

Written Homework 03

Assigned: Thu 29 Oct 2009

Due: Thu 5 Nov 2009

Instructions:

- The assignment is due at the *beginning* of class on the due date specified. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.
- We expect that you will study with friends and often work out problem solutions together; however, you must write up your own solutions, in your own words. Cheating will not be tolerated.
- We expect your homework to be neat, organized, and legible. If your handwriting is unreadable, please type your solutions. Use 8.5in by 11in loose-leaf or printer paper, and please do not hand in sheets that have been ripped from spiral bound notebooks.

Problem 1 [28 pts, (4,4,4,4,6,6)]: Picture Arrangements

The College of Computer and Information Science has 30 faculty members: 13 Full Professors, 11 Associate Professors, and 6 Assistant Professors. Four of the professors are women and 26 are men. Among the professors are 1 Dean (full professor, male) and 2 Associate Deans (full professors, one male and one female). Photos of the faculty have been taken, and they will be displayed in a row along the entry hallway.

- In how many ways can the photos be displayed so that all of the women are together?
- In how many ways can the photos be displayed so that all of the women are together and all of the men are together?
- In how many ways can the photos be displayed so that all of the women are together or all of the men are together (or both)?
- In how many ways can the photos be displayed with the Dean appearing first, followed by the Associate Deans, the (remaining) Full Professors, the Associate Professors, and finally the Assistant Professors, in that order?
- In how many ways can the photos be displayed if the Dean appears first, and the remaining photos are displayed in such a way that the Associate Deans are both together, the (remaining) Full Professors are all together, the Associate Professors are all together, and the Assistant Professors are all together?
- In how many ways can the photos be displayed so that no two women are adjacent?

Hint: Consider arranging the men's photos first and then placing the women's photos among the men's photos so as to ensure that no two women are adjacent.

Show your work.

Problem 2 [16 pts, (3,5,8)]: **Exams**

Consider an exam consisting of 16 problems divided into two groups of 8 problems each. Students are required to solve 10 of the 16 problems.

- i. In how many ways can a student choose 10 of the 16 problems?
- ii. Suppose that students are required to solve 5 problems from the first group and 5 problems from the second group. In how many ways can a student choose the 10 problems to solve?
- iii. Now suppose that students are allowed to solve any 10 problems out of the 16 but no more than 6 from either group. In how many ways can a student choose the 10 problems to solve?

Show your work.

Problem 3 [12 pts, (6 each)]: **Surveys**

A marketing firm has been hired to conduct a survey of automobile purchases in the greater Boston area. The survey consists of recording, for each household surveyed, the number of cars presently owned by that household among the following six major manufacturers: GM, Ford, Chrysler, Toyota, Honda, and Nissan. Thus, one household may have two GMs and a Ford, another household may have one Honda and one Nissan, etc.

- i. Suppose that no household surveyed has more than four cars. In how many different ways can the survey sheet be filled out?
- ii. There are approximately 1,200,000 households in the greater Boston area. Suppose that all households are surveyed, the resulting survey sheets are organized and tallied by type (e.g., two GMs and a Ford vs. one Honda and one Nissan, etc.), and the survey results are sorted by popularity (e.g., two GMs and a Ford is the most common, one Honda and one Nissan is the second most common, etc.). At a minimum, how large must be the count associated with the most popular survey result?

Show your work.

Problem 4 [18 pts, (3,3,6,6)]: **Pseudorandom Number Generators**

Pseudorandom number generators are programs that generate sequences of 0s and 1s that “look” random. These sequences have a wide range of applications, including in cryptography, cellular networks, and simulations of large physical systems. Many pseudorandom generators used in practice generate sequences with some specific properties, as discussed below.

Consider a pseudorandom number generator that generates 24-bit sequences; i.e., each sequence is of length 24 and consists of 0s and 1s. For each of the following questions, complete all the calculations and show all your work.

- i. What is the total number of different 24-bit sequences?
- ii. One criterion that a sequence generator may desire is to have an equal number of 1s and 0s. What number of 24-bit sequences contain an equal number of 0s and 1s?

- iii. Another commonly-used criterion is limiting the length of *runs*. A run is a maximal contiguous sequence of identical bits. That is, a run of 0s is a contiguous sequence of 0s, which is preceded by a 1 or is at the start of the sequence, and succeeded by a 1 or at the end of the sequence. Similarly, a run of 1s is a contiguous sequence of 1s, which is preceded by a 0 or is at the start of the sequence, and succeeded by a 0 or is at the end of the sequence. For example, the sequence 001111100001110000111 has a run of 0s of length 2, two runs of 0s of length 4, one run of 1s of length 5, and two runs of 1s of length 3.

How many 24-bit sequences contain a run of 0s of length 12 or a run of 1s of length 12 (or both)?

- iv. How many 24-bit sequences contain at least one run of 1s of length 11 ?

Show your work.

Problem 5 [26 pts, (6,3,6,3,4,4)]: **The World Series**

The New York Yankees are playing the Philadelphia Phillies in a seven game World Series that ends when one team has won four games. Suppose we record the outcome of a game with a **W** for a Yankees win and an **L** for a Yankees loss. Then, the outcome of a series is a sequence that ends as soon as we get exactly four **Ws** or exactly four **Ls**. Thus, for example, **WWWWW**, **WLWLWLW**, **WWLLLL**, and **LLLWL** are all possible outcomes of the series, while **LLWLLL** and **WWWLWLW** are not possible outcomes of the series.

For each of these parts, show all your work.

- i. How many possible outcomes of the series are there?
- ii. How many series would have to be played to be sure that some series outcome happens twice?
- iii. The first two and the last two games of the series are scheduled to be held in New York while the middle three are scheduled to be held in Philadelphia. How many possible outcomes are there in which New York wins the series yet loses at least two of its home games? (New York has already lost Game 1 at home.)

The World Series uses a 2-3-2 home-away-home format, as described above. Let's denote this format HHAAAHH. Another popular playoff format is 2-2-1-1-1, i.e., HHAAHAH.

- iv. How many formats are there consisting of four home games and three away games?
- v. How many formats are there consisting of four home games and three away games where the series format must start and end with a home game? (This is a common constraint.)
- vi. How many formats are there consisting of four home games and three away games where the four home games cannot be consecutive? (This is another common constraint.)