Written Homework 03

Assigned: Wed 07 Oct 2015
Due: Wed 21 Oct 2015

Instructions:

• The assignment has to be uploaded to blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them by hand.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [18pts, 6pts each]
I once gave a 20 question True/False exam.

i. In how many ways can you answer the questions (pretend you answer each question True or False)?

ii. If you decide to leave some questions blank, in how many ways can you answer the questions?

iii. If you decide to answer one-half of the questions True and one-half False, in how many ways can you answer the questions?

Problem 2 [12pts, 6pts each]: Sets
Let A, B, and C be sets, using the laws of set theory show that:

i. $A \cap B \cap C = A \cup B \cup C$

ii. $(B - A) \cup (C - A) = (B \cup C) - A$

Problem 3 [15 pts (4,5,6)]: Sets

i. Let $A = \{ x \mid x \in \mathbb{N} \land 10 \leq x^2 + 5 \leq 41 \}$. (Here $\mathbb{N}$ includes 0.) What are the elements of $A$?
ii. Let \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) be the universe. Let \( A = \{1, 4, 6\} \) and \( B = \{1, 2, 3, 7, 9\} \) be two subsets of \( U \). What are the elements of the set \( A \cap B \)?

iii. Let \( A = \{2\} \). What are the elements of \( \mathcal{P}(\mathcal{P}(A)) \)? (Note that \( \mathcal{P} \) is the notation for power set.)

**Problem 4** [30 pts, (2,4,4,4,4,4,4,4)]: Divisibility

Consider the set of positive even integers \( S = \{2, 4, \ldots, 6000\} \).

i. What is the cardinality of this set?

ii. How many of these integers are divisible by 5?

iii. How many of these integers are divisible by 6?

iv. How many of these integers are divisible by 5 AND by 6?

v. How many of these integers are not divisible by 5 OR by 6?

vi. What is the least number of distinct integers that must be chosen from \( S \) to be sure that at least one of them is divisible by 5?

vii. What is the least number of distinct integers that must be chosen from \( S \) to be sure that at least one of them is divisible by 6?

viii. What is the least number of distinct integers that must be chosen from \( S \) to be sure that at least one of them is divisible by 5 or 6?

**Problem 5** [25 pts, (4,4,4,13)]: The Elections

On November, 2016, America goes to the polls to elect a president, 33 members of the Senate, and 435 members of the House of Representatives. Assume that for each seat up for election, we have exactly one Republican and one Democratic candidate, and there are no other candidates.

Show all your work for each of the following parts. For the first four parts, you need not complete all the calculations to yield a single number, but simplify each answer to the extent possible.

i. Taking into account all of the elections (1 President seat, 33 Senate seats, and 435 House seats), how many possible outcomes are there?

ii. The Senate has a total of 100 seats, of which 67 will be held by continuing members. Of these 67, 30 are Democrats and 37 are Republicans. One concern among Republican supporters is that the Democrats may end up with at least 60 seats, potentially ruling out filibuster in Senate debates. How many possible outcomes are there for the Democrats to end up with at least 60 seats in the Senate after the elections? Is this more than half of the possible outcomes for the Senate elections?
iii. According to the electoral college system for presidential elections, all the electoral votes allocated to a state are won by the candidate who gets the most votes in the state. How many different ways can the 50 states be distributed between two presidential candidates?

iv. A pollster surveys 100,000 voters at Northeastern University. Of the 100,000 voters, 75,000 are Democrats and the rest are Republicans, 85,000 are students (the rest are faculty/staff members), and 58000 are women. The pollster further finds that there are 18,000 Republican students, 30,000 male Democrats, 38,000 male students, and 6,000 female Republican students. What is the number of male Democrat faculty/staff members? What is the number of student Democrats? Show all your work, including any Venn diagrams that clearly specify the counts of relevant sets.

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1This is not really true, but we will assume this for our problem. Do you know which states do not follow this rule?