of you all:
    - Be in a **location** conducive to learning
    - Set your zoom profile picture to a picture **of yourself**
    - When we are in breakout rooms, **turn on your cameras**
    - When we are in breakout rooms, each group will pick one person to **screenshare**

Remote lectures: expectations

- Remote learning can be weird! We'll be doing our best to reduce weirdness.
- Here are my expectations of you all:
  - Use the chat or "raise hand" features to ask me questions!
  - Wear a fun hat
  - Pets are absolutely welcome
  - Tell me about your music preferences every week

Rachel: likes Calculus! Economics

Charvi: differentiation!

Divya: health info, statistics!

Swati: probability, Bayes!

Icebreaker #1

meta: I'll never start lecture until the timer ends

- In your breakout rooms:
- Turn on your cameras
- Then, share:
  - Name (& pronouns if you'd like)
  - one relaxing moment that you got to have over break
- (write this down) Finally, as a group, one (or more) thing(s) that you'd like to learn about math or how we can apply math to a certain area of CS

✓ – matrix mult
✓ – prob, stat
? – Vision
? – crypto
✓ – ML
✓ – weather
✓ – covid

Review

- How do we know if an equation is linear?

↳ scaling : $f(\alpha x) = \alpha f(x)$

↳ addition : $f(x+y) = f(x) + f(y)$

↳ formally $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$

Review

- What is a system of equations?

↳ multiple eq'ns (w/ shared solution?)

↳ shared variables

↳ linear system

↳ set of (lines, planes) in n-dim. space

↳ dimensions are shared

$$x + y = 3$$

$$2x + 4y = 3$$

Review - Gauss's method

a system w/ same solution set

- Gauss's method is a strategy for solving a system of linear equations where we change the system into an equivalent system that is easier to solve.

What operations can we do on our system of equations?

A. Swap two rows

~~B. Add a scalar to the left side of all rows~~

C. Multiply one row by a scalar (>1)

D. Divide one row by a scalar (<1)

"scale"

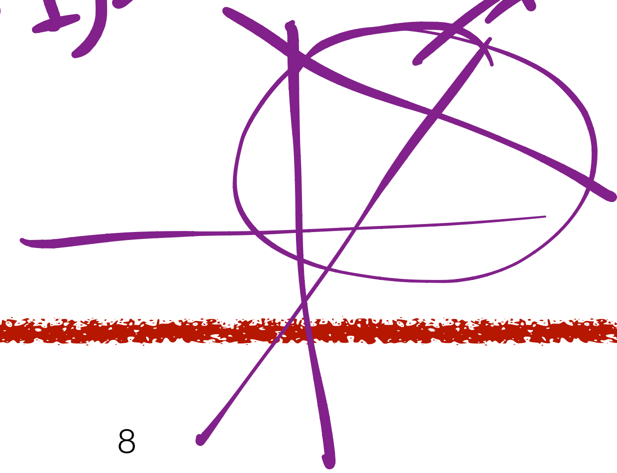
E. Add two rows together

$x + y + 7 =$

$2x + 4y + 7 =$

$x = \_\_$

$y = \_\_$



Review - Gauss's method

- For n = 0 to n = number of equations - 1: 0 2 -1
  - scale the leading coefficient of eq'n N to 1
  - add (the correct multiple) of eq'n N to others → get rid of x

n = 0

0 $10x + 20y = 30$ $\xrightarrow{r_0' = \frac{1}{10} r_0}$ $x + 2y = 3$

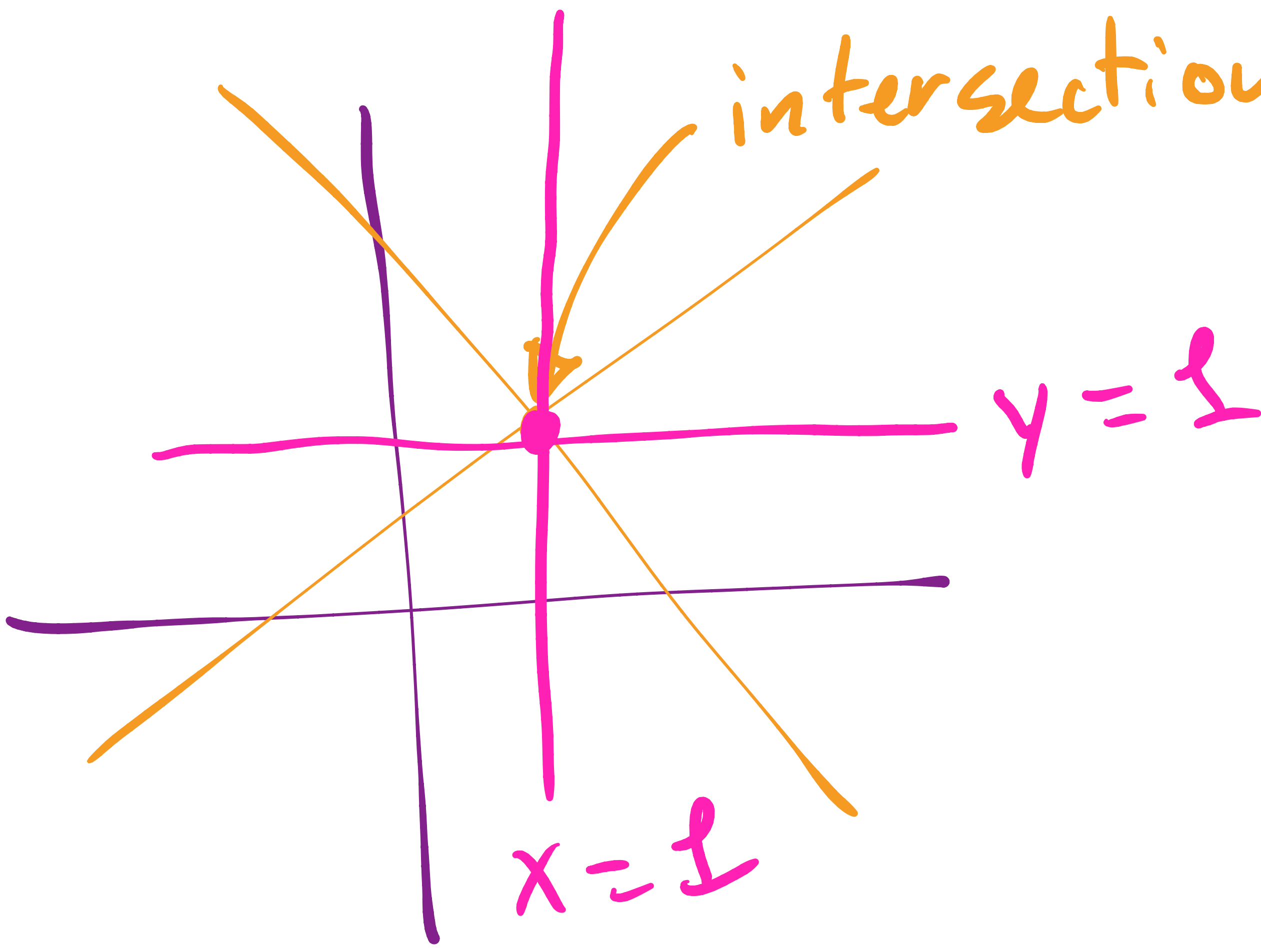
1 $2x - 3y = 13$ $2x - 3y = 13$ $\xrightarrow{r_1' = r_1 - 2r_0}$ $x + 2y = 3$, $0x - 7y = 7$

n = 1

$\xrightarrow{r_1' = -\frac{1}{7} r_1}$ $x + 2y = 3$, $0x + y = -1$ → plug + chug $x = 5$, $y = -1$

Equivalent Linear Systems

- High level: what are we trying to do when we solve a system of linear equations?



Solving linear systems

ICA 2, Question 8

ICA Question 1: solve the following linear system by row reduction, documenting your row operations as shown (e.g. $r_1' = r_1 + 4r_0$)

$$4x - y = 6$$
$$2x + y = 0$$

$r_0' = \frac{1}{4}r_0$

$$x - \frac{1}{4}y = \frac{6}{4}$$
$$2x + y = 0$$

$r_1' = r_1 - 2r_0$

$$x - \frac{1}{4}y = \frac{6}{4}$$
$$0x + \frac{3}{2}y = -3$$

A: done

B: part way

C: stuck

Can I plug in var. values when I find them? → yes

Matrices (& Augmented Matrices)

- Matrices help us avoid re-writing variable names

$$4x - y = 6$$

$$2x + y = 0$$

$$\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 4 & -1 & 6 \\ 2 & 1 & 0 \end{array}\right] \end{array}$$

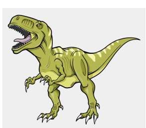
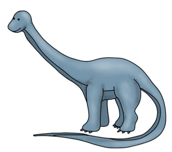
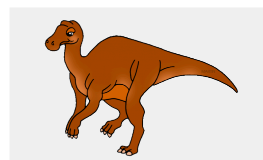
matrix

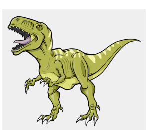
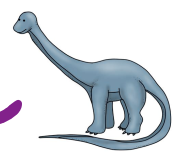
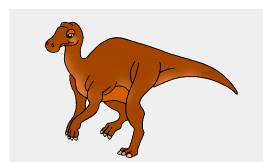
augment column

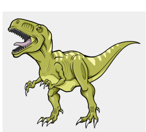
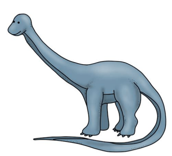
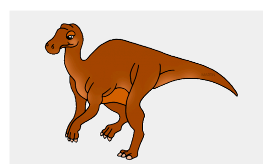
augmented matrix all together

Matrices (& Augmented Matrices)

- Matrices help us avoid re-writing variable names

 +  +  $= 10$

 $+ 2$  +  $= 12$

3  $+ 4$  -5  $= 8$

$$\begin{array}{cccc} \text{trex} & \text{apat.} & \text{duckbill} & \\ \downarrow & \downarrow & \downarrow & \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 1 & 2 & 1 & 12 \\ 3 & 4 & -5 & 8 \end{array}\right]$$

Solving linear systems

A: done
B: partway
C: stuck

ICA Question 2: solve the following linear system by row reduction, documenting your row operations as shown (e.g. $r_1' = r_1 + 4r_0$), **and representing each step as a matrix**

$2x + y = 1$

$-x + y = 0$

ICA Question 3: do you think that the following systems of equations are solvable? why? why not?

A

$10x + z = 2$

$-x + y = 0$

2 eqns

3 variables

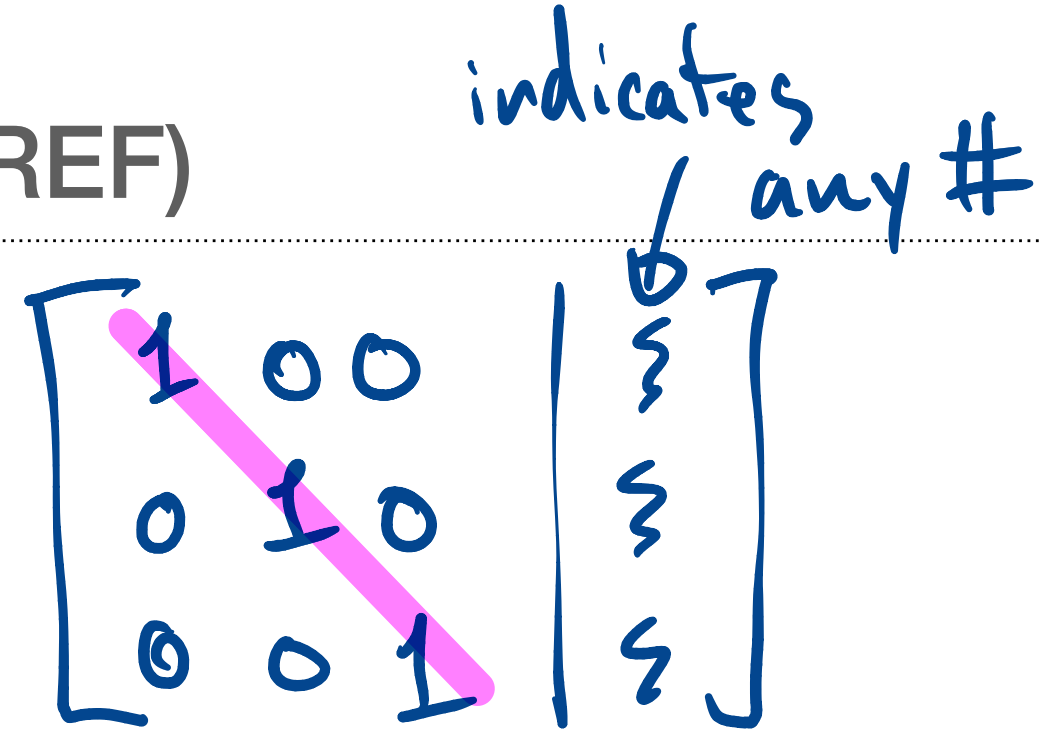
$\ne$

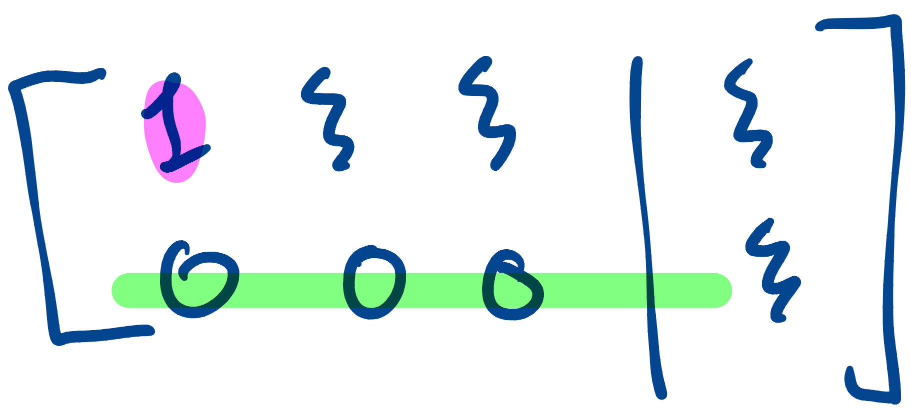
$10x + z = 2$

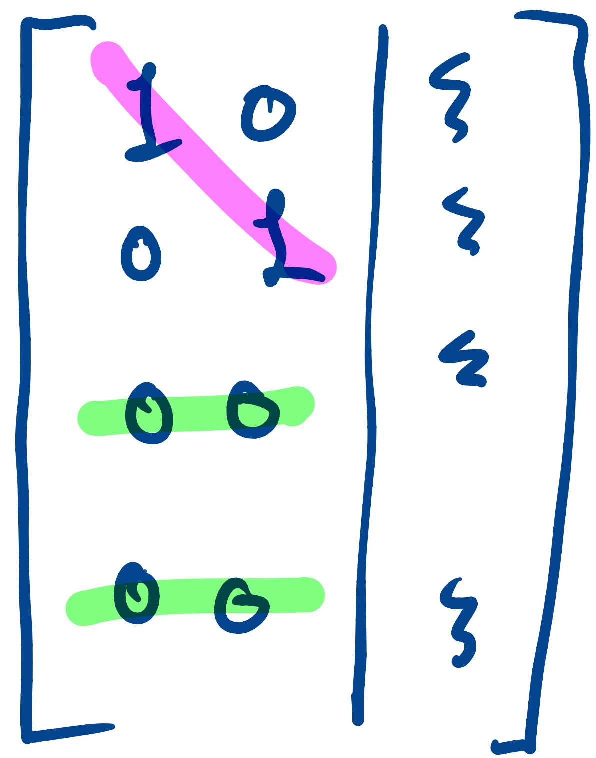
$z = -3$

Reduced Row Echelon Form (RREF)

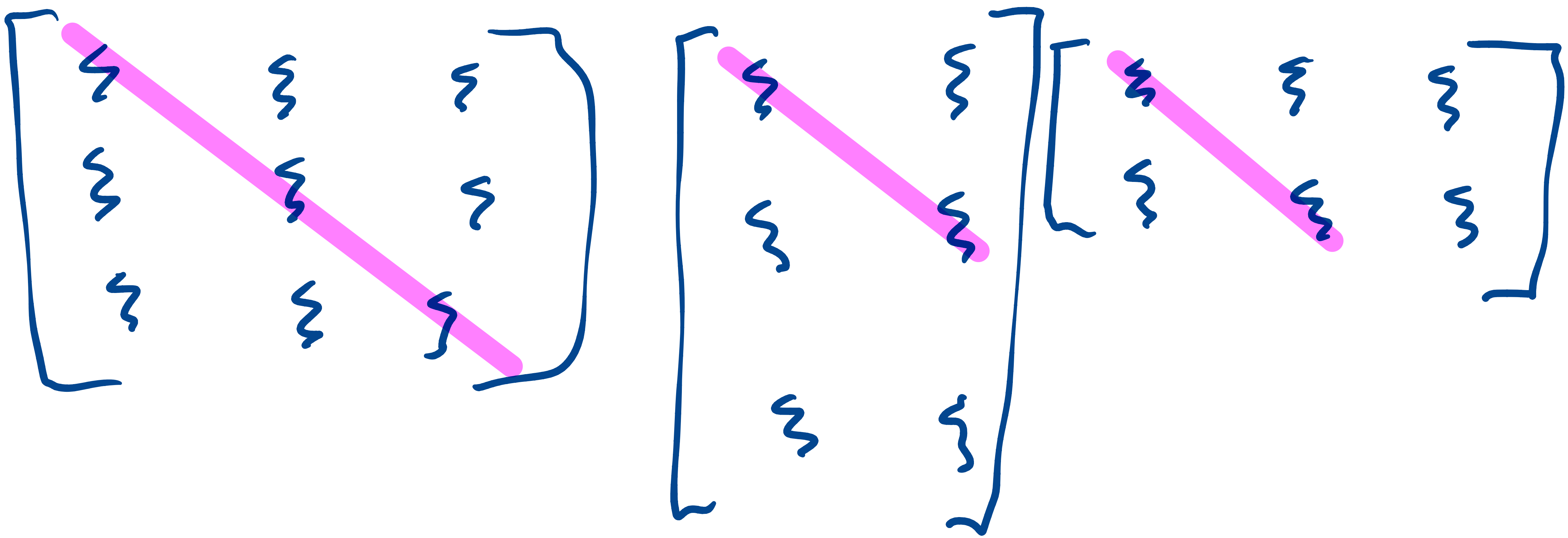
- All "zero rows" at the bottom
- 1s on the diagonal of the remaining matrix
- 0s above/below all the 1s







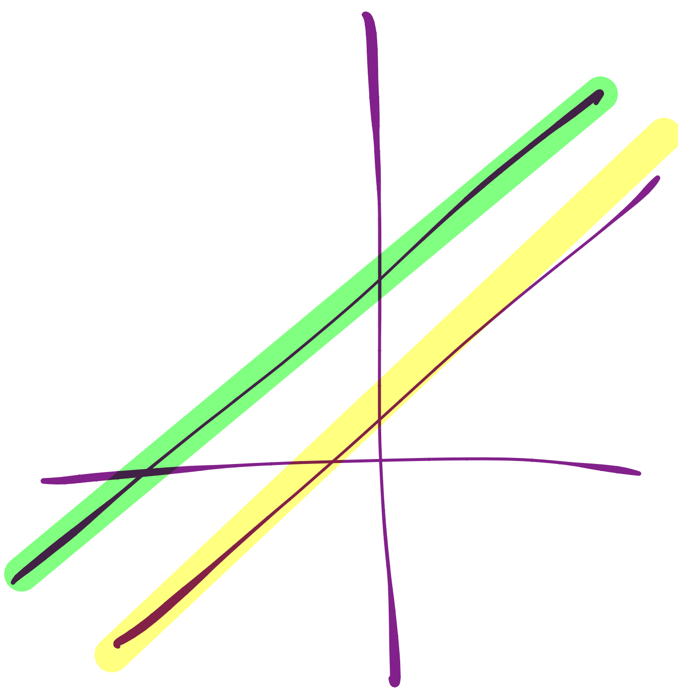
Matrix Anatomy: diagonal



Linear Systems & Solutions

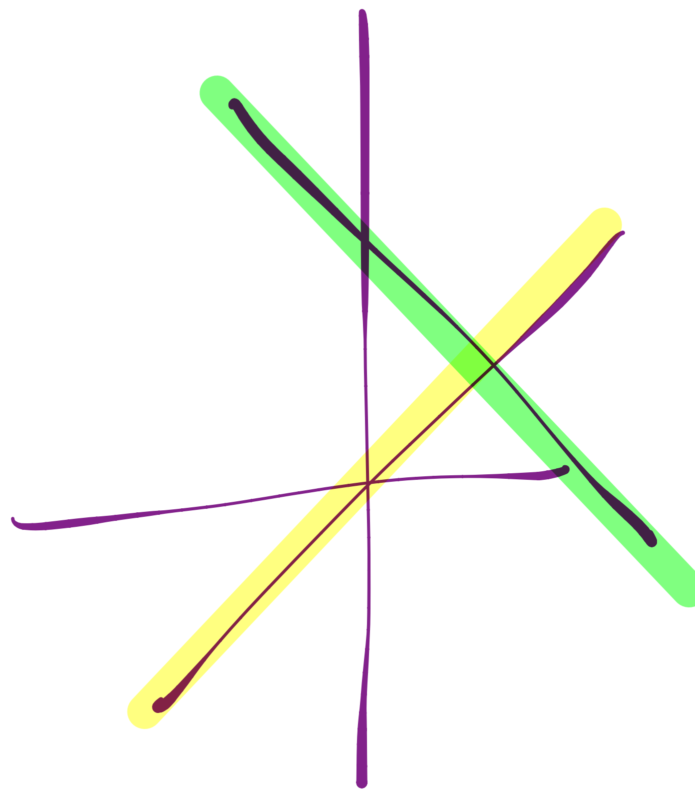
- Recall: when we are solving linear systems, we're looking for **the intersection of lines**

1)



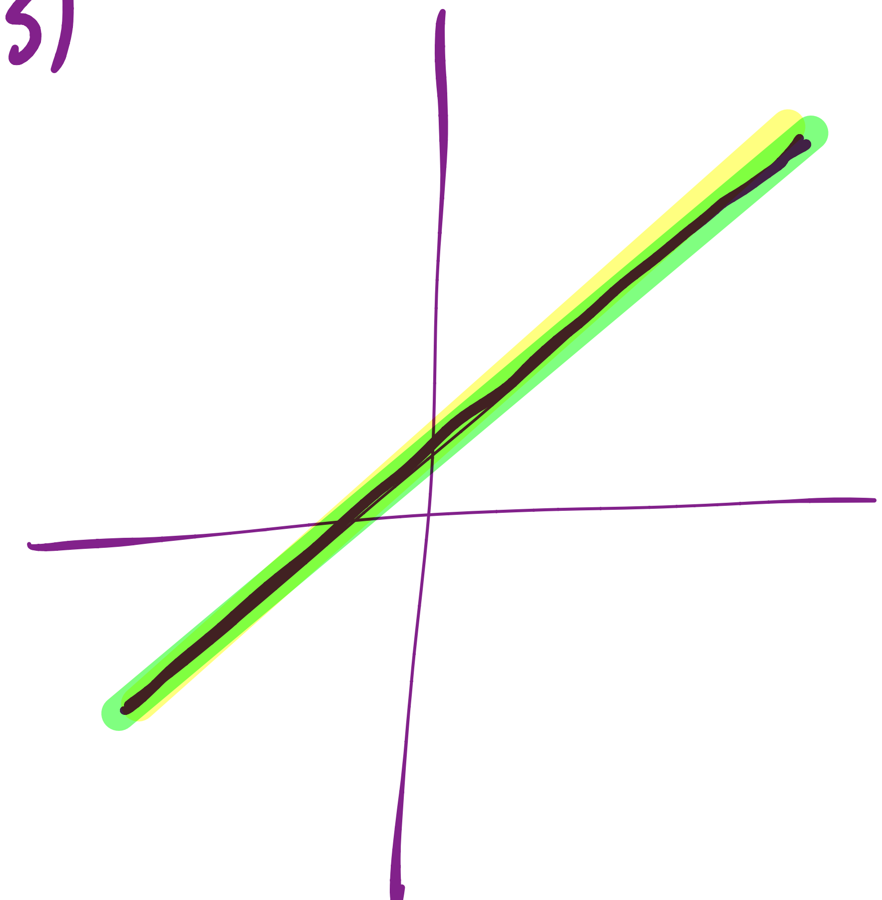
no intersection

2)



a point

3)



same line

Linear Systems & Solutions

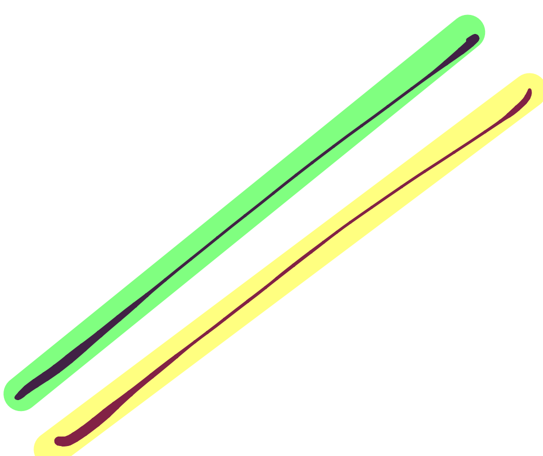
⚠ math for RREF forms on the next slides ⚠

- We can also write these equations in RREF form

$y = x + 1$

$y = x + 2$

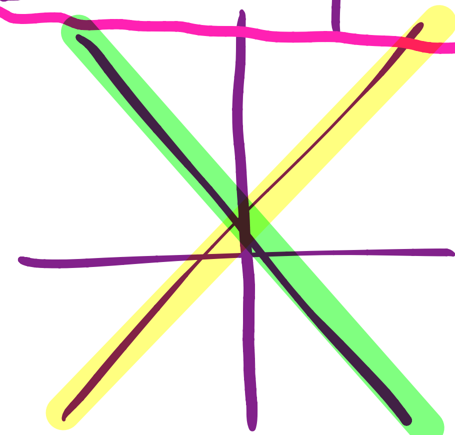
$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$



$y = x + 1$

$y = -x + 1$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$



if we had a 3^{rd} eq'n that defined the same line as #1 or #2

$y = x + 1$

$2y = 2x + 2$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$y = x + 1$

$y = x + 2$

$-x + y = 1$
$-x + y = 2$

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ -1 & 1 & 2 \end{array}\right]$$

$r_1' = r_1 - r_0$

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

$y = x + 1$

$y = -x + 1$

$-x + y = 1$
$x + y = 1$

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 1 & 1 & 1 \end{array}\right]$$

$r_0' = -1 r_0$

$r_1' = r_1 + r_0$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 2 & 2 \end{array}\right]$$

$r_1' = \frac{1}{2} r_1$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

$r_0' = r_0 + r_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & \end{array}\right]$$

$$y = x + 1$$
$$2y = 2x + 2$$

$$x - y = -1$$
$$2x - 2y = -2$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 2 & -2 & -2 \end{array}\right]$$

$$r_1' = r_1 - 2r_0$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

Linear Systems & Solutions

- .... and we can use these RREFs to see what's going on in the underlying system

no solution

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

zero row w/ non-zero augment

unique solution

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

RREF w/ as many 1s as columns in matrix

Many solution

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

has a row in matrix w/ more than 1 non-zero entry

all zero rows have zero augment

Solving linear systems

on Monday as warm-up

ICA Question 4: for each matrix, write down the following:

a) is it in RREF?

b) if no, identify a specific reason why not

c) if yes, identify whether the system has:

- no solutions
- one unique solution
- many solutions

Admin stuff....

A : yes
B : no
C : idK

- Office hours: we'll be using khoury office hours this semester

↳ you'll need a Khoury account

  - expect office hours to begin next week (we'll be releasing HW 1 on Monday)

- Felix will have two kinds of office hours:

  - Calendly: reserve in advance on Tuesdays (https://calendly.com/muzny)

↳ higher-level questions, not HW questions

  - Khoury Office Hours: on Thursdays 2 - 4pm

Khouryofficehours.com

Schedule

Qs 1–3

Turn in ICA 2 on Gradescope → under item "ICA 2" → do this now!

We are remote until Feb 5th

Mon	Tue	Wed	Thu	Fri	Sat	Sun
January 17th **MLK Day**	**Felix OH Calendly**		Lecture 2 - Vector Algebra **Felix OH Khoury Office Hours**			
January 24th Lecture 3 - Matrices & vector geometry **HW 1 released**	**Felix OH Calendly**		Lecture 4 - ML, linear perceptron **Felix OH Khoury Office Hours**			