Written Homework 04

Due: Wed 4 Nov 2015

Instructions:

• The assignment has to be uploaded to blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them with your hand.

• To get full credit, show intermediate steps leading to your answers, throughout.

NOTE: In questions below give complete numerical answers (for example, \( \ldots = \binom{10}{2} \) is considered incomplete—you should write \( \ldots = \binom{10}{2} = 45 \). Similarly, you should write \( \ldots = 2^7 = 128 \).

Problem 1 [21 pts (5, 3, 5, 8)]: Words & Numbers

i. A palindrome is a word (or a number) which does not change when read from left or right. Disregarding the fact that some letter sequences may not have a meaning in English, how many 7 letter palindromes are there which consist of the small letters of the alphabet (the obtained words do not have to have a meaning).

ii. How many 9-digit numbers are there whose all digits are prime?

iii. Consider products

\[ a_1a_2...a_{10} \]

where \( a_i = x \) or \( a_i = y \) for some distinct real numbers \( x, y \). How many of such products simplify to \( x^4y^6 \)? For example, \( xyxyxyxyyy \) and \( xxyyyxxyyy \) are two possibilities.

iv. How many distinguishable words can be formed from the word CHIHUAHUA.

Problem 2 [21 pts (5, 8, 8)]: Restaurant
i. A restaurant received an order for delivery of 3 different appetizers, 3 different main courses and 3 different desserts. However the connection on the phone broke off and the caller did not give any additional information. In how many ways such a meal can be assembled if the restaurant offers 5 appetizers, 10 main courses, 7 desserts?

ii. Assume that before breaking off, the caller managed to convey an additional restriction that one of the main courses must be vegetarian (no restriction on the others). In how many ways could the restaurant fulfill the order if out of the 10 main courses 4 are vegetarian?

iii. Consider the same problem (i.e. part i) from the perspective of 3 friends who visit the same restaurant and would like to eat 3 different appetizers, 3 different main courses and 3 different desserts. In how many distinguishable ways can they have their meal?

Problem 3 [8 pts]: Symbols
In how many ways can you arrange the symbols

\[ \bullet \quad \blacksquare \quad \blacktriangle \quad \blacktriangledown \quad \blacklozenge \quad ? \]

in a row under the restriction that the symbol \( \bullet \) cannot appear at the beginning or at the end of the sequence.

Problem 4 [28 pts (5, 5, 8, 5)]: Probabilities

i. Suppose that you are choosing a 6-digit number at random (numbers that are < 100,000 are considered 6-digit; the first digits are assumed to be 0s). What is the probability that the chosen number consists of distinct digits? (Note. the number 12345 is represented here as 012345 and consists of 6 distinct digits.)

ii. In a lottery, 6 balls are drawn from a pool of 49 balls painted with numbers 1 through 49. What is the probability that the numbers on the chosen balls are all squares of integers?

iii. What’s the probability that all the chosen numbers in ii. are composite (since 1 is not considered prime, we will view 1 as a composite number for the purpose of this question).

iv. Consider a set \( S \) of 9x9 matrices of 0’s and 1’s in which is each row contains exactly one 1. What is the probability a randomly chosen matrix from \( S \) has the property that there is exactly one 1 in each of its columns?

v. Suppose you tossed a coin 6 times. What is the probability that you obtained at least 1 tails?

Problem 5 [22 pts (5, 5, 2, 10)]: Events & Bayes Theorem
Suppose that you are throwing a die twice. Consider the following events

\[ A = \text{the event that you get an odd number in the first throw} \]

\[ B = \text{the event that you get an odd number in the second throw} \]
i. What is the event $A \cap B$? Calculate its probability.

ii. Explain what is the event $A \cup B$ and calculate its probability.

iii. Verify that $P(A \cap B) = P(A)P(B)$ (i.e. that the events $A$ and $B$ are independent).

iv. In a certain town where all cars are either green or blue., there are 3 times more green cars than the blue ones. Studies indicate that at a particularly dimly lighted pedestrian crossing at night the probability that a person correctly identifies the color of a green car is 0.68. The probability that they correctly identify the color of a blue car is 0.45. You were just hit by a car at that crossing and believe that the car was green. What is the probability that you were actually hit by a green car?