Written Homework 03

Assigned: Wed 10 Feb 2016  
Due: Wed 24 Feb 2016  

Instructions:

- The assignment has to be uploaded to blackboard by the due date. *NO* assignment will be accepted after 11:59pm on that day.
- We expect that you will study with friends and often work out problem solutions together, but *you must write up your own solutions, in your own words*. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.
- We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them by hand.
- *To get full credit*, **show INTERMEDIATE steps** leading to your answers, throughout.

**Problem 1** [18pts, 6pts each]

I once gave a 30 question True/False exam.

i. In how many ways can you answer the questions (pretend you answer each question True or False)?

ii. If you decide to leave some questions blank, in how many ways can you answer the questions?

iii. If you decide to answer 10 of the questions True and the rest of the questions False, in how many ways can you answer the questions?

**Problem 2** [15 pts (4,5,6)]: Sets

i. Let $A = \{x \mid x \in \mathbb{N} \land 4 \leq x^2 - 5 \leq 39\}$. (Here $\mathbb{N}$ includes 0.) What are the elements of $A$?

ii. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universe. Let $A = \{1, 4, 6\}$ and $B = \{1, 2, 3, 7, 9\}$ be two subsets of $U$. What are the elements of the set $A \cup \overline{B}$?

iii. Let $A = \{5\}$. What are the elements of $\mathcal{P}(\mathcal{P}(A))$? (Note that $\mathcal{P}$ is the notation for power set.)

**Problem 3** [30 pts; (2,4,4,4,4,4,4,4,4)]: Divisibility

Consider the set of positive even integers $S = \{2, 4, \ldots, 4000\}$. 
i. What is the cardinality of this set?

ii. How many of these integers are divisible by 3?

iii. How many of these integers are divisible by 5?

iv. How many of these integers are divisible by 3 AND by 5?

v. How many of these integers are not divisible by 3 OR by 5?

vi. What is the least number of distinct integers that must be chosen from $S$ to be sure that at least one of them is divisible by 3?

vii. What is the least number of distinct integers that must be chosen from $S$ to be sure that at least one of them is divisible by 5?

viii. What is the least number of distinct integers that must be chosen from $S$ to be sure that at least one of them is divisible by 3 or 5?

**Problem 4** [18pts, 6pts each]

There are 39 3rd graders in a school, each of whom likes at least one genre of books. There are 3 popular genres among the students - fiction, fantasy, and mystery:

- 16 like Realistic Fiction;
- 25 like Fantasy;
- 18 like Mystery;
- 12 like Realistic fiction and Fantasy;
- 5 like Realistic Fiction and Mystery;
- 8 like fantasy and mystery.

Answer the following questions: (It may help to draw a Venn diagram to capture the above data.)

1. How many students like either Realistic Fiction, Mystery or both?

2. How many children like all 3 genres of books?

3. How many students like exactly 2 genres?

4. How many students like exactly 1 genre?