CS 2810 Day 1 Jan 18

Data Models
- what it takes to build a good one

Admin
- ask me anything about administration of course

Linearity

Gauss Jordan Elimination (beggining)

What is prob someone in class is exposing the rest of us to covid, right now?

Assume:
-100 peope in this room
- prob contraction is uniform per person
- one who tests positive is contagious for 7 after test

how to get prob contracting covid
"use the average rate in Mass"
- 7 million people in Mass
- 20k new cases a day

$$\underbrace{\overbrace{1-(1-p)^{100}}^{\text{PROB } \geq 1 \text{ WAS COVID}}}_{\text{PROB NO-ONE WAS COVID}} = 86\%$$

$$p = \frac{20k}{7000k} \cdot 7$$

ICA 1: Discuss whether you trust the model, and identify 2-3 most important assumptions which you disagree with

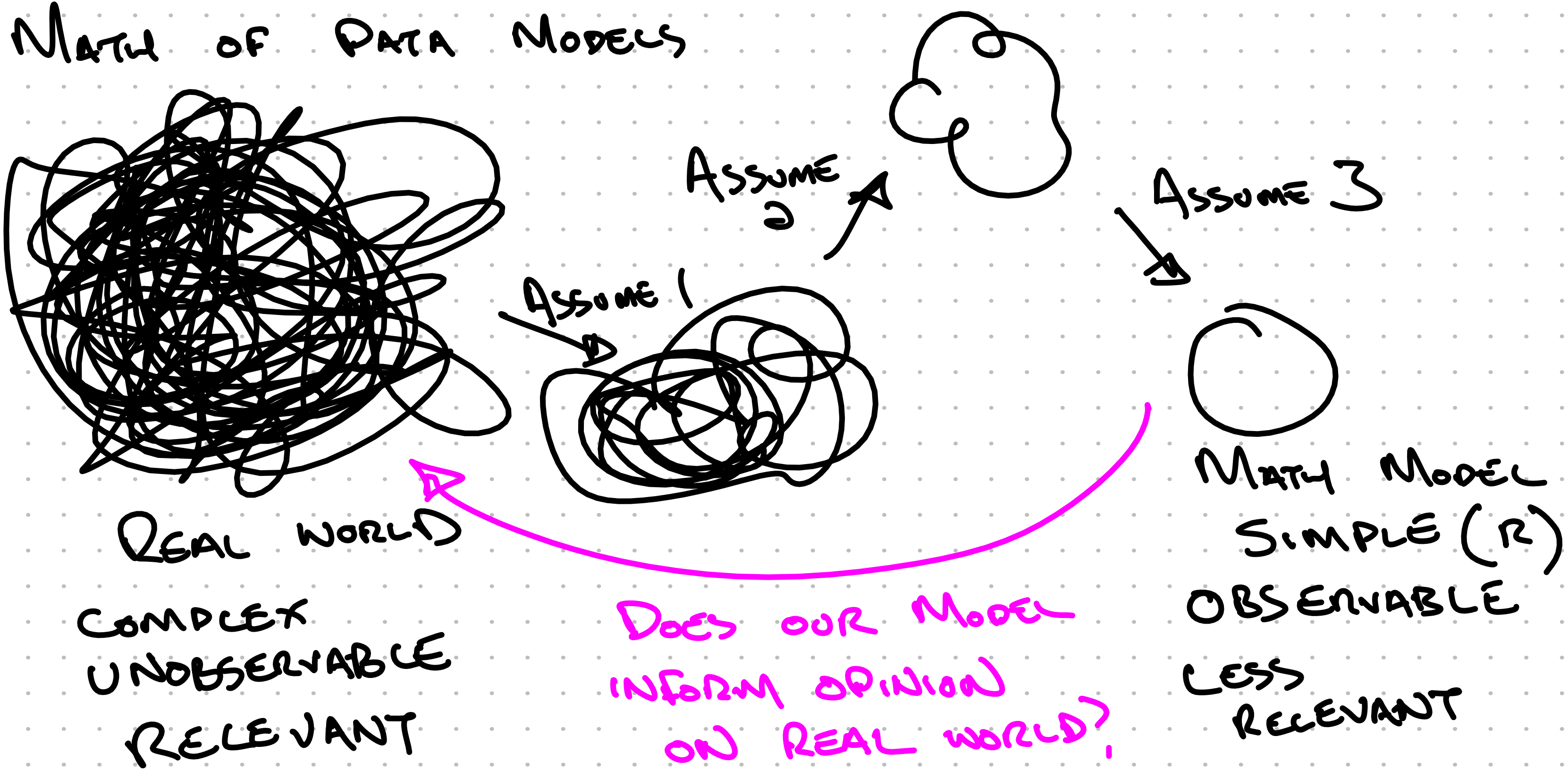
Critiques:

(should be lower)
NU covid stats don't match Mass stats
NU is insulated because students interact among themselves (mostly)
NU testing: if you tested positive, you're staying home

(should be higher)
testing rate doesn't count people who are positive, but haven't been tested
- false negative on testing
mixing globally, we pull covid from all corners of earth (beggining of semester)

(generally wrong, not necessarily that estimate is too high or low)
uniform probability of contracting covid

Math of Data Models



what it takes to build a "good" data model

- a breadth of model models to choose from in one's mental library

    (we'll learn about many models from linear algebra & prob/stats)

- the ability to be creative / rigorouus in making and evaluating assumptions
    - how "false" are my assumptions?
    - is model strong enough to draw conclusions on?

- a keen sense of which aspects of the application we seek to model most accurately

"all models are wrong, some are useful"

Linearity (Intuitive Definition: use this one to understand meaning)

A function is linear if
- scaling, applied before or after the function, has equivilent effect
-addition, applied before or after the function, has equivilent effect

for all $\alpha \in \mathbb{R}$

$x \in \text{Domain}(f)$

$$f(\alpha x) = \alpha f(x)$$

Scale before fnc

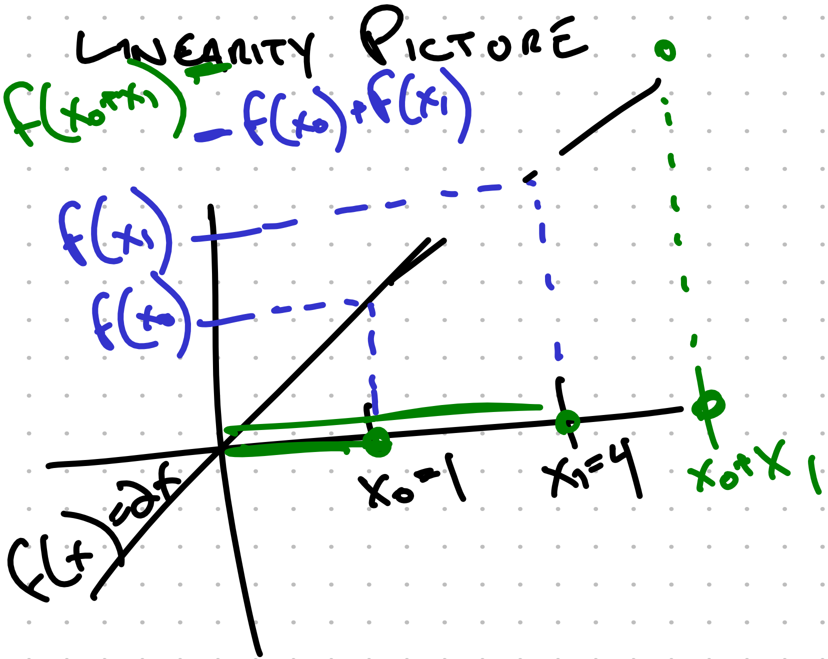
Scale after fnc

$x, y \in \text{Domain}(f)$

$$f(x+y) = f(x) + f(y)$$

Add before fnc

Add after fnc

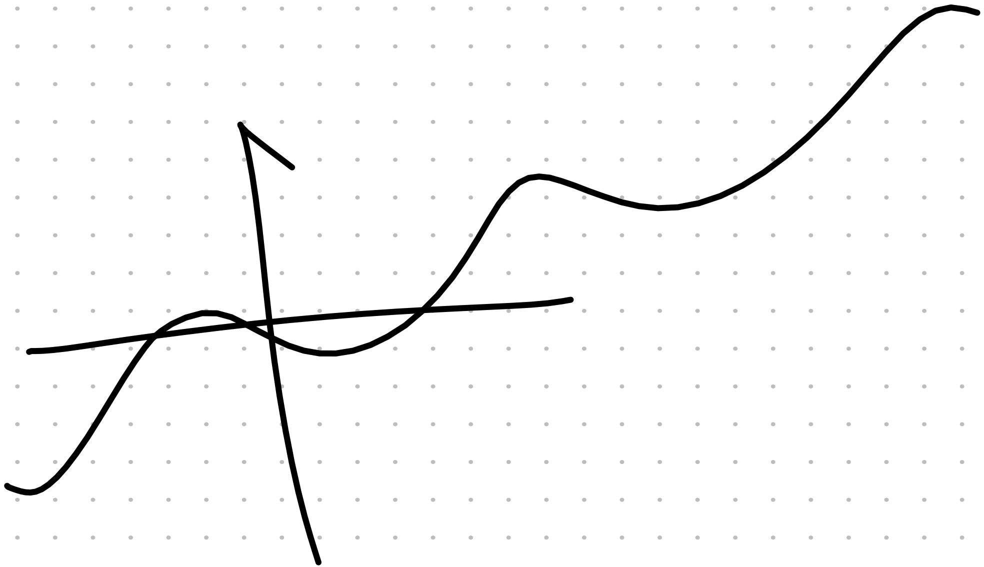


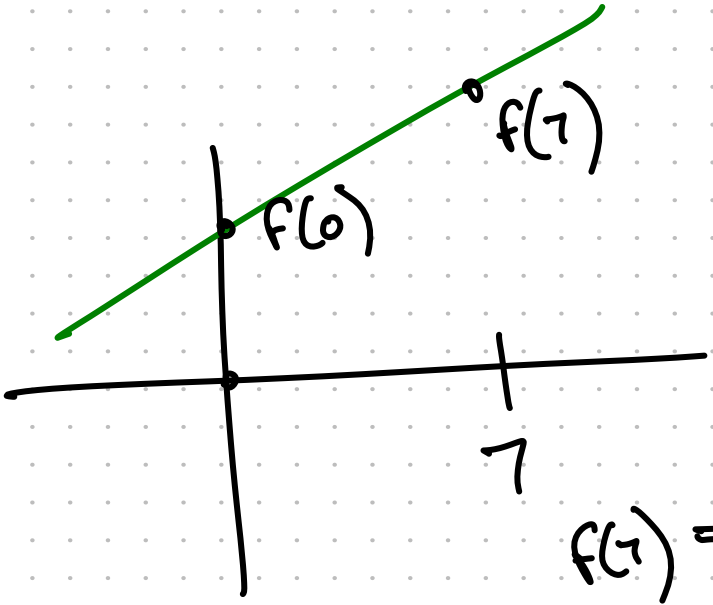
ADDITION BEFORE FNC

$f(x_0+x_1) = f(1+4) = f(5)$
$= 10$

ADDITION AFTER FNC

$f(x_0)+f(x_1) = f(1) + f(4)$
$= 2 + 8$
$= 10$





$f(0)$

$$f(7) = f(0+7) \stackrel{?}{=} f(0) + f(7)$$

$$0 = f(0)$$

Linearity (working definition: useful to prove something is / is not linear)

A function is linear if:

FOR ANY $\alpha, \beta \in \mathbb{R}$ ← SCALARS

$x, y \in \text{DOMAIN}(f)$ ← INPUTS TO f

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

GOAL: PROVE A FUNCTION IS LINEAR

$f(x) = 10x$

CHOOSE $\alpha, \beta \in \mathbb{R}$ CHOOSE $x, y \in \text{DOMAIN}(f) = \mathbb{R}$

$$f(\alpha x + \beta y) = 10(\alpha x + \beta y)$$
$$= \alpha 10x + \beta 10y$$
$$= \alpha f(x) + \beta f(y)$$

Goal: Proving something is non-linear

$f(x) = x^2$

$\alpha = \beta = 1 \qquad x = y = 1$

$$f(\alpha x + \beta y) = f(1 \cdot 1 + 1 \cdot 1)$$
$$= f(2)$$
$$= 4$$

$$\alpha f(x) + \beta f(y) = 1 \cdot 1^2 + 1 \cdot 1^2$$
$$= 2$$

so there exists alpha, beta, x, y with
f(alpha x + beta y) != alpha f(x) + beta f(y)

Why all the fuss about defining a "linear" equation?

TLDR:

many real world things are linear, some that may not seem linear can be re-cast as linear

- with this class of equalities, it is possible to either ...
    - find all solutions of a set of equalities
    - find value which gets "closest" (line of best fit)

- the infite linear function outputs are all defined by the linear system's behavior on a set of basis inputs

- all linear functions can be expressed as matrix multiplications

Solving system of linear equations

definition: system of linear equations is a set of linear function equations
solutions satisfy all equalties

IS $x=1 \quad y=-1 \quad z=1$ A SOLUTION?

$$x + y = 0 \quad \checkmark \qquad 1 - 1 = 0$$

$$2x - y + 3z = 3 \quad \times \qquad 2 \cdot 1 + 1 + 3 \neq 3$$

$$2 + 1 + 3$$

$$x - 2y - z = 3$$

NO

ICA 2

Solve the previous linear system using any method known from your algebra experience

$x + y = 0$

$2x - y + 3z = 3$

$x - 2y - z = 3$

Thinker:
How might you teach a computer to solve every possible linear system?
(no matter what coefficients are given to x, y, z and constant they're equal to,
your method provides a solution)

SOLVING LINEAR SYSTEM

WE "TRANSFORM SYSTEM TO ANOTHER SYSTEM WITH THE SAME SOLUTION)

$r_0: x + y = 0$

$r_1: 2x - y + 3z = 3$

$r_2: x - 2y - z = 3$

① SCALE A ROW

$r_0' = 2r_0$

$$2x + 2y = 0$$
$$2x - y + 3z = 3$$
$$x - 2y - z = 3$$

② SUM TWO ROWS

$r_1' = r_1 + r_0$

$$x + y = 0$$
$$3x + 3z = 3$$
$$x - 2y - z = 3$$

③ SWAP TWO ROWS

$r_0' = r_1$

$r_1' = r_0$

$$2x - y + 3z = 3$$
$$x + y = 0$$
$$x - 2y - z = 3$$